Soliton switch using birefringent optical fibers

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Received October 16, 1989; accepted February 18, 1990

The use of solitons is proposed for interferometric switching in birefringent optical fibers. Pulse propagation in the fiber is modeled by the coupled nonlinear Schrödinger equation, in which the ratio between the self-coupling and cross-coupling terms depends on the ellipticity of the fiber eigenmodes. It is shown that shadows occur unless the self-coupling and the cross-coupling are equal and that these shadows seriously affect the contrast ratio attainable by a switch. These two couplings become equal at an ellipticity angle θ = 35°, and, within a tolerance of θ = ±5°, a contrast ratio of 10 or more can be achieved.

Optical solitons, nonlinear pulses that can propagate in single-mode optical fibers without dispersion, were first predicted theoretically by Hasegawa and Tappert1 and observed experimentally by Mollenauer et al.2 Since the research of Mollenauer et al., these solitons have been the continual focus of both theoretical and experimental activity. Most research has been aimed at application to long-distance communications and short-pulse production, but recently interest has grown in using these solitons for switching. Solitons are of interest because they tend to act as a unit—switching nearly entirely or not at all from one channel to another.

Configurations proposed include loop mirrors,3 directional couplers,4 and interferometers.5 Here we consider a Mach–Zehnder interferometer configuration.6 A signal pulse of one polarization is divided into two solitons, with each soliton going down one arm of the interferometer. In one of the arms a switching pulse may be introduced at the other polarization. This switching pulse then shifts the phase of the soliton in that arm so that, when the solitons in both arms recombine, they interfere destructively rather than constructively. In the optical fiber implementation,7 the two arms of the interferometer are temporally, rather than spatially, separated in order to avoid the effect of different parameter fluctuations in the two arms. One of the solitons is temporally delayed, the two solitons go through the same fiber, and then the delayed soliton is advanced before the two solitons are recombined. For one of them to be advanced and delayed conveniently without affecting the other, the solitons should have orthogonal polarizations. The switching pulse must have the polarization orthogonal to that of the soliton whose phase it is altering.

Menyuk6,7 has shown that birefringent, single-mode fibers can be modeled by the coupled nonlinear Schrödinger equation in the form

\[
\begin{align*}
&i \frac{\partial u}{\partial \xi} + i \delta \frac{\partial u}{\partial s} + \frac{1}{2} \frac{\partial^2 u}{\partial s^2} + (|u|^2 + B|v|^2)u = 0, \\
&i \frac{\partial v}{\partial \xi} - i \delta \frac{\partial v}{\partial s} + \frac{1}{2} \frac{\partial^2 v}{\partial s^2} + (B|u|^2 + |v|^2)v = 0,
\end{align*}
\]

(1)

where \( u \) and \( v \) are normalized complex envelopes of the two polarization eigenmodes, \( \beta \) is the normalized strength of the linear birefringence, \( \xi \) is the normalized distance, and \( s \) is the normalized time. The normalizations are standard.6 The cross-coupling coefficient \( B \) depends on the ellipticity angle \( \theta \) of the eigenmodes: it equals 2/3 for a linearly birefringent fiber and equals 2 for a circularly birefringent fiber. More generally, one finds that

\[
B = \frac{2a + 2b \sin^2 \theta}{2a + b \cos^2 \theta},
\]

(2)

where \( a \) and \( b \) are material coefficients. In silica fibers, \( a = b \), which is used in following discussion.

If either \( u \) or \( v \) vanishes, Eqs. (1) reduce to the nonlinear Schrödinger equation, whose solution is well known. Additionally, when \( B = 1 \), this coupled equation becomes Manakov’s equation8 and belongs to the special set of equations, including the nonlinear Schrödinger equation, that are integrable by using nonlinear spectral transform methods.9 For \( B \neq 1 \), Eqs. (1) are not integrable, and their solutions must be determined numerically.

To investigate the switching operation, we consider the following initial conditions:

\[
\begin{align*}
u &= A_1 \text{sech}[A_1(s - s_1 - \delta \xi)] \exp \left[ i \left( \frac{A_1^2}{2} - \xi + \phi_1 \right) \right], \\
v &= A_2 \text{sech}[A_2(s - s_2 + \delta \xi)] \exp \left[ i \left( \frac{A_2^2}{2} - \xi + \phi_2 \right) \right],
\end{align*}
\]

(3)

where \( A_1 \) and \( A_2 \) are amplitudes and \( \phi_1, \phi_2 \) and \( s_1, s_2 \) are the phase and time offsets, respectively. The difference \( |s_1 - s_2| \) is assumed to be large compared with the widths of the solitons. Thus in both polarizations we begin with a soliton. One may identify \( u \) as the signal pulse and \( v \) as the switching pulse. In principle, the switching pulse does not have to consist only of solitons. However, we have found that use of other switching pulses leads to a complicated pulse evolution and, hence, to a sensitive dependence of the switch’s contrast ratio on the details of the pulse and fiber parameters.

Figure 1 shows the simulated results of the interac-
Fig. 1. Signal pulse during soliton interaction for $B = 2/3$ (a) before interaction and (b) after interaction. The $u$ polarization is shown as a solid curve, and the $v$ polarization is shown as a dashed curve.

The interaction of solitons in a linearly birefringent fiber ($B = 2/3$) when $A_1 = 1$ and $A_2 = 2$. Before collision, the soliton consists of a single polarization [Fig. 1(a)]. After collision, the soliton picks up a shadow, a nonzero component in the orthogonal polarization [Fig. 1(b)]. Shadows are detrimental to our proposed switch. They affect the phase of the signal pulse, leading to a steadily changing phase shift, and thus degrade the switch's performance. To show this effect clearly, we define $s_{\text{max}}(s)$ to be the $s$ value where $u$ reaches its maximum as $\xi$ evolves and $\phi_0(\xi, s) = \int_s^{s_{\text{max}}(s)} u(\xi, s') ds' / \int_s^{s_{\text{max}}(s)} u(\xi, s) ds$, where $\phi_0(\xi, s)$ is the phase of $u(\xi, s)$. The time shift and the phase shift are shown in Fig. 2 by plotting $s_{\text{max}}(s) - s_{\text{max}}(s)$ and $\phi_0(s) - \phi_0(0) = A_1^2 \xi^2 - \phi_0$ versus $\xi$. The shifts change abruptly during interaction and then keep changing linearly with $\xi$ after the formation of a shadow.

When $B = 1$, corresponding to an ellipticity angle $\theta \approx 35^\circ$, the additional time shift and phase shift due to the interaction can be expressed in terms of $s_0$, $s_{\text{max}}$, and $\delta$ (Refs. 6, 8):

$$\Delta s = \frac{1}{2A_1} \ln \left[ \frac{4\delta^2 + (A_1 + A_2)^2}{4\delta^2 + (A_1 - A_2)^2} \right],$$

$$\Delta \phi = \arg \left[ \frac{2\delta + i(A_1 + A_2)}{2\delta + i(A_1 - A_2)} \right].$$

In particular, no shadows form.

Figure 3 shows the results of a simulation with the same parameters as those in Fig. 2 except that $B = 1$. The phase and time shifts change only during the interaction and are constant afterward. The numerical results are in good agreement with Eqs. (4) and (5).

From Eq. (5), $|\Delta \phi| < \pi$. Therefore, it is necessary to use two solitons in the switching pulse in order to obtain a phase shift of $\pi$. In addition, from Eq. (4), $\Delta s > 0$; hence an ideal switching operation is not possible.

Consider now a signal soliton of amplitude $\alpha$ that interacts with two consecutive switching solitons of amplitudes $\alpha_1$ and $\alpha_2$. These switching solitons are separated enough so that the time shifts and phase shifts of the signal soliton are additive; they are denoted $\Delta s_{1,2}$ and $\Delta \phi_{1,2}$, respectively. The amplitudes $\alpha_1$ and $\alpha_2$ are chosen to satisfy $\Delta \phi_1 + \Delta \phi_2 = \pi$. Then it can be shown that $\Delta s = \Delta s_1 + \Delta s_2$ satisfies

$$\Delta s = \frac{1}{\alpha} \ln \frac{\alpha_1 + \alpha_2 + 2a}{\alpha_1 + \alpha_2 - 2a} \leq \frac{2}{\alpha} \ln \alpha + \frac{(\alpha^2 + 4\delta^2)^{1/2}}{\alpha} = \frac{2}{\alpha} \ln \alpha + \frac{(\alpha^2 + 4\delta^2)^{1/2}}{\alpha},$$

where $4\delta^2 + \alpha^2 = \alpha_1 \alpha_2$. The time shift approaches zero if either $\alpha_1$ or $\alpha_2$ becomes large. The right-hand side of this expression is $\Delta s_{\text{max}}$. The equality holds only when $\alpha_1 = \alpha_2 = (4\delta^2 + \alpha^2)^{1/2}$.

After interaction with the switching pulses, the signal pulse can be written as

$$u = \alpha \sech(\alpha[s - (s_0 + \Delta s) - \xi \delta]) \times \exp \left[ i \left( \frac{\alpha^2}{2} \xi + \pi + \epsilon \right) \right].$$

where $s_0$ is the initial offset, while $\Delta s$ and $\pi + \epsilon$ are the time shift and phase shift produced by the switching operation. The parameter $\epsilon$ is assumed to be uniform in $s$. This assumption is supported by simulation results for the ellipticity angle that we have considered. The output power at the end of the interferometer where pulses recombine is

Fig. 2. Phase and time shifts of the signal soliton for $B = 2/3$.

Fig. 3. Phase and time shifts of the signal soliton for $B = 1$. 
The contrast ratio, i.e., the ratio of the output power of a constructive interference to that of a destructive one, is given by

$$r = \frac{2}{\left[1 - \frac{\Delta s \cos \epsilon}{\sinh(\Delta s)}\right]}$$  \hspace{1cm} (9)$$

Two factors affect the performance of the switch, namely, the deviation of the signal pulses from solitons and the deviation of the ellipticity angle from 35°. From previous numerical research,\(^\text{10}\) we know that the former factor will lead to a slight, slowly spreading pedestal on the solitons and will not otherwise affect the outcome. The latter factor is studied numerically by considering as an example the parameters \(\alpha = 1, \delta = 1\), and \(\alpha_1 = \alpha_2 = \sqrt{2}\) to make a \(\pi\) phase shift. This represents the worst-case scenario for the contrast ratio at \(B = 1\). From Fig. 4 the contrast ratio for \(\theta \neq 35°\) is lower than that for \(\theta = 35°\) owing to the formation of shadows. From Eqs. (4), (5), and (9), \(r = 14.4\) when \(\theta = 35°\). For \(\theta \neq 35°\), we find that Eq. (9) predicts the contrast ratios well (within 1%) when the simulated phase and time shifts are used. Assuming that \(\theta = 35° \pm 5°\) and requiring that \(r \geq 10\), we find that the tolerance length of the switch is greater than 30 soliton periods.

In this Letter we have shown the feasibility of interferometric switching using birefringent fibers of an ellipticity angle near 35°. Ideal switching, i.e., complete destructive interference, is not possible because the signal pulse suffers a time shift as well as a phase shift owing to collision with the switching pulse. An equation for the contrast ratio in terms of time and phase shifts of solitons is derived for \(\theta = 35°\) and found to agree with numerical results if simulated time and phase shifts are used for \(\theta = 35° \pm 5°\).

This research was supported by the U.S. Department of Energy. Computational work was carried out at the San Diego Supercomputer Center.


References