Q1.

\[ Y = S + N(1 - z^{-1}) \]
\[ S = \frac{1}{1 - z^{-1}} R \]
\[ R = X - z^{-1} Y \]

\[ (1 - z^{-1}) Y = (1 - z^{-1}) S + N(1 - z^{-1})^2 \]
\[ = R + N(1 - z^{-1})^2 \]
\[ = X - z^{-1} Y + N(1 - z^{-1})^2 \]
\[ Y = X + N(1 - z^{-1})^2 \]

\[ H_2(z) \]
\[ |H_2(z)| = (|H_1(z)|)^2 = 4 \sin^2 \left( \frac{\pi f}{f_s} \right) \]

\( f_s \) is the sampling frequency.
Q2. Transfer functions of Fig Q2a:

\[ R = X - z^{-1}Y \]
\[ S = \frac{1}{1-z^{-1}}R \]
\[ Y = S + N \]

\[ R = X - z^{-1}Y \]
\[ S = \frac{1}{1-z^{-1}}R \]
\[ Y = S + N \]

where \( N \) = requantization noise,
\( T_n \) = wavelength of noise shaper sampling frequency.

Transfer function of Fig Q2b:

\[ Y = N + A \]
\[ X - (Y - A) z^{-1} = A \]
\[ X = N z^{-1} = (Y - N) \]
\[ Y = X + N (1 - z^{-1}) \]

Their transfer functions are identical.

Q3. (i)

<table>
<thead>
<tr>
<th>Input to the quantizer</th>
<th>Output from the quantizer</th>
<th>Error signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>0.6-0.4 = -0.2</td>
<td>0</td>
<td>-0.2</td>
</tr>
<tr>
<td>0.6+0.2 = 0.8</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.6-0.2 = -0.4</td>
<td>0</td>
<td>-0.4</td>
</tr>
<tr>
<td>0.6+0.4 = 1.0</td>
<td>1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

(ii) It repeats pattern “10101”. The period is 5.

(iii) A constant level input results in a regular output pattern. If the period of the repetition of such patterns is long enough, they may be audible as a deterministic or oscillatory tone, rather than as noise.

Solution: Dithering the input fed into the quantizer as follows.

A noise shaping circuit must employ dithering to minimize distortion.