Soliton shadows in birefringent optical fibers


Department of Electrical Engineering, University of Maryland, Baltimore, Maryland 21228

Received May 18, 1992

The shadows generated by the interaction of a pair of solitons in a birefringent optical fiber are studied numerically. The amplitudes, widths, and shapes of the shadows are calculated.

Soliton-dragging logic gates have been proposed to do all-optical, ultrafast switching in optical fibers. In these gates, a partial collision can take place between a control pulse that is in one polarization eigenstate of a linearly birefringent optical fiber and a signal pulse that is in the other polarization eigenstate. When a collision takes place, shifts in the central frequencies and hence the group velocities of both pulses result. As a consequence, the control pulse is shifted out of the time slot that it would have occupied had there been no collision. The control pulse has to be a soliton in order to make the gate cascadable. When solitons that are initially in opposite polarization eigenstates collide, each emerges from the collision in a mixed polarization state. The largest contribution to the polarization state of each soliton comes from the initial polarization eigenstate of that soliton. The contribution from the other polarization eigenstate is referred to as a shadow because it is typically small, less than 10% of the integrated intensity when the solitons are far away from the trapping limit, and because in plots of the intensity in each polarization eigenstate it has the appearance of a shadow following the main contribution. Radiation is also generated in these collisions since the system of equations that describes linearly birefringent fibers is not integrable. However, under the circumstances present in switching devices, in which collisions take place over distances comparable to a soliton period, the amplitudes of the shadows can be substantially larger than the radiation amplitude. Hence the shadows can play a significant role. In this Letter we report on shadow formation owing to the collision of a pair of identical solitons in linearly birefringent optical fibers.

The equation governing the pulse evolution in a birefringent optical fiber, written in soliton units, is given by the coupled nonlinear Schrödinger equation,

\[ i \frac{\partial u}{\partial \xi} + i \delta \frac{\partial u}{\partial s} + \frac{1}{2} \frac{\partial^2 u}{\partial s^2} + (|u|^2 + B|v|^2)u = 0, \]

\[ i \frac{\partial v}{\partial \xi} - i \delta \frac{\partial v}{\partial s} + \frac{1}{2} \frac{\partial^2 v}{\partial s^2} + (|u|^2 + |v|^2)v = 0, \]

where \( s \) is the normalized local time, \( \xi \) is the normalized distance along the fiber, \( u \) and \( v \) are complex envelopes of the two polarization eigenmodes, \( \delta \) is the normalized birefringence, and \( B \) is the cross-phase modulation coefficient. In a linearly birefringent fiber, one finds that \( B = 2/3, \) and this is the value that we use. The initial pulses are assumed to be two identical solitons, i.e., \( u(\xi = 0, s) = A_0 \text{sech}[A_0(s + s_0)] \) and \( v(\xi = 0, s) = A_0 \text{sech}[A_0(s - s_0)], \) where \( A_0 \) is the initial amplitude and \( 2s_0 \) is the initial separation. Without loss of generality, we may also assume that \( \delta = 1 \) because the shadows for other values of \( \delta \) can be obtained by using the following scaling transformation of Eq. (1): \( u = \alpha u, v = \alpha v, s' = s/\alpha, \xi' = \xi/\alpha^2, \delta' = \delta \alpha, \) and \( B' = B, \) where \( \alpha \) is a scaling parameter.

A soliton that is initially in \( u \) polarization is shown in Fig. 1(a). The \( u \)-polarization intensity profile is shown as a solid curve and the shadow, the \( v \)-polarization intensity profile, is shown as a dashed curve. After 50 soliton periods, the radiation that is generated in the collision has left the soliton and its shadow. Fig. 1(b), the amplitude of the shadow is compared with the following analytical form,

\[ \phi = A_{\text{shadow}} \left[ \text{sech}[A_{\text{soliton}}(s - s_p)] \right]^{(1 + \sqrt{4B - 1})/2}, \]

which is obtained by linearizing Eqs. (1). The time delay of the shadow \( s_p \), the amplitude of the shadow \( A_{\text{shadow}}, \) and the amplitude of the soliton \( A_{\text{soliton}} \) in Eq. (2) are directly obtained from the numerical simulations. The comparison shows that Eq. (2) is a good approximation to the shape of the shadow. The average relative error, \( \langle \int ds |\phi - \phi|^2 / \int ds |\phi|^2 \rangle^{1/2} \), is less than 3.4% in this example. More generally, Eq. (2) approximates well the shape of the shadow except when the two solitons are near the trapping limit. We note that the shadow and the soliton do not have the same width. The FWHM of the shadow is approximately 1.2 times that of the soliton.

The shadow amplitudes for different initial separations and \( A_0 = 1 \) are shown in Fig. 2. Positive values of \( s_0 \) mean that the two solitons pass through each other, while negative values of \( s_0 \) mean that the solitons do not pass through each other. For large negative values of \( s_0 \), the solitons do not interact, and hence no shadows are formed. The shadow amplitude increases when \( s_0 \) increases and reaches its maximum at \( 2s_0 = 0.254\tau, \) where \( \tau \) is the FWHM of the initial soliton. For \( 2s_0 > 0.254\tau, \) the shadow
Fig. 1. (a) Soliton and its shadow and (b) the shadow and its approximation shown at a distance of 50 soliton periods with $A_0 = 1$ and $s_0 = 0$. The $u$ polarization is shown as a solid curve, the $v$ polarization is shown as a dashed curve, and the approximation is shown as a dashed-dotted curve.

Fig. 2. Amplitudes of the shadows versus initial separation with $A_0 = 1$. The points correspond to the curves in Fig. 4.

amplitude decreases until it reaches 0.0169 for large positive values of $2s_0$, which correspond to a complete collision. The analytical theories developed to date are not consistent with Fig. 2, which indicates a need for further theoretical development.

For other values of $A_0 \leq 1.387$, similar curves are obtained. We find that the amplitude of the shadow for a complete collision is approximately 10–15% of that for a half-collision that occurs when $s_0 = 0$. However, when $A_0 \geq 1.387$, only part of the curve exists because the soliton self-trap for some initial separations but not for others. The larger the value of $A_0$, the larger the range of $s_0$ values for which bound states form. Conversely, at each value of $s_0$, there is some value of $A_0$, called the self-trapping limit, beyond which the solitons always self-trap because the cross-phase modulation between the solitons is strong enough to shift their central frequenies and compensate for the initial velocity difference. Likewise, there is some value of $A_0$, called the separation limit, below which the solitons always separate. These two limits coincide when $s_0 < 0$ and tend toward infinity as $s_0 \to -\infty$, as shown in Fig. 3. However, these two limits are different when $s_0 > 0$. Between the two limits is a complicated transition region. As $s_0 \to \infty$, the self-trapping limit saturates at 3.6, while the separation limit saturates at 3.2. In the transition region, the collisions are complex and extremely sensitive to $A_0$. A small change in the value of $A_0$ can cause a large change in the shadow amplitude. The solitons may pass through each other, reflect back, or even self-trap after a few oscillations of their positions. The dependence of the shadow amplitudes on $A_0$ for different values of $s_0$ is plotted in Fig. 4. The solid curves are for negative values of $s_0$, and the dashed curves are for positive values of $s_0$. The transition regions are shown as horizontal line segments. The largest value of $A_0$ at each value of $s_0$ is the self-trapping limit.

In summary, the amplitudes, widths, and shapes of shadows have been studied numerically. The shapes of the shadows can be described by a simple

Fig. 3. Asymptotic behavior of colliding solitons as functions of $A_0$ and $s_0$. There is a region in which solitons always self-trap, a region in which solitons always pass through each other, and a transition region.

Fig. 4. Amplitudes of the shadows versus the initial amplitudes of the solitons. The solid curves are for $s_0 \leq 0$, and the dashed curves are for $s_0 > 0$. The flat part of the curves indicates the transition regions, beyond which bound states form. The curves correspond to the points in Fig. 2.
analytic formula. The FWHM of the shadow is approximately 1.2 times that of the soliton. The amplitudes of the shadows for solitons with different initial amplitudes and separations are calculated numerically over a wide range of initial soliton separations and amplitudes. There is no analytical theory to date that successfully predicts the shadow amplitudes.

This research was supported by Department of Energy grant DE-FG05-89ER-14090 and National Science Foundation grant ECS-9113382. Computational work was carried out at the San Diego Supercomputer Center.

P. K. A. Wai is also with the Laboratory of Plasma Research, University of Maryland, College Park, Maryland 20742.

References