An image is divided into two data blocks. It is noted that the rate-distortion curve of the two data blocks is as follows:

The exact value of the operating points is as follows:

<table>
<thead>
<tr>
<th>Block 1</th>
<th>Block 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate (bpp)</td>
<td>Distortion (MSE)</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2.2</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>15</td>
</tr>
<tr>
<td>0.9</td>
<td>33</td>
</tr>
<tr>
<td>0.6</td>
<td>60</td>
</tr>
<tr>
<td>0.1</td>
<td>200</td>
</tr>
</tbody>
</table>

If \( R_{\text{budget}} \) for coding these two data blocks is set at 3.5 bpp, use the Lagrangian optimization technique to determine the optimum operating point for the two blocks that gives the lowest total distortion. Clearly show the steps to obtain the results.
Solution

A. Initialization:
   a. Set $\lambda_l = 0$.  $R_l = 5+5 = 10$;  $D_l = 1+2 = 3$
   b. Set $\lambda_u = \infty$.  $R_u = 0.1+0.1 = 0.2$;  $D_u = 200+140 = 340$
   c. $R_u \leq R_{budget} = 3.5 \leq R_l$

B. Iteration:
   i. Set $\lambda = (D_l - D_u) / (R_u - R_l) = 34.4$
      $R = 1.2+1.2 = 2.4$;  $D = 15+40 = 55$
      Since $R < R_{budget}$ hence
      Set $\lambda_u = 34.4$
      $R_l = 10$;  $D_l = 3$  $R_u = 2.4$;  $D_u = 55$
   ii. Set $\lambda = (D_l - D_u) / (R_u - R_l) = 6.8$.
      $R = 2.2+2.2 = 4.4$;  $D = 7+20 = 27$
      Since $R > R_{budget}$ hence
      Set $\lambda_l = 6.8$.
      $R_l = 4.4$;  $D_l = 27$  $R_u = 2.4$;  $D_u = 55$
   iii. Set $\lambda = (D_l - D_u) / (R_u - R_l) = 14$.
      $R = 1.2+2.2 = 3.4$;  $D = 15+20 = 35$
      Since $R < R_{budget}$ hence
      Set $\lambda_u = 14$.
      $R_l = 4.4$;  $D_l = 27$  $R_u = 3.4$;  $D_u = 35$
   iv. Set $\lambda = (D_l - D_u) / (R_u - R_l) = 8$.
      $R = 1.2+2.2 = 3.4$;  $D = 15+20 = 35$
      Since $R = R_u$ hence stops

The operation points are: Block 1 – (1.2, 15);  Block 2 – (2.2, 20).