Effects of randomly varying birefringence on soliton interactions in optical fibers

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The effects of randomly varying birefringence on soliton interactions in optical fibers are studied. It is shown that for initial separations of less than 10 pulse widths, the phase-dependent short-range interaction dominates. For separations larger than 10 pulse widths, the soliton interacts through the dispersive radiation that they generate. This interaction is too weak to explain the phase-independent long-range soliton interaction observed experimentally.

Increasing attention is being paid to the effects of birefringence in single-mode optical fibers. In the case of constant birefringence, it has been shown that pulse walk-off can be avoided by making use of the Kerr nonlinearity. However, in real communication fiber, the axes of birefringence tend to shift randomly, scattering light from one polarization to another in the process. This polarization dispersion has potentially detrimental effects in a long-distance optical communication system. In a previous paper we studied the stability of solitons in a fiber with randomly oriented birefringence. It was observed that if the random variation length is much shorter than the soliton period, the soliton does not split even at high values of average birefringence. Nevertheless, the polarization dispersion tends to produce dispersive wave radiation from the soliton. This radiation can lead to interaction between copropagating solitons, thus limiting the bit rate of an optical communication system.

In this Letter we study the effect of randomly varying birefringence with randomly varying orientation on the interaction of solitons. The fiber is assumed to undergo periodically a sudden, random rotation $\theta$. This angle is uniformly spaced over the interval $[0, 2\pi]$. Between each rotation the fiber is assumed to maintain a constant linear birefringence, so that the state of polarization at each point in time is steady and rapidly changing. The state of polarization is characterized by the angle $\theta$ and the complex field amplitudes in the two polarizations. We will assume that when $\theta$ changes, a random phase factor $\phi$ is added to the phase difference between the pulse envelopes $U$ and $V$, which correspond to $E_1$ and $E_2$. We do so because in real fibers the state of polarization changes with great rapidity, and slight variations in the birefringence over the sections that are being treated as constant will lead to complete randomization of the state of polarization. The overall change in the evolution equation is thus given by the transformation

$$\Psi' = \Psi \Psi,$$  

where

$$\Psi = \left[ \begin{array}{cc} \cos \theta & \sin \theta e^{i\phi} \\ -\sin \theta e^{-i\phi} & \cos \theta \end{array} \right]$$

and $\Psi = (U,V)^T$ is the polarization-state envelope vector. The matrix $\Psi(\theta, \phi)$ in Eq. (2) is unitary, and its determinant equals 1. It is a member of the mathematical group SU(2), and its action is equivalent to an arbitrary spatial rotation on a sphere. With the Poincaré sphere representation used for the polarization-state vector, Eq. (2) is a rotation of the corresponding Stokes vector. From another point of view, the effect of the sudden change in the birefringence axes can be viewed as rotation of a Poincaré sphere on which the current state of polarization is the North Pole. If the angles $\theta$ and $\phi$ are uniformly distributed random variables, the representative point on the Poincaré sphere is also uniformly distributed. If Eq. (2) is applied to the coupled nonlinear Schrödinger equation with linear birefringence and the premise is dropped, the resulting equation of motion is given by

$$i\frac{\partial \Psi}{\partial z} + i\sigma \frac{\partial \Psi}{\partial t} + \frac{1}{2} \frac{\partial^2 \Psi}{\partial t^2} + \frac{5}{6} |\Psi|^2 \Psi$$

$$+ \frac{1}{6} (\Psi^* \sigma \Psi) \sigma \Psi = 0,$$

where $\Psi' = (U^*V^*)$, and the matrix $\sigma$ is given by

$$\sigma = \left[ \begin{array}{cc} \cos 2\theta & -\sin 2\theta e^{i\phi} \\ -\sin 2\theta e^{-i\phi} & -\cos 2\theta \end{array} \right].$$

In the following analysis, the angles $\theta$ and $\phi$ are assumed to vary randomly at intervals of $z_0$, which is typically of length 100 m or so. For the 50-ps solitons that would be used in a long-distance communication system, the nonlinear length scale $z_0$ is of the order of thousands of kilometers. Thus the Poincaré sphere would rotate many times in the course of a soliton period. The rotations are so fast that the fiber appears to be isotropic to the lowest order of approximation in a perturbation series.
That is, pulses launched with different polarization states evolve identically. The equation describing the averaged evolution of the pulse can be obtained by averaging Eq. (3) on the Poincaré sphere. Using the pulse amplitudes $U$ and $V$ explicitly instead of $\Psi$, we have

$$
\frac{i}{\varepsilon} \frac{\partial U}{\partial z} + \frac{1}{2} \frac{\partial^2 U}{\partial t^2} + \frac{8}{9} (|U|^2 + |V|^2) U = 0,
$$

$$
\frac{i}{\varepsilon} \frac{\partial V}{\partial z} + \frac{1}{2} \frac{\partial^2 V}{\partial t^2} + \frac{8}{9} (|U|^2 + |V|^2) V = 0. \tag{5}
$$

From Eq. (5), the effect of the rapid random rotations is to strengthen the cross-phase modulation while weakening the self-phase modulation such that they are equal in magnitude, rather than in a $2/3$ ratio as in the case of constant linear birefringence. Manakov\(^5\) has shown that Eq. (5) can be solved by the spectral transform method and that it possesses soliton solutions. If the pulses are initially in the same polarization as a function of time, i.e., $U(z = 0, t) = q(t)\cos \chi$ and $V(z = 0, t) = q(t)\sin \chi \exp(i\alpha)$, where $\chi$ and $\alpha$ are constants, then Eq. (5) can be further reduced to the nonlinear Schrödinger equation. Therefore the soliton interaction in a fiber with randomly oriented birefringence is described by the nonlinear Schrödinger equation to the lowest order in a perturbation expansion.\(^3\) Physically, this result implies that the pulses remain in a single state of polarization as a function of time, although this state rapidly changes as the pulses propagate along the fiber. At the next highest order in a perturbation expansion, the state of polarization is no longer the same at every point in time, and depolarization results.\(^6\)

We study the effect of this random variation on a 2-soliton or breather of Eq. (5). A 2-soliton can be obtained by injecting a hyperbolic-secant pulse with twice the amplitude of a fundamental soliton. It can be viewed as consisting of two 1-solitons traveling at the same velocity with relative amplitudes 1 and 3, respectively. Interaction of the two 1-solitons leads to the periodic behavior of the 2-soliton. The two 1-solitons are not actually bound together because the binding energy is zero; i.e., the sum of intensities of the two 1-solitons is equal to that of the 2-soliton. As a result, a 2-soliton is unstable to most perturbations. We integrate Eq. (3) using $[U(0, t) = 2\sqrt{9/8\text{sech}} \, t, \ V(0, t) = 0]$ as the initial condition over 20 soliton periods for five different values of $\delta$ ranging from 1.25 to 7.5. For each $\delta$ value, five different sets of random numbers are used. The parameters $z_0$ and the soliton period are taken to be 100 m and 55 km, respectively. The results are found to be qualitatively the same. In all cases studied, the 2-soliton breaks into its constituent solitons after a few oscillations for $\delta < 4$ and only one oscillation for $\delta > 4$. It is observed that larger amounts of power are transferred from the $U$ polarization to the $V$ polarization when compared with that of a 1-soliton; thus more depolarization results.

Next, the evolution of a pair of 1-solitons initially $T_0$ apart is studied numerically for distances of as

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**Fig. 1.** Normalized power in the plane of polarization of the input pulse for a 1-soliton (circles) and a 2-soliton (crosses) as a function of the distance traveled. The birefringence axis rotates every $z_0/500, \, \delta = 2.5$.

**Fig. 2.** Change in pulse separation for a pair of solitons as a function of the distance traveled for both constructive (circles) and destructive (crosses) interference. The solitons oscillate once and then separate in the former case and repel each other in the latter case.

**Fig. 3.** Change in pulse separation ($T = T_0/\tau$) (crosses) as a function of distance traveled for $\delta = 5$. The change in time delay of individual solitons is also plotted (circles and dotted curve).
much as $40z_0$. The pulse separation $T_0$ ranges from $2\tau$ to $70\tau$, where $\tau$ is the initial pulse width of the soliton. For $T_0 \leq 10\tau$, the phase-dependent short-range interaction dominates. If the solitons are in phase, they initially attract each other and then separate after one oscillation owing to the randomly varying birefringence. If the solitons are out of phase, they repel each other. In Fig. 2, the pulse separation for a pair of solitons is plotted versus the distance traveled for both constructive and destructive interference. The solitons are initially $4\tau$ apart and $\delta = 5$. This short-range interaction has been shown to fall off exponentially with pulse separation.\(^6\)

For $T_0 \geq 10\tau$, the solitons interact through the dispersive wave components they generate. In Fig. 3 the change in pulse separation is plotted for 40 soliton periods with $\delta = 5$. Although the time delay for each individual soliton jitters,\(^3\) their separation remains unaffected until $20z_0$, which is the distance necessary for the dispersive waves emitted by one soliton to reach the other. In all cases studied with $T_0 > 10\tau$, the change in pulse separation $\Delta T$ is less than $0.3\tau$ even at $\delta$ values as high as 5. In Fig. 4 the change in pulse separation is plotted as a function of $T_0$ for a fiber length of $40z_0$. The variation does not display any periodic behavior. Therefore, although the randomly varying birefringence causes a well-separated pair of solitons to interact, its variation and magnitude do not account for the long-range interaction observed experimentally.\(^8\) Recently Dianov et al.\(^9\) have shown that the soliton long-range interaction could be the result of an electrostrictive excitation of acoustic oscillations by a periodic sequence of solitons.

In summary, we have studied numerically the effects of a randomly oriented birefringence on soliton interaction. A breather or 2-soliton breaks into its constituent solitons after a few oscillations. Single solitons initially less than $10\tau$ apart are dominated by the phase-dependent short-range interaction. The random variation causes solitons with the same initial phase to separate. For $T_0 > 10\tau$, the solitons interact through the dispersive waves that they generate, but the resulting variation in pulse separation cannot account for the long-range interaction between solitons that has been observed experimentally.

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References