\[ <x_k^* > = <y_k^* > = <v_k^* > \]

\[ - \sum_{n=1}^{p-1} \frac{p!}{(p-n)!} < (y_k - v_k)^{n-n} > <v_k^* > \]  

(3)

The learning is achieved so that every moment of \( \hat{x}_k \) and \( \hat{v}_k \) may be equal to those of \( x_k \) (\( = y_k - v_k \)) and \( v_k \), respectively. The learning of weighting parameters is achieved so that the following criterion function \( E_{cr} \) becomes minimum:

\[ E_{cr}(\hat{x}_k, \hat{v}_k) = \sum_p \left( e_{xp}(\hat{x}_k) + e_{vp}(\hat{v}_k) \right) \]  

(4)

with \( e_{xp}(\hat{x}_k) = <(x_k^p - \hat{x}_k^p)^2> \) and \( e_{vp}(\hat{v}_k) = <(v_k^p - \hat{v}_k^p)^2> \).

The usual gradient descent method is modified and employed here for the learning of the proposed filter. Learning is achieved by eqn. 5 for every \( 2N \) observations:

\[ w_{n+1}^{u,v} = I \left( w_{n}^{u,v} - \eta \frac{\partial E_{cr}(\hat{x}_k, \hat{v}_k)}{\partial w_{n}^{u,v}} \right) \]  

(5)

where \( \eta \) is a learning rate and \( h(u) \) is a nonlinear function to keep \( u \) in the range [0,1]. \( h(u) = 1 \) for \( 1 < u < 0 \) for \( 0 \leq u \leq 1 \), and 0 for \( u < 0 \).

**Simulation results:** Here we employ 0.7\( \sin(3\omega t) + \sin(9\omega t) + \sin(15\omega t) \) as \( x_k \) and we employ the following three types of signal as \( v_k \):

- **case 1:** \( \sin(2\omega t) + \sin(6\omega t) \)
- **case 2:** A Gaussian noise \( N(0,0.1) \)
- **case 3:** A uniform type noise \( U(0.5,0.5) \)

In the experiments, the Fourier window width \( 2N \) is 256, and the moments of \( v_k \) up to the fourth order are used as a priori information.

**Fig. 2 Simulation results**

a) Noisy signal \( y_k \) in case 1  
b) Estimated results \( \hat{x}_k \) and original signal \( x_k \) in case 1  
c) Noisy signal \( y_k \) in case 2  
d) Estimated results \( \hat{x}_k \) and original signal \( x_k \) in case 2

**Table 1:** Estimation performance \( I \) (dB) for proposed filter, average filter and midrange filter for noisy signal

<table>
<thead>
<tr>
<th>Type of noise</th>
<th>Demixing filter</th>
<th>Average filter</th>
<th>Midrange filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>11.8</td>
<td>6.2</td>
<td>5.0</td>
</tr>
<tr>
<td>Uniform</td>
<td>7.81</td>
<td>4.9</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Observation mechanism is \( y_k = x_k + v_k \)

Fig. 2 shows the experimental results for cases 1 and 2. In case 1, it is observed that the filtering is performed almost perfectly. In case 2, filtering performance is also good. Table 1 shows the average estimation performance \( I \) for the proposed filter for the results of cases 2 and 3. \( I \) is defined as \( I = 10 \log_2 (\sigma_0^2 / \sigma_1^2) \), with \( \sigma_0^2 = <(y_k - \hat{x}_k)^2> \) and \( \sigma_1^2 = <(y_k - \hat{v}_k)^2> \), which indicates the degree of improvement in the signal-to-noise ratio. For comparison, the results of an average filter and a midrange filter are also given in Table 1. It is known that the former is effective for a signal corrupted by Gaussian noise, and the latter for uniform-type noise. We can see that the proposed filter is superior to other filters.

**Conclusion:** The proposed filter does not require a complex algorithm, and its architecture is very simple. It has potential applications to a wide range of practical nonlinear and non-Gaussian filtering problems.

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**General criterion for decomposition of exact blocking computation in circuit-switched networks**

C.Y. Li and P.K.A. Wai

The criterion for the decomposition of the computation of blocking probabilities in circuit-switched networks is given. As the criterion does not depend on the network topology or the traffic distribution, more efficient decomposition can be obtained accordingly.

**Introduction:** Although the blocking probabilities of calls in circuit-switched networks with Poisson calls arrival and fixed routing can be solved explicitly from the simple product form call status probability, the computation is often intractable unless the network size is small [1]. Fortunately, the computation can be decomposed into smaller sub-problems in many situations. As a result, the computation is simplified [2, 3]. To date, the decomposition is based on the topology and/or the traffic distribution of the network. There is no general criterion to determine when the blocking computation is decomposable. In this Letter, we present such a criterion.

**Blocking model:** The network is defined as a set of \( p \) links labelled from 1 to \( p \). Link \( i \) has a capacity of \( N_i \) channels. Let \( L = \{1, ..., p\} \) be the set of links and \( N = (N_1, ..., N_p) \) be the corresponding link capacity vector. We assume that the routing paths of calls are fixed. Calls are classified according to their requirements of links, channel capacity, and holding time. The \( r \) classes of calls in the networks are labelled from 1 to \( r \). We define \( \mathbf{R} = \{1, ..., r\} \) as the set of calls. Class \( j \) calls arrive at the system according to a Poisson process with offered load \( p_j \) and request a non-negative number \( a_j \) of channels on link \( i \) for \( j \in \mathbf{R} \) and \( i \in L \). The demand matrix \( \mathbf{A} \) is defined as \( (a_j; i \in L, j \in R) \). If any of the links that a new call requests has insufficient free channels, the call is blocked immediately. We let \( n_i \) be the number of class \( j \) calls in progress, and \( n = (n_1, ..., n_r) \) be the call state vector. We further define \( \mathbf{T}(N) = \{0 \leq 0 \leq \mathbf{N} \leq N \} \) as the set of valid states for \( n \). Let \( X(t) \) be the call state at time \( t \). The stochastic process \( \{X(t)\} \) is a reversible...
Markov process defined on Γ(N). Its stationary probability distribution is given by
\[ \pi(n) = \frac{1}{G(N)} \prod_{j \in B_1} \rho_j^{n_j} \quad n \in \Gamma(N) \]  
(1)
where \( G(N) = \sum_{n \in \Gamma(N)} \prod_{j \in B_1} \rho_j^{n_j} \) is the normalisation constant [4]. The probability of a class \( j \) call being blocked is given by
\[ B_j = 1 - \frac{G(N - A_0)}{G(N)} \]  
(2)
where \( I \in \Gamma(N) \) is a unit vector which represents only one class \( j \) call in progress. In general, direct computation of \( G(N) \) is impractical except when the network size is small or when the problem can be reduced to a number of tractable subproblems.

Decomposition: Assume that a network can be partitioned into \( K \) subnetworks, and \( I_k \) is the set of links in subnetwork \( k \) for \( k = 1, ..., K \). The set \( I \) is defined as the set of links interconnecting these subnetworks. The set \( I_k \) is the set of classes of calls that only use links in subnetwork \( k \), for \( k = 1, ..., K \). The set \( I \) is the set of all classes of calls which request any interconnection links, i.e. links that belong to \( I \). Clearly, \( I = I_1 + I_2 + ... + I_k + R \) and \( R = \bigcup_{k=1}^{K} R_k \). Let \( \Gamma(N) = (n \in \Gamma(N): n_i \neq 0 \text{ or } j \notin \Gamma(N)) \) be the set of reduced set of states where arrivals of calls not belonging to \( R \) are suspended. The normalisation constant when only the traffic in subnetwork \( k \) is considered is given by
\[ G_k(N) = \sum_{n \in \Gamma(N)} \prod_{j \in R_k} \rho_j^{n_j} \]  
(3)
\[ G(N) = \sum_{n \in \Gamma(N)} G_k(N - A_n) \prod_{j \in R_k} \rho_j^{n_j} \]  
(4)
After \( G(N) \) is computed, the blocking probabilities can be determined accordingly. The details of the procedure and computational reduction with the decomposition method can be found in [3].

Criterion of decomposition: The efficiency of the decomposition method stated above depends on the way the network is partitioned. Traditionally, the partition is based on the network topology and/or the traffic distribution. In many instances, however, the blocking computation can be decomposed even when the network cannot be partitioned according to its topology and/or traffic distribution. In the following, we shall present a criterion for the decomposition of the computation of the normalisation constant.

Let \( n(0) \) be the extracted status vector in which all the components of \( n(0) \) that do not represent the status of the classes belonging to \( R_k \) for \( k = 0, ..., K \) are deleted. Hence, \( n = (n(0), ..., n(K)) \) for \( R = \bigcup_{k=1}^{K} R_k \). Let \( n(k) \) be the \( n \)-dimensional random variable for the status of the classes belonging to \( R_k \), and \( P[n(0)] \) be the probability of the classes in status \( n(0) \).

Proposition: Let \( R = R_1 + ... + R_K \). The normalisation constant is given by eqn. 4 and only if 
\[ P[n(0), ..., n(K) = n(0)] = 0 \text{ for all valid } n(0). \]

Proof:
(i) Necessity: Assume that eqn. 4 is applicable. For \( k = 1, ..., K \), classes belonging to different \( R_k \) behave as if they were in different separate networks when the status of classes in \( R_k \) is fixed. The corresponding random variables \( n(k) \) are independent with each other once \( n(0) \) is fixed. Hence, \( P[n(0), ..., n(K) = n(0)] \) is equal to 
\[ \Pi_{k=1}^{K} P[n(0), ..., n(K) = n(0)] \]  
for all valid \( n(0) \).

(ii) Sufficiency: Assume that \( P[n(0), ..., n(K) = n(0)] \) is equal to 
\[ \Pi_{k=1}^{K} P[n(0), ..., n(K) = n(0)] \]  
for all valid \( n(0) \). For simplicity, we denote \( P[. \bigcap_{k=1}^{K} P[n(0), ..., n(K) = n(0)] \). As \( n = (n(0), ..., n(K)) \), we have \( P(n) = P[n(0), ..., n(K)] \). From Bayes theorem, \( P(A) = P(A|B)P(B) \), \( P(n) = P[n(0), ..., n(K)] \). Defining \( R_k = R - R_k \) as \( k \). Similarly, \( P[K(n)|n(0)] \) is equal to \( \Pi_{k=1}^{K} P[n(0), ..., n(K) = n(0)] \). We define \( R_k = R - R_k \) as the complement set of \( R_k \) for \( k = 0, ..., K \). Similarly, \( P[K(n)|n(0)] \) is defined as the corresponding status vector for \( R_k \). Note that \( \Pi_{k=1}^{K} P[n(0), ..., n(K) = n(0)] = P[n(0), ..., n(K) = n(0)] \). In terms of \( n(k) \), we have
\[ P(n)K^{-1} = \frac{1}{P[n(0)]} \prod_{k=1}^{K} P[n(0), ..., n(K) = n(0)] \]  
(5)
Let \( P(n)K^{-1} \) be a projection operator on \( n \) such that only the components representing the status of classes belonging to \( R_k \) are retained. As \( P[K(n)|n(0)] \) is the marginal probability on \( n \), it is equal to 
\[ P[n]K^{-1} \bigcap_{k=1}^{K} P[n(0), ..., n(K) = n(0)] \]  
(6)
Eliminating \( P[n(0)] \) on both sides of eqn. 6, we obtain an expression for \( P(n)K^{-1} \). Note that eqn. 6 is valid for any \( n \in \Gamma(N) \). Hence, we set \( n = n(0), ..., n(K) \) and the condition inside the summation on the right hand side of eqn. 6 can therefore be simplified to \( \sum_{k=1}^{K} n(k) \). From eqn. 5, we finally have
\[ P[n]K^{-1} = \frac{1}{G} \prod_{k=1}^{K} \sum_{n \in \Gamma(N-n(0))} \prod_{j \in R_k} \rho_j^{n_j} \]  
(7)
where \( n = (n(0), ..., n(K)) \). Summing up \( P[n(0)] \) in eqn. 7 for all valid \( n(0) \), the result is the normalisation constant \( G(N) \) as given in eqn. 4. The proof is completed.

Conclusion: We have given a criterion for the decomposition of the blocking computation of a circuit-switched network. The criterion does not rely on the physical partitioning of the network which is required by current decomposition methods [2, 3]. A more efficient decomposition method can therefore be developed.

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