1. Introduction

- The objective is to represent and describe the resulting aggregate of segmented pixels in a form suitable for further computer processing after segmenting an image into regions.

- Two choices for representing a region:
  - *External characteristics*: its boundary;
  - *Internal characteristics*: the pixels comprising the region.

- For example, a region may be represented by (a) its boundary with the boundary described by features such as its length, (b) the orientation of the straight line joining the extreme points, and (c) the number of concavities in the boundary.

- An *external representation* is chosen when the primary focus is on shape characteristics.

- An *internal representation* is selected when the primary focus is on reflectivity properties, such as color and texture.

- In either case, the features selected as descriptors should be as insensitive as possible to variations such as change in size, translation and rotation.
2. Representation schemes

• The segmentation techniques yield raw data in the form of pixels along a boundary or pixels contained in a region.

• Although these data are sometimes used directly to obtain descriptors (as in determining the texture of a region), standard practice is to use schemes that compact the data into representations that are considerably more useful in the computation of descriptors.

• This section introduces some basic representation schemes for this purpose.

2.1 Chain codes

• To represent a boundary by a connected sequence of straight line segments of specified length and direction.

• The direction of each segment is coded by using a numbering scheme such as the ones shown below.

Fig 1. Directions for (a) 4-directional chain code and (b) 8-direction chain code.

• This method generally is unacceptable to apply for the chain codes to pixels:

(a) The resulting chain of codes usually is quite long;

(b) Sensitive to noise: any small disturbances along the boundary owing to noise or imperfect segmentation cause changes in the code that may not necessarily be related to the shape of the boundary.

• A frequently used method to solve the problem is to resample the boundary by selecting a larger grid spacing.
Depending on the proximity of the original boundary to the nodes, a point is assigned to those nodes.

The accuracy of the resulting code representation depends on the spacing of the sampling grid.

- **Normalization for starting point:**
  Treat the code as a circular sequence and redefine the starting point so that the resulting sequence of numbers forms an integer of minimum magnitude.

- **Normalization for rotation:**
  Use the first difference of the chain code instead of the code itself. The difference is simply by counting (counter-clockwise) the number of directions that separate two adjacent elements of the code.

  Example: The first difference of the 4-direction chain code 10103322 is 33133030.

- **Normalization for size:**
  Alter the size of the resampling grid.

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*Fig 2.* (a) Digital boundary with resampling grid; (b) result of resampling; (c) 4-directional chain-code; (d) 8-directional chain code.
2.2 Polygonal approximation

- The objective is to capture the essence of the boundary shape with the fewest possible polygonal segments.
- This problem in general is not trivial and can quickly turn into a time-consuming iterative search.

(a) Minimum-perimeter Polygons
- A given boundary is enclosed by cells. We can visualize this enclosure as consisting of two walls corresponding to the inside and outside boundaries of the cells. If the boundary is a rubber band, it will shrink and take the shape as in (b).
- The error in each cell would be at most $\sqrt{2}d$, where $d$ is the pixel distance.

(b) Merging Technique
- It is based on error or other criteria have been applied to the problem of polygonal approximation.
- One approach is to merge points along a boundary until the least square error line fit of the points merged so far exceeds a preset threshold.
- Vertices do not corresponding to corners in the boundary.
(c) **Splitting Techniques**

- To subdivide a segment successively into two parts until a given criterion is satisfied.
- Example: a line $a$ drawn between two end points of a boundary. The perpendicular distance from the line to the boundary must not exceed a preset threshold. If it does, the farthest points becomes a vertex.

![Diagram of splitting techniques](image)

**Fig 4.** (a) Original boundary (b) boundary divided into segments based on distance computations (c) joining of vertices (d) resulting polygon

2.3 **The skeleton of a region**

- The structural shape of a plane region can be reduced to a graph.
- This reduction can be accomplished by obtaining the skeleton of the region via a thinning algorithm.

(a) **Medial axis transformation**

- The skeleton of a region may be defined via the medial axis transformation (MAT) proposed by Blum in 1967.

![Diagram of medial axis transformation](image)

**Fig 5.** Medial axes of 3 simple regions

- Given a region $R$ and a border $B$:

  For each point $p$ in $R$, we find its closest neighbor in $B$. If $p$ has more that one such neighbor, it belongs to the medial axis (skeleton of $R$).
• The concept of "closest" depends on the definition of a distance.

• Although the MAT of a region yields an intuitively pleasing skeleton, direct implementation of that definition is typically prohibitive computationally.

• Implementation potentially involves calculating the distance from every interior point to every point on the boundary of a region.

(b) **Thinning algorithm for binary regions**

- Assume region points have value 1 and background points 0.
- A contour point is any pixel with value 1 and having at least one 8-neighbor valued 0.

<table>
<thead>
<tr>
<th>P_9</th>
<th>P_2</th>
<th>P_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_8</td>
<td>P_1</td>
<td>P_4</td>
</tr>
<tr>
<td>P_7</td>
<td>P_6</td>
<td>P_5</td>
</tr>
</tbody>
</table>

Fig 6. Neighborhood arrangement used by a thinning algorithm

• The thinning method consists of successive passes of two steps applied to the contour points.

  □ **Step 1**

  Flag a contour point p for deletion if the following conditions are satisfied:

  (a) \( 2 \leq N(p_i) \leq 6 \), where \( N(p_i) = \sum_{i=2}^{9} p_i \)

  (b) \( S(p_i) = 1 \), where \( S(p_i) \) is the 0-1 transitions in the ordered sequence of \( p_2, p_3, \ldots, p_8, p_9 \).

  (c) \( p_2 \cdot p_4 \cdot p_6 = 0 \)

  (d) \( p_4 \cdot p_6 \cdot p_8 = 0 \)

  To keep the structure during this step, points are not deleted until all border points have been processed.

□ **Step 2**

In the second step, condition (a) and (b) remain the same, and,

(c') \( p_2 \cdot p_4 \cdot p_8 = 0 \)

(d') \( p_2 \cdot p_6 \cdot p_8 = 0 \)
The whole procedure for one iteration
- Applying step 1 to flag border points for deletion
- Deleting the flagged points
- Applying step 2 to flag the remaining border points for deletion
- Deleting the flagged points.

The basic procedure is applied iteratively until no further points are deleted, at which time the algorithm terminates; yielding the skeleton of the region.

Physical meaning of the conditions:
- Condition (a) is violated when contour point p has only one or seven 8-neighbors valued 1, which implies that p is the end point of a skeleton strobe and should not be deleted.

  \[
  \begin{pmatrix}
  0 & 0 & 0 \\
  0 & 0 & 0 \\
  \end{pmatrix}
  \]

  e.g. Deleting p in \( 0 \ p \ 1 \) shortens the skeleton.

- Condition (b) is violated when it is applied to points on a stroke 1 pixel thick. Hence this condition prevents disconnection of segments of a skeleton during the thinning operation.

  \[
  \begin{pmatrix}
  0 & 0 & 1 \\
  0 & 0 & 1 \\
  \end{pmatrix}
  \]

  e.g. Deleting p in \( 1 \ p \ 0 \) disconnects the skeleton.

- Conditions (c) or (d) is violated when at least 3 of the 4-neighbors of p1 are connected to p1. In such cases, p1 is so critical that it can't be deleted.

  \[
  \begin{pmatrix}
  0 & 1 & 0 \\
  0 & 1 & 0 \\
  \end{pmatrix}
  \]

  e.g. Deleting p in \( 0 \ p \ 1 \) disconnects the skeleton.
3. Boundary descriptors

3.1 Some simple descriptors

- Area of the object
- Length of a contour
- Diameter of a boundary: The maximum distance between any 2 points on the boundary.

3.2 Shape numbers

- Shape number of a boundary is defined as the first difference of a chain code of the smallest magnitude.
- The order n of a shape number is the number of digits in its representation.
- The following figures shows all shapes of order 4 and 6 in a 4-directional chain code:

Fig 8. (a) Result of step 1 of the thinning algorithm during the first iteration through 1 region (b) result of step 2 and (c) final result.
Fig 9. All shapes of order 4 and 6. The dot indicates the starting point.

Note that the first difference is calculated by treating the chain codes as a circular sequence.

Fig 10. Steps in the generation of a shape number
### 3.3 Fourier Descriptors:

- Coordinate pairs of points encountered in traversing an N-point boundary in the xy plane are recorded as a sequence of complex numbers.

  Example: \{ (1,2), (2,3), (2,4), \ldots \} \Rightarrow \{ 1+2i, 2+3i, 2+4i, \ldots x+yi, \ldots \}.

- An N-point DFT is performed to the sequence and the complex coefficients obtained are called the **Fourier descriptors** of the boundary.

- In general, only the first few coefficients are of significant magnitude and are pretty enough to describe the general shape of the boundary.

- **Fourier descriptors** are not directly insensitive to geometrical changes such as translation, rotation and scale changes, but the changes can be related to simple transformations on the descriptors.

### Some basic properties of Fourier Descriptors:

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Boundary</th>
<th>Fourier Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity</td>
<td>s(k)</td>
<td>S(u)</td>
</tr>
<tr>
<td>Rotation</td>
<td>s(k)e^{j\theta}</td>
<td>S(u)e^{j\theta}</td>
</tr>
<tr>
<td>Translation</td>
<td>s(k)+d</td>
<td>S(u)+d\delta(u)</td>
</tr>
<tr>
<td>Scaling</td>
<td>c s(k)</td>
<td>c S(u)</td>
</tr>
<tr>
<td>Starting point</td>
<td>s(k-k_0)</td>
<td>S(u)e^{j2\pi k_0 u/N}</td>
</tr>
</tbody>
</table>

**Fig 11.** Examples of reconstructions from Fourier descriptors for various values of M.
3.4 Moments

- Coordinate pairs of points encountered in traversing an N-point boundary in the xy plane are recorded as a sequence of complex numbers \{g(i)\: i=1,..N\}.

Example: \{(1,2), (2,3), (2,4),..(x,y),..\} \Rightarrow \{1+2i, 2+3i, 2+4i,..x+yi,..\}.

- Normalize the area of the object to unit area.

- The moments of the boundary is given as

\[ \mu_n = \sum_{i=1}^{N} (g(i) - m)^n \]

where \( m = \sum_{i=1}^{N} g(i) \)

4. Regional descriptors

4.1 Some simple descriptors

- The area of a region is defined as the number of pixels contained within its boundary.

- The perimeter of a region is the length of its boundary

4.2 Texture descriptors

- Descriptors providing measures of properties such as smoothness, coarseness and regularity are used to quantify the texture content of an object.

Fig 12. Examples of (a) regular and (b) coarse textures.
There are 3 principal approaches used in image processing to describe the texture of a region: statistical, structural and spectral approaches.

(a) *Statistical approaches:*
- Statistical approaches yield characterizations of textures as smooth, coarse, grainy, and so on.
- One of the simplest approaches for describing texture is to use moments of the gray-level histogram of an image or region.

(b) *Structural approaches:*
- Structural techniques deal with the arrangement of image primitives.
- They use a set of predefined texture primitives and a set of construction rules to define how a texture region is constructed with the primitives and the rules.

(c) *Spectral approaches:*
- Spectral techniques are based on properties of the Fourier spectrum and are used primarily to detect global periodicity in an image by identifying high-energy, narrow peaks in the spectrum.
- The Fourier spectrum is ideally suited for describing the directionality of periodic or almost periodic 2-D patterns in an image.
- Three features of the spectrum are useful for texture description:
  1. prominent peaks give the principal direction of the patterns;
(2) the location of the peaks gives the fundamental spatial period of the patterns;

(3) eliminating any periodic components via filtering leaves nonperiodic image elements, which can be described by statistical techniques.

- The spectrum can be expressed in polar coordinates to yield a function $S(r, \theta)$.
- Two functions can then be used to describe the texture accordingly:

$$S(\theta) = \sum_{r=0}^{R} S(r, \theta)$$

$$S(r) = \sum_{\theta=0}^{\pi} S(r, \theta)$$

Fig 14. Image showing periodic texture; (b) spectrum; (c) plot of $S(r)$; (d) plot of $S(\theta)$. 