RCS SCATTERING ANALYSIS USING THE THREE-DIMENSIONAL MRTD SCHEME

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Abstract—A three-dimensional electromagnetic scattering model based on the multiresolution time-domain (MRTD) scheme is presented, in which we apply an anisotropic perfectly matched layer absorber for open boundary truncation to the MRTD scattering analysis. With use of an initial-condition excitation technique based on a pair of one-dimensional MRTD update equations, we develop an MRTD near-to-far-zone field transform to derive the scattered fields. We also adopt the pure scattered field formulation in order to obtain effective incident and scattered fields. With applications of the MRTD scheme, we construct the surface equivalent currents in the near field region and further derive the radar cross section (RCS) in the far-zone region for different scattering targets including perfectly electric conductors, lossless, and lossy dielectric targets. We show that the results derived from the MRTD scheme are in good agreement with those of the finite-difference time-domain (FDTD) method as well as the method of moments (MoM), while the MRTD scheme requires much less computer memory and CPU time for the same level of accuracy.
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1. INTRODUCTION

The finite difference time domain (FDTD) method has been used extensively to calculate electromagnetic scattering from both dielectrics and perfect conductor targets [1-3]. It typically requires large amount of computer resource and computational time, because the FDTD method requires a relatively large number of discretization nodes (usually 10-20 nodes per minimum wavelength for a problem of moderate size) to obtain reasonable good accuracy [1-3]. To improve the efficiency of the FDTD techniques, the multiresolution time domain (MRTD) scheme, a high-order expansion technique, has recently been developed to analyze practical electromagnetic problems [4]. A two-dimensional (2D) MRTD scheme has been successfully developed for 2D target scattering analyses [5].

In this paper, we further develop and extend to a three-dimensional (3D) MRTD scattering scheme, and apply the scheme for the scattering analysis of various perfectly electric conducting and dielectric scattering targets. In order to overcome a difficulty in constructing incident and scattered fields, we adopt the pure scattered field formulation [6] so that the incident fields appear only inside the scattering targets. Starting from a set of the Maxwell’s governing equations, we derive the generalized MRTD field update equations involving both the perfectly electric conducting and dielectric scattering targets, and develop an MRTD near-to-far-zone
field transformation for the calculation of the radar cross section (RCS).

2. GOVERNING EQUATIONS OF MRTD

Similar to the technique employed in the FDTD method [6], we adopt the pure scattered field formulation in the MRTD method. Inside the target region, there are scattered and incident fields while outside the target there is only scattered field. Inside the target region the governing equations are given as

\[ j \omega E_{z}^{\text{scat}} = -\sigma E_{z}^{\text{scat}} - \sigma E_{x}^{\text{inc}} - j\omega(\varepsilon - \varepsilon_{0})E_{x}^{\text{inc}} + \nabla \times \bar{H}_{x}^{\text{scat}} \]  
\[ j \mu \mu H_{z}^{\text{scat}} = -\sigma H_{z}^{\text{scat}} - \sigma H_{x}^{\text{inc}} - j\omega(\mu - \mu_{0})H_{x}^{\text{inc}} - \nabla \times \bar{E}_{x}^{\text{scat}} \]  

where \((E_{x}^{\text{inc}}, H_{x}^{\text{inc}})\) and \((E_{x}^{\text{scat}}, H_{x}^{\text{scat}})\) are the incident and scattered fields. \(\varepsilon, \mu, \sigma\) and \(\sigma^{*}\) denote permittivity, permeability, electric conductivity and magnetic conductivity for the scattering target, respectively. The \(x\)-components of the governing equations, for example, are given by:

\[ j \omega E_{x}^{\text{scat}} = -\sigma E_{x}^{\text{scat}} - \sigma E_{y}^{\text{inc}} - j\omega(\varepsilon - \varepsilon_{0})E_{y}^{\text{inc}} \]
\[ + \left( \frac{\partial H_{y}^{\text{scat}}}{\partial y} - \frac{\partial H_{y}^{\text{scat}}}{\partial z} \right) \]  
\[ j \omega \mu H_{x}^{\text{scat}} = -\sigma^{*} H_{x}^{\text{scat}} - \sigma^{*} H_{y}^{\text{inc}} - j\omega(\mu - \mu_{0})H_{y}^{\text{inc}} \]
\[ + \left( \frac{\partial E_{y}^{\text{scat}}}{\partial y} - \frac{\partial E_{y}^{\text{scat}}}{\partial z} \right) \]  

Outside the target region there is only scattered field, and we apply an anisotropic perfectly matched layer (APML) [7] as truncation boundary condition for the computational region. We express the governing equations outside of the target and APML regions as

\[ j \omega \varepsilon |[\Lambda]|E_{x}^{\text{scat}} = \nabla \times \bar{H}_{x}^{\text{scat}} \]  
\[ j \omega \mu |[\Lambda]|H_{x}^{\text{scat}} = -\nabla \times \bar{E}_{x}^{\text{scat}} \]

where

\[ [\Lambda] = [\Lambda_{x}][\Lambda_{y}][\Lambda_{z}] \]

\[ = \begin{bmatrix} s_{x}^{-1} & 0 & 0 \\ 0 & s_{x} & 0 \\ 0 & 0 & s_{x} \end{bmatrix} \begin{bmatrix} s_{y} & 0 & 0 \\ 0 & s_{y}^{-1} & 0 \\ 0 & 0 & s_{y} \end{bmatrix} \begin{bmatrix} s_{z} & 0 & 0 \\ 0 & s_{z} & 0 \\ 0 & 0 & s_{z}^{-1} \end{bmatrix} \]
\[
\begin{pmatrix}
    s_x^{-1} & s_y & 0 & 0 \\
    0 & s_x s_y^{-1} & s_y & 0 \\
    0 & 0 & s_x s_y s_z^{-1} & 0 \\
\end{pmatrix}
\]  
\( (4) \)

with

\[
s_\alpha = \begin{cases}
    \left(1 + \frac{\sigma_\alpha^r}{j\omega\varepsilon_0}\right), & \text{APML regions} \\
    1, & \text{non-APML region (}\sigma_\alpha^r = 0\text{)}
\end{cases} \quad (\alpha = x, y, z) \quad (5)
\]

and \( \sigma_\alpha^r \) is the spatially varying conductivity along the absorption axis. In order to improve the performance in the APML regions, we choose \( \sigma_\alpha^r \) as

\[
\sigma_\alpha^r = \sigma_{\alpha_{\text{max}}}^r \left| \frac{\alpha - \alpha_0}{d} \right|^2, \quad \alpha = x, y, z \quad (6)
\]

where \( d \) is the thickness of the APML region. The constant \( \sigma_{\alpha_{\text{max}}}^r \) is determined by the expectant reflection error with normal incidence on an APML wall [7]:

\[
H(0) = \exp \left[ -\frac{2}{\varepsilon_0 V} \int_0^d \sigma_\alpha^r(\alpha) d\alpha \right]
\]

\[
= \exp \left[ -\frac{2\sigma_{\alpha_{\text{max}}}^r d}{3\varepsilon_0 V} \right]
\]

\[
= \exp \left[ -\frac{2\sigma_{\alpha_{\text{max}}}^r d \sqrt{\varepsilon_{\text{refl}}}}{3\varepsilon_0 V} \right] \quad (7)
\]

Equation (7) can be solved for \( \sigma_{\alpha_{\text{max}}}^r \) as

\[
\sigma_{\alpha_{\text{max}}}^r = -\frac{3\varepsilon_0 V}{2d \sqrt{\varepsilon_{\text{refl}}}} \ln(H(0)). \quad (\alpha = x, y, z) \quad (8)
\]

where \( V \) denotes the speed of electromagnetic wave propagation and \( \varepsilon_{\text{refl}} \) is the effective relative permittivity.

3. IMPLEMENTATION

3.1. MRTD Update Equations Inside the Target

In the MRTD scheme, we expand all the scattered field quantities in terms of the scaling functions in space and the pulse functions in time.
[4]; for example, the $x$-components are:

$$E_{x}^{\text{scat}}(x, y, z, t) = \sum_{n, i, j, k = -\infty}^{+\infty} \frac{\text{scat}}{\phi_x} E_{i+j+\frac{1}{2}, j, k}^{n} \phi_i(x) \phi_j(y) \phi_k(z) h_n(t)$$

(9a)

$$H_{x}^{\text{scat}}(x, y, z, t) = \sum_{n, i, j, k = -\infty}^{+\infty} \frac{\text{scat}}{\phi_x} H_{i+j+\frac{1}{2}, j+\frac{1}{2}, k}^{n+\frac{1}{2}} \phi_i(x) \phi_j(y) \phi_{k+\frac{1}{2}}(z) h_{n+\frac{1}{2}}(t)$$

(9b)

where $\phi_i(x)$, $\phi_j(y)$ and $\phi_k(z)$ denote the space cubic spline Battle-Lemarie scaling functions, and $h_n(t)$ is a rectangular pulse function. Substitution of Eq. (9) into the time-domain Maxwell’s equations leads to the update equations for the $E$- and $H$-fields. For instance, the $x$-component of the update equations for the $H$-fields reads

$$\frac{\text{scat}}{\phi_x} H_{i+j+\frac{1}{2}, k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{\text{scat}}{\phi_x} H_{i+j+\frac{1}{2}, k+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{1}{\mu_0} \sum_{\nu} a(\nu) \times \left( \frac{\text{scat}}{\phi_y} E_{i+j+\frac{1}{2}, k+\nu+1}^{n} \frac{\Delta t}{\Delta z} - \frac{\text{scat}}{\phi_z} E_{i+j+\nu+1, k+\frac{1}{2}}^{n} \frac{\Delta t}{\Delta y} \right)$$

(10)

where the summation coefficients $a(\nu)$ are listed in [4] and [8].

In the target region, due to the non-local property of the Battle-Lemarie scaling functions, the MRTD expansions may cover both the target and free space. Consequently, the derivation of the update equations for the $E$-fields is rather tedious. We will first consider a lossless dielectric target ($\sigma = 0$). Starting from the time domain equations, the $x$-components of the governing equations, for example, are

$$\frac{\partial E_{x}^{\text{scat}}}{\partial t} = \varepsilon_0 \sigma_t E_{x}^{\text{scat}},$$

$$\frac{\partial E_{x}^{\text{scat}}}{\partial t} = -\varepsilon_0 (\varepsilon_r - 1) \frac{\partial \varepsilon_{\text{inc}}^{\text{scat}}}{\partial t} + \left[ \frac{\partial H_{y}^{\text{scat}}}{\partial y} - \frac{\partial H_{z}^{\text{scat}}}{\partial z} \right].$$

(11b)

From (11), we can derive the update equations for the $E$-field component as

$$\frac{\text{scat}}{\phi_x} D_{i+j+\frac{1}{2}, j, k}^{n+1} = \frac{\text{scat}}{\phi_x} D_{i+j+\frac{1}{2}, j, k}^{n} - e_0 \varepsilon_r \frac{\text{scat}}{\phi_x} D_{i+j+\frac{1}{2}, j, k}^{n} \sum_{i', j', k'} \alpha_{i+j+\frac{1}{2}, j', k'} \kappa_{i', j', k'}$$

$$\times \left( \frac{\text{inc}}{\phi_x} E_{i'+\frac{1}{2}, j', k'}^{n+1} \frac{\Delta t}{\Delta z} - \frac{\text{inc}}{\phi_x} E_{i'+\frac{1}{2}, j', k'}^{n} \frac{\Delta t}{\Delta y} \right)$$
\[ + \sum_{\nu} a(\nu) \left( \text{scatter} \; H^{n+\frac{1}{2}}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \frac{\Delta t}{\Delta y} - \text{scatter} \; H^{n+\frac{1}{2}}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \frac{\Delta t}{\Delta z} \right) \]  

\[ \text{scatter} \; D^{n+\frac{1}{2}}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} = \varepsilon_0 \sum_{\nu', j', k'}^{\infty} \left[ \delta_{\nu, \nu'} \delta_{j, j'} \delta_{k, k'} \right] \text{scatter} \; E^{n+\frac{1}{2}}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \]  

where the coefficients \( \alpha_{i+\frac{1}{2}, j+\frac{1}{2}, \nu+\frac{1}{2}} \) and \( \gamma_{k, k'} \) are defined as:

\[ \alpha_{i+\frac{1}{2}, j+\frac{1}{2}, \nu+\frac{1}{2}} = \int_{x_1}^{x_2} \psi_{j+\frac{1}{2}}(x) \psi_{\nu+\frac{1}{2}}(x) \frac{dx}{\Delta x} \]  

\[ \beta_{j, j'} = \int_{y_1}^{y_2} \phi_j(y) \phi_{j'}(y) \frac{dy}{\Delta y} \]  

\[ \gamma_{k, k'} = \int_{z_1}^{z_2} \phi_k(z) \phi_{k'}(z) \frac{dz}{\Delta z} \]

The parameters \( (x_1, x_2), (y_1, y_2) \) and \( (z_1, z_2) \) are the lower and upper limits of the dielectric target region defined along the \( x \)-, \( y \)- and \( z \)-directions, respectively. For example, the coefficients \( \alpha_{i+1/2,j+1/2,\nu+1/2} \) and \( \alpha_{i+1/2,j+1/2,\nu+1/2} \) with \( x_1 = 0, x_2 = 5\Delta x \) and different values of \( i \) and \( m \) are given in Table 1.

In the above derivations, we can use the diagonal approximation, i.e., we only choose the main terms in the summation running, so that \( \alpha_{i+1/2,j+1/2,\nu+1/2} \delta_{j, j'} \gamma_{k, k'} \delta_{\nu, \nu'} \) is instead of \( \alpha_{i+1/2,j+1/2,\nu+1/2} \delta_{j, j'} \gamma_{k, k'} \) in Eq. (12b). This approximation treatment is justified because of the compact support of the basis functions. Finally, the update equations are:

\[ \text{scatter} \; E^{n+1}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} = \text{scatter} \; E^{n}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \frac{\Delta t}{C} \]  

\[ + \sum_{\nu', j', k'}^{\infty} \alpha_{i+\frac{1}{2}, j+\frac{1}{2}, \nu+\frac{1}{2}} \beta_{j, j'} \gamma_{k, k'} \left( \text{scatter} \; E^{n+1}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} - \text{scatter} \; E^{n}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \right) \]  

\[ + \frac{1}{C \varepsilon_0} \sum_{\nu} a(\nu) \left( \text{scatter} \; H^{n+\frac{1}{2}}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \frac{\Delta t}{\Delta y} - \text{scatter} \; H^{n+\frac{1}{2}}_{i+\frac{1}{2}j+\nu+\frac{1}{2}k} \frac{\Delta t}{\Delta z} \right) \]  

where

\[ C = \left[ 1 + (\varepsilon_r - 1) \alpha_{i+\frac{1}{2}, j+\frac{1}{2}, \nu+\frac{1}{2}} \delta_{j, j'} \gamma_{k, k'} \right] \]
Table 1a. Coefficient $\alpha_{i+1} (x_1 = 0, x_2 = 5 \Delta x)$.

<table>
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<tr>
<th>$\alpha_{i+1}$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
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<td>-5.3702x10^{-4}</td>
<td>1.3714x10^{-1}</td>
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</tr>
<tr>
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<td>1.6445x10^{-1}</td>
<td>9.9716x10^{-1}</td>
<td>2.9886x10^{-1}</td>
<td>-4.3347x10^{-1}</td>
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<td>2.9886x10^{-1}</td>
<td>9.9418x10^{-1}</td>
<td>1.0052x10^{-2}</td>
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</tr>
<tr>
<td>4</td>
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<td>9.6086x10^{-2}</td>
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</tr>
<tr>
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<td>9.6086x10^{-2}</td>
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<td>9.4821x10^{-3}</td>
<td>-4.0709x10^{-1}</td>
</tr>
</tbody>
</table>

Table 1b. Coefficient $\alpha_{i+1/2, i+1/2-m} (x_1 = 0, x_2 = 5 \Delta x)$.

<table>
<thead>
<tr>
<th>$\alpha_{i+1/2, i+1/2}$</th>
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<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<td>2.1435x10^{-4}</td>
</tr>
</tbody>
</table>

Next, we consider a perfectly electric conducting (PEC) target. The $x$-component of the time domain equation is

\[
\frac{\partial E_{\text{scat}}^n}{\partial t} = -\frac{\sigma}{\varepsilon_0} E_{\text{inc}}^n + \frac{1}{\varepsilon_0} \left( \frac{\partial H_{\text{scat}}^n}{\partial y} - \frac{\partial H_{\text{scat}}^n}{\partial z} \right). \tag{15}
\]

Using the MRTD expansions and diagonal approximation, the corresponding update equation becomes

\[
s_{\text{scat}} E_{i+1/2,j,k}^n = \frac{C_0}{C_+} s_{\text{scat}} E_{i+1/2,j,k}^n - \frac{C_0}{C_+} \sum_{j',k'} \alpha_{i+1/2,j',k'} \gamma_{j',k'} \frac{\gamma_{j',k'}^{i+1/2,j',k'}}{\gamma_{j',k'}^{i+1/2,j',k'}} \times \left( \frac{E_{i+1/2,j',k'}^{n+1}}{\gamma_{j',k'}^{i+1/2,j',k'}} + \frac{E_{i+1/2,j',k'}^{n+1}}{\gamma_{j',k'}^{i+1/2,j',k'}} \right) + \frac{C_0}{C_+} \sum_{j'} a(j') \left( \frac{H_{i+1/2,j'+1/2,k}^{n+1}}{\Delta y} - \frac{H_{i+1/2,j'+1/2,k}^{n+1}}{\Delta y} \right) \epsilon_{\text{scat}} H_{i+1/2,j,k}^{n+1/2} + \frac{\Delta t}{\gamma_{j'}^{i+1/2,j',k'}} - \frac{\Delta t}{\gamma_{j'}^{i+1/2,j',k'}} \right) \tag{16a}
\]
with

\[ C_- = 1 - \frac{\sigma \alpha_{s+1/2, s+1/2} \Delta t}{2 \varepsilon_0}, \]
\[ C_+ = 1 + \frac{\sigma \alpha_{s+1/2, s+1/2} \Delta t}{2 \varepsilon_0}, \quad C_0 = \frac{\sigma \Delta t}{2 \varepsilon_0} \]  \hspace{1cm} (16b)

In fact, because of a large value of electric conductivity, the Eq. (16a) can be reduced into:

\[ \frac{\partial}{\partial x} E^{n+1}_{s+1/2, j, k} = -i \omega E^n_{s+1/2, j, k} \]  \hspace{1cm} (16c)

for the PEC target.

Finally, considering a lossy dielectric target (\( \sigma \neq \infty \)) and using a similar procedure in developing the update equations for the lossless dielectric and the PEC cases, we have, for example, the update equation of the \( x \)-component of \( E \)-fields for the lossy dielectric medium as given in Eq. (2a):

\[ \begin{array}{l}
\frac{\partial}{\partial x} E^{n+1}_{s+1/2, j, k} = \frac{C_-}{C_+ - \varepsilon_0} \frac{\partial}{\partial x} E^n_{s+1/2, j, k} \\
- \frac{C_0}{C_+} \sum_{j', k'} \alpha_{s+1/2, j', k'}^2 \beta_{s+1/2, j', k'} \gamma_{s+1/2, j', k'} \left( \frac{\partial}{\partial x} E^{n+1}_{s+1/2, j', k'} + i \omega E^n_{s+1/2, j', k'} \right) \\
- \frac{(\varepsilon_x - 1)}{C_+} \sum_{j', k'} \alpha_{s+1/2, j', k'}^2 \beta_{s+1/2, j', k'} \gamma_{s+1/2, j', k'} \left( \frac{\partial}{\partial x} E^{n+1}_{s+1/2, j', k'} - i \omega E^n_{s+1/2, j', k'} \right) \\
+ \frac{1}{\varepsilon_0 C_+} \sum_{j', \Delta y} u(j') \left( \frac{\partial}{\partial y} H^{n+1/2}_{s+1/2, j', j, k} - \frac{\Delta t}{\varepsilon y} H^{n+1/2}_{s+1/2, j', j, k} \right), \hspace{1cm} (17a)
\end{array} \]

where

\[ C_- = \left[ 1 + \left( (\varepsilon_x - 1) - \frac{\sigma \Delta t}{2 \varepsilon_0} \right) \alpha_{s+1/2, s+1/2} \beta_{s+1/2, s+1/2} \gamma_{s+1/2, s+1/2} \right], \]
\[ C_+ = \left[ 1 + \left( (\varepsilon_x - 1) + \frac{\sigma \Delta t}{2 \varepsilon_0} \right) \alpha_{s+1/2, s+1/2} \beta_{s+1/2, s+1/2} \gamma_{s+1/2, s+1/2} \right], \quad C_0 = \frac{\sigma \Delta t}{2 \varepsilon_0}. \]  \hspace{1cm} (17b)

### 3.2. MRTD Update Equations Outside the Target

The computational domain outside the target region consists of both the free space and the APML regions. We can derive the time domain equations from Eq. (3) using the frequency-time conversion \( \partial_t \leftrightarrow j \omega \).
In the APML regions, there are six face-APMLs, twelve edge-APMLs, and eight corner-APMLs. As an illustration, we consider a corner-APML and derive the MRTD update equations for the x-component of the E-fields:

\[
\begin{align*}
\frac{j_{\omega \varepsilon_0}}{1 + \frac{\sigma_y^c}{j_{\omega \varepsilon_0}}} \left( 1 + \frac{\sigma_y^c}{j_{\omega \varepsilon_0}} \right) E_x^{\text{scat}} &= \frac{\partial H_y^{\text{scat}}}{\partial y} - \frac{\partial H_y^{\text{scat}}}{\partial z} .
\end{align*}
\] (18)

Similar to the FDTD approach used in [7], we have used a two-step method in the regions outside the target region, such that,

\[
\begin{align*}
\phi_x D_n^{n+\frac{1}{2},j,k} &= \frac{1 - \sigma_x^c \Delta t}{2\varepsilon_0} \phi_x D_n^{n,\frac{1}{2},j,k} + \frac{1}{1 + \frac{\sigma_y^c \Delta t}{2\varepsilon_0}} \\
&\times \sum_{\nu} a(\nu) \left( \phi_x H_n^{n+\frac{1}{2},j,k} + \frac{\Delta t}{\Delta y} \phi_y H_n^{n+\frac{1}{2},j,k} - \frac{\Delta t}{\Delta z} \phi_z H_n^{n+\frac{1}{2},j,k} \right) ,
\end{align*}
\] (19a)

\[
\begin{align*}
\phi_x E_n^{n+\frac{1}{2},j,k} &= \frac{1 - \sigma_x^c \Delta t}{2\varepsilon_0} \phi_x E_n^{n,\frac{1}{2},j,k} + \frac{1}{1 + \frac{\sigma_y^c \Delta t}{2\varepsilon_0}} \\
&\times \left[ \left( 1 + \frac{\sigma_y^c \Delta t}{2\varepsilon_0} \right) \phi_x D_n^{n+1,\frac{1}{2},j,k} - \left( 1 - \frac{\sigma_y^c \Delta t}{2\varepsilon_0} \right) \phi_x D_n^{n,\frac{1}{2},j,k} \right] .
\end{align*}
\] (19b)

In free space, we can simply set \( \sigma_x^c = \sigma_y^c = \sigma_z^c = 0 \) in Eq. (19), while inside an APML region, we need to set \( \sigma_x^c (a = x,y,z) \) along the wave propagation direction.

Finally, we can extract the radar cross section (RCS) for the scattering target as [1, 3]:

\[
RCS = \lim_{n \to \infty} \left( 4\pi r^2 \frac{E_y^2 + E_z^2}{(E_{inc})^2} \right) .
\] (20)

4. NUMERICAL RESULTS

In this section, we compare the results obtained from the MRTD scheme to those derived from the conventional FDTD method and the MoM technique [1-3, 9, 10]. We compute the surface equivalent currents and derive far-zone scattered fields for canonical three-dimensional scattered objects such as PEC, lossless, and lossy dielectric flat plates.
4.1. Validation of Near-Fields for a PEC Cube

In the first example, we compute the magnitude of surface electric current along the perfectly conducting cube using the MRTD scheme with the validation of the FDTD method [9]. Each side of the PEC cube was divided into 20 intervals for both the FDTD and the MRTD. The loci are along two different straight lines traversing the cube surface as shown in [9]. The field components of the plane-wave excitation are $E_{xmc}^m = E_{xmc}^m$ and $H_{xmc}^m = H_{xmc}^m$. The incident wave propagates in the positive y-direction, i.e., $\theta_i = 90^\circ$, $\phi_i = 90^\circ$, and $\psi = 90^\circ$. At the frequency of 3 GHz, the PEC cube whose electric size $ke = 2$ ($l$: the side length of the PEC cube), and the total computational domain (excluding APML) contain $30 \times 30 \times 30$ cells for both cases and the target dimensions are $20 \times 20 \times 20$ cells. The surface current is given by $\vec{J} = \vec{h} \times \vec{H}$ where $\vec{H}$ is the total magnetic field similar to that used in [5]. For the loop $\vec{d}\vec{e}\vec{c}\vec{d}$ current density, the tangential magnetic field $H_{tan}^m$ is taken as $H_x$ along the entire path. For the loop $\vec{a}\vec{b}\vec{c}\vec{d}$ z-direction current density $I_z$, the tangential magnetic fields $H_{tan}^m$ are taken as $H_x$, $H_y$, $H_z$ for path $\vec{a}\vec{b}$, $\vec{c}\vec{d}$, and $\vec{a}\vec{d}$, respectively. Figures 1a and 1b show the looping currents on surface and phases of electric current on surface for two loops, respectively. The results of the MRTD agree with that derived from the FDTD method [9] at the same accuracy level along the two loops $\vec{a}\vec{b}\vec{c}\vec{d}$ and $\vec{a}\vec{b}\vec{c}\vec{d}$.

4.2. Scattering Analysis for a Perfectly Conducting Plate

We now analyze a perfectly conducting plate to illustrate the application of the MRTD method. The dimension of the plate is $29 \text{ cm} \times 29 \text{ cm} \times 1 \text{ cm}$ as shown in [3]. Table 2 lists the dimensions of the target discretization and the computation grid size used in the MRTD scheme and FDTD method, respectively [3]. We first consider a Gaussian pulse incident along the +z direction. The incident wave is normal to the plate, i.e., $\theta_i = 0^\circ$, $\phi_i = 0^\circ$, $\psi = 90^\circ$, $\theta_s = 0^\circ$, and $\psi_s = 180^\circ$, and Fig. 2a gives the monostatic RCS versus frequency. Next, we consider an incident plane wave with $\theta_i = 45^\circ$, $\phi_i = 30^\circ$, $\psi = 0^\circ$ and calculate the bistatic RCS with $\theta_s = 45^\circ$ and $\phi_s = 210^\circ$. Figures 2b and 2c compare the MRTD co-polarized and cross-polarized frequency domain RCS with those of the FDTD method and the MoM method [10], respectively, and show good agreement among all the methods. Here, a total of 8-layer of APML is used in the boundary truncation. The computational CPU time used for normal incident on the plate is about 1125 seconds for the MRTD, and 1208 seconds for the FDTD method using a 500 MHz Alpha digital workstation.
Figure 1. (a) The magnitude and phase of the loop $\overline{ABCDE}$ current; (b) the magnitude and phase of the looping $\overline{ABCD}$ current.
Figure 2. RCS as a function of frequency (a) backscattering ($\theta_i = 0^\circ$, $\phi_i = 0^\circ$, $\psi = 90^\circ$, $\theta_s = 0^\circ$, and $\phi_s = 180^\circ$); (b) co-polarized backscattering ($\theta_i = 45^\circ$, $\phi_i = 30^\circ$, $\psi = 0^\circ$, $\theta_s = 45^\circ$, and $\phi_s = 210^\circ$); (c) cross-polarized backscattering ($\theta_i = 40^\circ$, $\phi_i = 30^\circ$, $\psi = 0^\circ$, $\theta_s = 45^\circ$, and $\phi_s = 210^\circ$).
RCS scattering analysis using the 3-D MRTD scheme

![Graph](image)

**Figure 2.**

Table 2 Discretization of PEC plate and dielectric flat plate using the MRTD scheme and FDTD method.

<table>
<thead>
<tr>
<th>Discretization</th>
<th>Target</th>
<th>PEC Plate ((29\times29\times1\text{cm}^3))</th>
<th>Lossless and Lossy Dielectric Plate ((18\times18\times6\text{cm}^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta x = \Delta y (\text{cm}), \Delta z (\text{cm}))</td>
<td>(\Delta t (10^{-11}\text{s}))</td>
<td>MRTD</td>
<td>FDTD</td>
</tr>
<tr>
<td>(2.64, 1.0)</td>
<td>1.18</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(1.92)</td>
<td>0.626</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Target Dimensions (cells)</td>
<td>(29\times29\times1)</td>
<td>20x20x1</td>
<td>20x20x7</td>
</tr>
<tr>
<td>Total computation space (Containing APML) (cells)</td>
<td>(35\times35\times25)</td>
<td>(55\times55\times25)</td>
<td>44x44x31</td>
</tr>
</tbody>
</table>

The total number of time steps in both cases is 1000. Although the CPU times in both cases are at the same level, the MRTD uses less memory space \((V_{\text{MRTD}}/V_{\text{FDTD}} \approx 40.5\%)\), because the low sampling rate of the MRTD \((\Delta x = \Delta y \approx \lambda_{\text{min}}/4.74)\) in comparison to a larger one \((\Delta x = \Delta y = \lambda_{\text{min}}/10.0)\) in the FDTD, where \(\lambda_{\text{min}}\) is the wavelength of the highest frequency of interest. In fact, the cell size of FDTD method used is a rather 'coarse' cell size.
Figure 3. RCS as a function of frequency of forward scattering for a lossless ($\theta_t = 0^\circ$, $\phi_t = 0^\circ$, $\psi = 90^\circ$, $\theta_s = 0^\circ$, and $\phi_s = 0^\circ$); and a lossy ($\theta_t = 0^\circ$, $\phi_t = 0^\circ$, $\psi = 90^\circ$, $\theta_s = 30^\circ$, and $\phi_s = 0^\circ$) dielectric flat plates; (b) RCS as a function $\theta_s$ for a lossless (at $f = 3.03$ GHz and $\phi_s = 0^\circ$) and a lossy (at $f = 3.15$ GHz and $\phi_s = 0^\circ$) dielectric flat plates.
4.3. Scattering Analysis for a Lossless Dielectric Flat Plate

Next, we apply the MRTD to analyze a lossless dielectric ($\varepsilon_r = 2.0$) flat plate (18 cm $\times$ 18 cm $\times$ 6 cm). A Gaussian pulse, with frequency bandwidth of 5 GHz, is incident on the target with $\theta_i = 0^\circ$, $\phi_i = 0^\circ$, and $\psi = 90^\circ$. Table 2 also lists the dimensions of the structure discretization used in the MRTD scheme and the FDTD method, respectively. Figure 3a shows the forward scattering ($\theta_s = 0^\circ$ and $\phi_s = 0^\circ$) RCS results as a function of frequency, while Fig. 3b displays the RCS with the entire span angles ($\theta_s$). Again, we have used an 8-layer AMPL in boundary truncation. We find good agreement in the RCS results between the MRTD and FDTD methods. In the forward scattering ($\theta_s = \phi_s = 0^\circ$) case, the MRTD CPU time is about 2525 seconds and the FDTD CPU time is 4892 seconds using the same 500 MHz Alpha Digital Workstation. The total number of time steps in both cases is 1500. The CPU time and memory of MRTD are, respectively, about 54% and 19.3% of the FDTD.

4.4. Scattering Analysis for a Lossy Dielectric Flat Plate

Finally, we apply the MRTD scheme to the analysis of a lossy dielectric plate, which is characterized with $\varepsilon_r = 2.0$, $\sigma = 0.01$ s/m, and the dimensions of 18 cm $\times$ 18 cm $\times$ 6 cm. A Gaussian pulse, with the frequency bandwidth of 5 GHz, is incident in a direction with $\theta_i = 90^\circ$, $\phi_i = 0^\circ$, and $\psi = 90^\circ$. Table 2 lists the dimensions of the discretization used in the MRTD scheme and FDTD modeling, respectively. Figure 3a shows the monostatic RCS ($\theta_s = 30^\circ$ and $\phi_s = 0^\circ$) as a function of frequency obtained from the MRTD and FDTD, respectively. Fig. 3b compares the RCS results in the range of the entire span of angles ($\theta_s$) derived using the MRTD and FDTD, and again we observe a good consistency. We have also used an 8-layer AMPL in the boundary truncation. Here, the MRTD CPU time is about 2488 seconds and the FDTD one is 4734 seconds by using the same computer. The total number of time steps is 1000. The MRTD CPU time is used about 53% of that used in that of FDTD, and the MRTD computational space used is only about 19.3% of that of the FDTD.

5. CONCLUSIONS

In this work, we have developed a multiresolution time domain scheme based on cubic spline Battle-Lemarie wavelet base for the scattering analysis of three-dimensional targets. We have used the pure scattered field formulation to construct the incident fields inside the target.
region, and developed an MRTD near-to-far-zone field transformation
to determine the scattering RCS. We have validated the near-fields,
the far-fields, and the RCS by analyzing both the PEC and dielectric
(lossless and lossy) plates. The results from the MRTD method
are in good agreement with those derived from FDTD method. We
conclude that the MRTD scheme has good potential in efficiently
solving generalized electromagnetic problems, particularly for large
objects.

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