Ideal amplifier spacing for reduction of Gordon-Haus jitter in dispersion-managed soliton communications

J.N. Kutz and P.K.A. Wai

Based on a variational analysis, the authors demonstrate that the noise-induced Gordon-Haus timing jitter in a dispersion-managed soliton transmission system can be substantially reduced by appropriate placement of the amplifiers.

One of the factors limiting the performance of soliton transmission systems is the Gordon-Haus jitter [1] which arises from the amplified spontaneous emission noise of the amplifiers. Recently, dispersion-management has been shown both numerically [2] and experimentally [3] to naturally reduce Gordon-Haus jitter. In these dispersion-managed systems, it can be shown using a variational method [4, 5] that the induced timing jitter is proportional to the pulse peak power at the amplifier divided by the power enhancement factor [4, 6]. And because the peak power of a dispersion-managed soliton [4, 6] fluctuates periodically, the timing jitter can be substantially reduced by judicious placement of the amplifiers in a transmission line.

The evolution of the slowly varying envelope of the electric field normalised by standard soliton units [1] corresponding to the path-average dispersion is given by the modified nonlinear Schrödinger equation:

\[
\frac{\partial Q}{\partial Z} + \frac{\sigma(Z)}{2} \frac{\partial^2 Q}{\partial T^2} + \alpha^2(Z)|Q|^2 Q = S(Z, T)
\]

where \(S(Z, T) \ll 1\) is a white-noise process, \(\sigma(Z)\) is the dispersion map, and \(\alpha(Z)\) gives the loss and gain fluctuations so that \(d\alpha/dZ + \Gamma_\alpha = G \alpha[Z + \eta Z, \alpha]\), where \(\eta Z\) is the normalised amplifier spacing, \(\Gamma_\alpha\) is the loss rate, \(G\) is the gain, \(\eta\) is the total number of amplifiers, and \(\delta\) is the Dirac delta function (see Fig. 1).

The variational method uses the Lagrangian associated with the governing eqn. 1 [4, 5], along with an appropriate choice of ansatz which describes the amplitude, width, and chirp fluctuations [4, 5] and includes the centre-frequency and centre-position dynamics which are driven by the noise field. The ansatz of the pulse envelope is

\[
Q(Z, T) = A \eta \text{sech}[\eta (T - C)]
\times \exp \left( i \left( \Omega (T - C) + \beta (T - C)^2 - \frac{\phi}{2} \right) \right)
\]

where \(A\) is the power enhancement factor [4, 6] and \(\eta\), \(\beta\), \(C\), \(\Omega\), and \(\phi\) are evolution parameters determined by taking the variations of the Lagrangian with respect to each of the free parameters.

Similar to the approach of Gordon and Haus [1], we assume the appropriate noise field phase component which contributes to the frequency dynamics at the amplifier to be \(S(Z, T)\) = \(\text{sech}[\eta (T - C)]\). The resulting frequency shift at each amplifier is

\[
\Omega = -2s \frac{\eta}{A}
\]

Thus, the frequency shift, and corresponding Gordon-Haus jitter, can be reduced by minimising the quantity \(\eta/A\) at the amplifier. We note that the amplitude and chirp dynamics are governed by

\[
\frac{\partial \eta}{\partial Z} = -2s \eta \beta \eta
\]

\[
\frac{\partial^2 \beta}{\partial Z^2} = \sigma(Z) \left( \eta^2 - \beta^2 \right) - A^2 \sigma^2(Z) \eta^2
\]

which are studied in detail elsewhere [4, 5].

In this study, we assume a symmetric dispersion map [4, 6] (Fig. 1 shows a typical map with two amplifiers). Because of the large number of parameters in the problem, in the following, we limit the parameter space by fixing the pulsewidth to be 35 ps, the path-average dispersion to be \(D_{\text{ave}} = 0.2 \text{ ps/km}\), and the amplifier spacing to be 60 km. The total length of the anomalous dispersion fibre is fixed at 60 km with a dispersion value of \(D = 17.4 \text{ ps/km}\) (standard 50 fibre). We consider both two- and eight-amplifiers per map segment since the former has been studied extensively [6] and the latter is being investigated experimentally by Bergano et al. [7]. For completeness, we also study the cases of four and six amplifiers per map period. We do not consider an odd number of amplifiers here because periodic solutions in these cases are difficult to construct due to an asymmetry arising from \(a(Z)\) in eqn. 4. This issue will be explored elsewhere. The total length of the normal dispersion fibre segment is therefore 60, 180, 300, and 420 km, respectively, with the dispersion value chosen to give the fixed-path-average dispersion.

![Fig. 1](image1.png)

**Fig. 1** Typical dispersion map

Top: amplifier spacing and loss-gain intensity fluctuations | Middle: dispersion values | Bottom: average intensity fluctuations \(\eta\)

The location of the amplifiers is determined by the variable \(x\) (\(x = 10 \text{ km}\) in Fig. 1). Thus, we can calculate the sum of the jitter contribution, \(\eta/A\), from each amplifier by varying the value of \(x\) from 0 to 60 km (note that a typical periodic amplitude fluctuation over a single map period is given by the bottom diagram of Fig. 1). In Fig. 2, the average jitter contribution per amplifier \(\langle S^2 \rangle_{\eta/A} / m\), where \(m\) is the number of amplifiers per map period, is shown against \(x\) for the dispersion maps given above. The results from all four maps exhibit similar behaviours, i.e. the jitter is smallest when \(x = 40 \text{ km}\) and greatest when \(x = 5 \text{ km}\). In the two-amplifier case, the minimum jitter contribution is 3.5 times smaller than the maximum. In contrast, for the eight-amplifier case, the minimum jitter contribution is only 20% smaller than the maximum. Qualitatively (see Fig. 1), we expect the least amount of jitter when the amplitude, \(\eta\), is minimal at the amplifier (roughly just before and just after the anomalous dispersion fibre) and we expect the greatest amount of jitter when \(\eta\) is maximal at an amplifier (roughly at the mid-point of the anomalous dispersion fibre). In the case of many amplifiers per map period, the amplifiers effectively see the \(\eta\) value averaged over a dispersion period, whereas in the case of only two
amplifiers, a significant difference in both the enhancement factor $A$ and the value of $\eta$ can result, depending on the location of the amplifiers. This leads to a large variation of jitter generated against $x$. We note that the case of two-amplifiers per map period gives both the minimum and maximum amount of average jitter per amplifier among the dispersion maps studied. When the amplifiers are placed properly ($x = 40$ km), the two-amplifier dispersion map reduces the jitter by nearly 40% in comparison with the best jitter performance of the four-, six-, and eight-amplifier cases.

In conclusion, for the short maps which require only two amplifiers, the amplifier position can greatly reduce the induced Gordon-Haus timing jitter (by a factor of nearly 4 in the above examples) and increase system performance. In contrast, the use of many amplifiers per period has a relatively small (< 20%) impact on system performance due to the jitter, since the noise contribution effectively averages over the amplitude fluctuations per period. In either case, the optimal performance is achieved by keeping amplifiers away from the mid-point of the anomalous dispersion fibre where the pulse experiences peak power. Finally, the shorter map (two amplifiers) can boost performance by 40% per amplifier in comparison with the longer maps ($m > 4$) considered.

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J.N. Kutz and P.K.A. Wai (Department of Electronic Engineering, Hong Kong Polytechnic University, Hong Kong, Hong Kong)
E-mail: enwai@polyu.edu.hk
J.N. Kutz: Permanent address: Department of Applied Mathematics, University of Washington, Seattle, WA 98195-2420, USA

References

Impact of cell fanout distributions on multicast ATM switch performance

Yijun Xiong

For shared buffer multicast ATM switches, it is found that the highest cell loss ratio may be caused by the unicast plus broadcast traffic given the mean cell fanout, or roughly by the maximum coefficient of variation of cell fanout given the fanout distribution.

Multicast switch: An $N \times N$ multicast ATM switch with shared buffer output queuing is considered, where a simple bit-mapping approach is used to implement the multicast function which is conceptually illustrated in Fig. 1. At switch inlets, each incoming multicast ATM cell is converted to the internal cell format with a bit-mapping pattern (BMP) and a multicast connection identifier (MCI) as its header ("10" is an indicator of multicast cells). The BMP has $N$ bits, with the $i$th bit indicating whether switch outlet $i$ will receive a copy. In Fig. 1, the BMP of the multicast cell is 1011, which means that it will be copied to switch outlets 1, 2, and 3. The MCI is used to identify different multicast connections in the switch and will be used at switch outlets to convert copies back to ATM cells.

![Fig. 1](image_url) Cell replication in a $4 \times 4$ switch

SBM: shared buffer memory
AQ: address queue

For switches with a shared buffer, only the original multicast cell need be stored in the shared buffer memory (SBM) and its address in the SBM is sent to address queues of the switch outlets which expect to receive a copy (see Fig. 1). The last copy leaving the switch will erase the multicast cell and release the occupied space in the SBM. This storage mechanism can achieve high utilization of buffer memory and has been used in the design of multicast ATM switches [1, 2].

The performance of multicast ATM switches depends not only on the statistical characteristics of input traffic, but also on the cell fanout distributions. It was observed in [3] that multicast traffic with a truncated geometric cell fanout could have an impact on the switch performance worse than that of unicast traffic. Here we study the effect of cell fanout distributions on the switch performance, which, to our knowledge, has not been studied before. The complete buffer sharing strategy is considered.

Input traffic: Both Bernoulli random traffic and bursty traffic are considered [3]. For bursty traffic, cell arrivals on each switch inlet are in bursts. The burst lengths (in cells) are IID and are assumed to have a geometric distribution with mean $L$ ($\geq 1$). The idle period (in slots) between any two consecutive bursts is also geometrically distributed with mean $F$ ($\geq 0$). The average traffic load on a switch inlet is $\sigma = L / (L + F)$. Further, the fanout $X$ of each burst is IID and has an arbitrary probability distribution $f_j = P[X = j]$, $1 \leq j \leq N$, with mean $F$. For (truncated) geometric cell fanout, $f_j = (1 - \gamma \rho)^j (1 - \gamma \rho)^{-1}$, $1 \leq j \leq N$. The destinations of copies of a burst are uniformly distributed among different switch outlets. Moreover, traffic on different switch inlets is also IID. The offered traffic load on each switch outlet is $p = \sigma F$. It is easy to see that random traffic is just a special case of bursty traffic when $L = 1$. Unicast traffic is a special case of multicast traffic with $F = 1$.

Performance: Previous study shows that multicast traffic has an impact no worse than unicast traffic on the behaviour of address queues [3], and therefore its effect on the cell delay [4] will be no worse either. Consider now the cell loss ratio (CLR) which equals $(p-p_c)/p$, where $p_c$ is the carried load on a switch outlet. The multicast traffic could be regarded as a kind of bulk arrival with size $F$, which usually results in a high CLR as $F$ increases. However, since only the original multicast cells are stored in the SBM, the available buffer space is used efficiently and consequently the CLR is greatly reduced. However, each multicast cell may remain in the SBM longer than unicast cells, which could also increase the CLR. The interaction of these pro and con factors will determine the actual CLR. Because of the storage mechanism, the shared buffer behaviour becomes too complicated to study analytically,