EIE 339 DTSS

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Normal Office hour: 8:00am – 5:30pm
Assessment

Exam

Continuous assessment

Three Experiments
(Attendance, preparation and report writing)

Test  x 1
Quiz  x 2
Assignment  x 3
Textbook

Syllabus

Passband Data Transmission
  – PSK
  – AK
  – FSK

Multiplexing
  – FDM
  – TDM

Traffic Engineering
Circuit Switching
Modulation

- Modulating signal
  - Message
- Carrier wave

- Example:
  - mobile telephone system
  - radio
Modulation

- Modulator
  - “Combine” modulating signal and carrier wave
  - Modulated signal

- Demodulator
  - Extract the modulating signal from the modulated signal
Efficient transmission

- Example: Antenna size
  - For efficient radiation, physical dimensions > 0.1 wavelength
  - Audio signal contains frequency components down to 100Hz $\Rightarrow$ wavelength = 3km
  - Modulated frequency at 100MHz $\Rightarrow$ wavelength = 3m
Modulation benefits and applications

Modulation to overcome hardware limitations
  - Fractional bandwidth
    - Bandwidth/center frequency
    - 1-10%

  - Example: FM radio
    - Signal Bandwidth = 200kHz
    - Fractional bandwidth = 200kHz/ 100MHz = 0.2%

Intermediate frequency = 10.7MHz
  Fractional bandwidth = 200kHz/ 10.7MHz =1.9%
Modulation benefits and applications

Modulation to reduce noise and interference
  – Wideband noise reduction

Modulation for frequency assignment
Modulation for multiplexing

- Combining several signals for simultaneous transmission on one channel.

- Frequency-division multiplexing (FDM)
  - Each signal uses different carrier frequencies
Passband Data Transmission

In **baseband data transmission**, a data stream represented in the form of a discrete pulse-amplitude modulated (PAM) signal is transmitted over a **low-pass channel**.

**Example**: Nyquist channel

\[ H(f) , G(f) , C(f) \]

\[ W \]

\[ G(f) \rightarrow H(f) \rightarrow C(f) \]

OR
In **passband data transmission**, the incoming data stream is modulated onto a carrier with fixed frequency and then transmitted over a band-pass channel.

**Example:**

\[ H(f), G(f), C(f) \]

modulator \[ \cos 2\pi ft \]
Passband data transmission allows more efficient use of the allocated RF bandwidth, and flexibility in accommodating different baseband signal formats.

**Example**

– Mobile Telephone Systems
  - GSM: Gaussian Minimum Shift Keying (GMSK) is used (a variation of FSK)
  - IS-54: $\pi/4$-Differential Quaternary Phase Shift Keying (DQPSK) is used (a variation of PSK)
Types

The modulation process making the transmission possible involves switching (keying) the amplitude, frequency, or phase of a sinusoidal carrier in accordance with the incoming data.

There are three basic signaling schemes:

- Amplitude-shift keying (ASK)
- Frequency-shift keying (FSK)
- Phase-shift keying (PSK)
Two waveforms
Unlike ASK signals, both PSK and FSK signals have a **constant envelope**.

PSK and FSK are preferred to ASK signals for passband data transmission over **nonlinear channel** (amplitude nonlinearities) such as microwave link and satellite channels.
Classification of digital modulation techniques

Coherent and Noncoherent

Digital modulation techniques are classified into coherent and noncoherent techniques, depending on whether the receiver is equipped with a **phase-recovery circuit** or not.

The phase-recovery circuit ensures that the local oscillator in the receiver is synchronized to the incoming carrier wave (in both frequency and phase).
Phase Recovery (Carrier Synchronization)

Two ways in which a local oscillator can be synchronized with an incoming carrier wave

- Transmit a pilot carrier (similar to the DSB-LC modulation in analog communication)

- Use a carrier-recovery circuit such as a phase-locked loop (PPL)
**M-ary signaling**

In an *M*-ary signaling scheme, there are *M* possible signals during each signaling interval of duration *T*.

Usually, \( M = 2^n \) and \( T = nT_b \) where \( T_b \) is the bit duration.

\[
T = nT_b
\]

![Diagram](image)
**M-ary signaling**

In passband transmission, we have

- $M$-ary ASK
- $M$-ary PSK
- $M$-ary FSK

We can also combine different methods:

- $M$-ary quadrature-amplitude modulation (QAM)

(In baseband data transmission, we have $M$-ary PAM)
**M-ary signaling**

*M*-ary signaling schemes are preferred over binary signaling schemes for transmitting digital information over band-pass channels when the requirement is to conserve bandwidth at the expense of increased power.

The use of *M*-ary signaling enables a reduction in transmission bandwidth by the factor $n = \log_2 M$ over binary signaling.

\[ T = nT_b \]

10101...01 \hspace{1cm} M \text{ bits} \hspace{1cm} \text{modulator} \hspace{1cm} T
Coherent PSK

The functional model of passband data transmission system is

\[ m_i \rightarrow \text{Signal transmission encoder} \rightarrow s_i \rightarrow \text{Modulator} \rightarrow s_i(t) \rightarrow \text{Channel} \rightarrow x(t) \rightarrow \text{Detector} \rightarrow x \rightarrow \text{Signal transmission decoder} \rightarrow \hat{m} \]

- \( m_i \) is a sequence of symbol emitted from a message source.
- The channel is linear, with a bandwidth that is wide enough to transmit the modulated signal and the channel noise is Gaussian distributed with zero mean and power spectral density \( N_o / 2 \).
Coherent PSK

The following parameters are considered for a signaling scheme:

**Probability of error**

A major goal of passband data transmission systems is the optimum design of the receiver so as to minimize the average probability of symbol error in the presence of additive white Gaussian noise (AWGN)
Coherent PSK

Power spectra

Use to determine the signal bandwidth and co-channel interference in multiplexed systems.

In practice, the signalings are linear operation, therefore, it is sufficient to evaluate the baseband power spectral density.
Coherent PSK

**Bandwidth Efficiency**

- Bandwidth efficiency \( \rho = \frac{R_b}{B} \) bits/s/Hz

where \( R_b \) is the data rate and \( B \) is the used channel bandwidth.

**Example:** Nyquist channel for baseband data transmission

Bandwidth \( B = W = 1/2T_b \).

\[
\therefore \rho = \frac{R_b}{B} = \frac{1}{1/2T_b} = 2 \text{ bits/s/Hz}
\]
In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

where $0 \leq t \leq T_b$, and $E_b$ is the transmitted signal energy per bit.
Coherent PSK

Example:

\[ E = \int_{0}^{T_b} [s_1(t)]^2 \, dt = \frac{2E_b}{T_b} \int_{0}^{T_b} \cos^2 (2\pi f_c t) \, dt = \frac{2E_b}{T_b} \cdot \frac{T_b}{2} = E_b \]

To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency \( f_c \) is chosen equal to \( n / T_b \) for some fixed integer \( n \).
Coherent PSK

The transmitted signal can be written as

\[ s_1(t) = \sqrt{E_b} \phi(t) \quad \text{and} \]

\[ s_2(t) = -\sqrt{E_b} \phi(t) \]

where \( \phi(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t < T_b \)

Note: \( \overline{\phi^2(t)} = \int_0^{T_b} \left[ \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \right]^2 dt = 1 \)
Generation of coherent binary PSK signals

To generate a binary PSK signal, the first step is representing the input binary sequence in polar form with symbols 1 and 0 represented by constant amplitude levels of and , respectively.

This signal transmission encoder is performed by a polar nonreturn-to-zero (NRZ) encoder.

\[
 s_i = \begin{cases} 
 + \sqrt{E_b} & \text{input symbol is 1} \\
 - \sqrt{E_b} & \text{input symbol is 0} 
\end{cases}
\]

101011 \quad \text{Signal transmission encoder} \quad s_i 

101011
The second step is multiplying the carrier encoder output with the carrier

\[
s_i(t) = \begin{cases} 
  s_1(t) & = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{if } s_i = \sqrt{E_b} \\
  s_2(t) & = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{if } s_i = -\sqrt{E_b} 
\end{cases}
\]

\[
f_c = \frac{n}{T_b}
\]

\[
\phi(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)
\]
To detect the original binary sequence of 1s and 0s, we apply the noisy PSK signal to a correlator. The correlator output is compared with a threshold of zero volts.

\[
\int_{0}^{T_b} \phi(t) x(t) \, dt
\]

**Correlator**

**Decision device**

\[ x_1 < 0 \Rightarrow 0 \]
\[ 1 \text{ if } x_1 > 0 \]
\[ 0 \text{ if } x_1 < 0 \]
Detection of coherent binary PSK signals

Example

If the transmitted symbol is 1, \( x(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \)
and the correlator output is

\[
x_1 = \int_0^{T_b} x(t)\phi(t)dt
\]

\[
= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \cdot \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)dt
\]

\[
= \sqrt{E_b} \cdot \frac{2}{T_b} \int_0^{T_b} \cos^2 (2\pi f_c t)dt
\]

\[
= \sqrt{E_b}
\]

Similarly, if the transmitted symbol is 0, \( x_1 = -\sqrt{E_b} \)
We can represent a coherent binary system with a signal constellation consisting of two message points.

- The coordinates of the message points are all the possible correlator output under a noiseless condition.

- The coordinates for BPSK are $\sqrt{E_b}$ and $-\sqrt{E_b}$.

Decision boundary

$-\sqrt{E_b} \quad \sqrt{E_b}$
Error probability of binary PSK

There are two possible kinds of erroneous decision:

- Signal $s_2(t)$ is transmitted, but the noise is such that the received signal point inside region with $x_1 > 0$ and so the receiver decides in favor of signal $s_1(t)$.

- Signal $s_1(t)$ is transmitted, but the noise is such that the received signal point inside region with $x_1 < 0$ and so the receiver decides in favor of signal $s_2(t)$.

\[ \int_0^{T_b} \varphi(t) \]

Decision device

1 if $x_1 > 0$
0 if $x_1 < 0$
Error probability of binary PSK

For the first case, the observable element $x_1$ is related to the received signal $x(t)$ by

$$
x_1 = \int_0^{T_b} x(t) \phi(t) dt
$$

$$
= \int_0^{T_b} [s_i(t) + w(t)] \phi(t) dt
$$

$$
= -\sqrt{E_b} + \int_0^{T_b} w(t) \phi(t) dt
$$

$x_1$ is a Gaussian process with mean:

$$
\bar{x}_i = E[x_i]
$$

$$
= E[-\sqrt{E_b} + \int_0^{T_b} w(t) \phi(t) dt]
$$

$$
= -\sqrt{E_b}
$$
Error probability of binary PSK

Variance is

\[ \sigma^2 = E[(x_i - \bar{x}_i)^2] \]

\[ = E \left[ \left( \int_0^{T_b} w(t) \phi(t) dt \right)^2 \right] \]

\[ = E \left[ \int_0^{T_b} \int_0^{T_b} w(t)w(u)\phi(t)\phi(u)dtdu \right] \]

\[ = \int_0^{T_b} \int_0^{T_b} E[w(t)w(u)]\phi(t)\phi(u)dtdu \]

\[ = \int_0^{T_b} \int_0^{T_b} \frac{N_o}{2} \delta(t - u)\phi(t)\phi(u)dtdu \]

\[ = \frac{N_o}{2} \int_0^{T_b} \phi^2(t)dt \]

\[ = \frac{N_o}{2} \]
Error probability of binary PSK

Therefore, the conditional probability density function of $x_1$, given that symbol 0 was transmitted is

$$f(x_1 \mid 0) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[ -\frac{(x_1 - \bar{x}_1)^2}{2\sigma^2} \right]$$

$$= \frac{1}{\sqrt{\pi N_o}} \exp\left[ -\frac{(x_1 + \sqrt{E_b})^2}{N_o} \right]$$
and the probability of error is

\[ p_{10} = \int_{0}^{\infty} f(x_1 | 0) \, dx_1 \]

\[ = \frac{1}{\sqrt{\pi N_o}} \int_{0}^{\infty} \exp \left[ - \frac{(x_1 + \sqrt{E_b})^2}{N_o} \right] \, dx_1 \]

Putting \( z = \frac{1}{\sqrt{N_o}}(x + \sqrt{E_b}) \), we have

\[ p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_o}}^{\infty} \exp[-z^2] \, dz \]

\[ = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_o}} \right) \]
Similarly, the error of the second kind

\[ p_{01} = p_{10} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_o}} \right) \]

and hence

\[ p_e = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_o}} \right) \]
Quadriphase-shift keying (QPSK)

QPSK has twice the bandwidth efficiency of BPSK, since 2 bits are transmitted in a single modulation symbol. The data input $d_k(t)$ is divided into an in-phase stream $d_I(t)$, and a quadrature stream $d_Q(t)$.

$$d_k(t) : 1001$$

$$d_I(t) : 10$$

$$d_Q(t) : 01$$
QPSK

\[ d_k(t) \]

\[ d_I(t) \]

\[ d_Q(t) \]

\[ T = 2T_b \]
The phase of the carrier takes on one of four equally spaced values, such as $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$.

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + (2i - 1)\pi/4] & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}$$

where $i = 1, 2, 3, 4$.

$E$ is the transmitted signal energy per symbol;
$T$ is the symbol duration;
$f_c = n / T$;

(Note : $T = 2T_b$)
Each possible value of the phase corresponds to a unique dibit.

For example, 10 for $i=1$, 00 for $i=2$, 01 for $i=3$ and 11 for $i=4$.

(only a single bit is change from one dibit to the next)
The transmitted signal can be written as

\[ s_i(t) = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + (2i - 1)\pi / 4] \]

\[ = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t] \cos[(2i - 1)\pi / 4] \]

\[ - \sqrt{\frac{2E}{T}} \sin[2\pi f_c t] \sin[(2i - 1)\pi / 4] \]

\[ = s_{i1}\phi_1(t) + s_{i2}\phi_2(t) \]

where

\[ \phi_1(t) = \sqrt{\frac{2}{T}} \cos[2\pi f_c t]; \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin[2\pi f_c t] \]
### QPSK

<table>
<thead>
<tr>
<th>Input dibit</th>
<th>Phase of QPSK</th>
<th>$s_{i1}$</th>
<th>$s_{i2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$\pi/4$</td>
<td>$\sqrt{E/2}$</td>
<td>$-\sqrt{E/2}$</td>
</tr>
<tr>
<td>00</td>
<td>$3\pi/4$</td>
<td>$-\sqrt{E/2}$</td>
<td>$-\sqrt{E/2}$</td>
</tr>
<tr>
<td>01</td>
<td>$5\pi/4$</td>
<td>$-\sqrt{E/2}$</td>
<td>$\sqrt{E/2}$</td>
</tr>
<tr>
<td>11</td>
<td>$7\pi/4$</td>
<td>$\sqrt{E/2}$</td>
<td>$\sqrt{E/2}$</td>
</tr>
</tbody>
</table>

### Diagram

[Diagram showing QPSK symbols and phase relationships]
Input binary sequence

\[ \begin{align*}
0 & \quad 1 \\
\text{Dibit 01} & \quad \text{(a)} \\
1 & \quad 0 \\
\text{Dibit 10} & \quad \text{(b)} \\
1 & \quad 0 \\
\text{Dibit 10} & \quad \text{(c)} \\
0 & \quad 0 \\
\text{Dibit 00} & \quad \text{(d)} 
\end{align*} \]

Odd-numbered sequence 0
Polarity of coefficient \( s_{i1} \)

\[ s_{i1}\phi_1(t) \]

Even-numbered sequence
Polarity of coefficient \( s_{i2} \)

\[ s_{i2}\phi_2(t) \]

\[ s(t) \]
Generation of coherent QPSK signals

The incoming binary data sequence is first transformed into polar form by a nonreturn-to-zero level encoder. The binary wave is next divided by means of a demultiplexer into two separate binary sequences.

The result can be regarded as a pair of binary PSK signals, which may be detected independently due to the orthogonality of $\phi_1(t)$ and $\phi_2(t)$. 
\[
\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)
\]

\[
\phi_1(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)
\]

Diagram:

10101 \rightarrow \text{Polar NRZ} \rightarrow \text{Demultiplexer} \rightarrow s(t)

\[
X: \quad s_{1i} + s_{2i} = s(t)
\]
Detection of coherent QPSK signals

In-phase channel

Quadrature channel
The received signal is

\[ x(t) = s_i(t) + w(t) \]

and the observation elements are

\[ x_1 = \int_0^T x(t)\phi_1(t)dt \]
\[ = \pm \sqrt{E/2} + \int_0^T w(t)\phi_1(t)dt \]

\[ x_2 = \int_0^T x(t)\phi_2(t)dt \]
\[ = \pm \sqrt{E/2} + \int_0^T w(t)\phi_2(t)dt \]
As a coherent QPSK is equivalent to two coherent binary PSK systems working in parallel and using two carriers that are in phase quadrature.

Hence, the average probability of **bit error in each channel** of the coherent QPSK system is

\[
p = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E}{2 N_o}} \right) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E}{2 N_o}} \right)
\]
Error probability of QPSK

As the bit error in the in-phase and quadrature channels of the coherent QPSK system are statistically independent, the average probability of a correct decision resulting from the combined action of the two channels is

\[ p_c = (1 - p)^2 = \left(1 - \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E}{2N_o}} \right) \right)^2 \]

\[ = 1 - \text{erfc} \left( \sqrt{\frac{E}{2N_o}} \right) + \frac{1}{4} \text{erfc}^2 \left( \sqrt{\frac{E}{2N_o}} \right) \]
The average probability of symbol error for coherent QPSK is therefore

\[
p_e = 1 - p_c = \operatorname{erfc}\left(\sqrt{\frac{E}{2N_o}}\right) - \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_o}}\right) \\
\approx \operatorname{erfc}\left(\sqrt{\frac{E}{2N_o}}\right) \quad \text{if } \frac{E}{2N_o} \gg 1
\]
In a QPSK system, since there are two bits per symbol, the transmitted signal energy per symbol is twice the signal energy per bit,

\[ E = 2E_b \]

and then

\[ p_e \approx \text{erfc}\left(\sqrt{\frac{E_b}{2N_o}}\right) \]
With Gray encoding, the bit error rate of QPSK is

\[
\text{BER} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{2N_o}} \right)
\]

Therefore, a coherent QPSK system achieves the same average probability of bit error as a coherent binary PSK system for the same bit rate and the same \( E_b / N_o \) but uses only half the channel bandwidth.
M-ary PSK

During each signaling interval of duration $T$, one of the $M$ possible signals

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos \left( 2\pi f_c t + \frac{2\pi}{M} (i-1) \right) \quad i = 1, 2, \ldots$$

is sent.
M-ary PSK

The signal constellation of \( M \)-ary PSK consists of \( M \) message points which are equally spaced on a circle of radius \( \sqrt{E} \). For example, the constellation of octaphase-shift keying is

\[
P_e \approx \text{erfc}\left( \sqrt{\frac{E}{N_o}} \sin\left( \frac{\pi}{M} \right) \right)
\]

\( M \geq 4 \)
Power spectra of M-ary PSK signals

The symbol function is

\[ g(t) = \begin{cases} \sqrt{\frac{2E}{T}} & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases} \]

where \( T = T_b \log_2 M \) and \( T_b \) is the bit duration.

As the energy spectral density is the magnitude of the signal’s Fourier transform, the baseband power spectral density is

\[ S(f) = 2E \frac{\sin^2(\pi Tf)}{(\pi Tf)^2} \]

\[ = 2E_b \log_2 M \sinc^2(T_b f \log_2 M) \]
(Normalized to $fT_b$)
Bandwidth efficiency

The bandwidth required to pass M-ary signal (main lobe) is given by

\[
B = \frac{2}{T}
\]

\[
= \frac{2}{T_b \log_2 M}
\]

\[
= \frac{2R_b}{\log_2 M}
\]

Therefore, the bandwidth efficiency is

\[
\rho = \frac{R_b}{B} = \frac{\log_2 M}{2}
\]
QPSK: Change of phase is $\pm 90^\circ$ or $\pm 180^\circ$
Offset QPSK: Change of phase is ±90° only

- Envelope variation is reduced.

\[
\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t \quad 0 \leq t \leq T
\]

\[
\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t \quad 0 \leq t \leq 3T / 2
\]
The carrier phase used for the transmission of successive symbols is alternately picked from one of the two QPSK constellation shown below.
Eight possible phase states for the $\pi/4$-shifted QPSK

- phase change: $\pm 45^\circ$ and $\pm 135^\circ$
- envelope variation is reduced.
- Unlike offset QPSK, it can be noncoherently detected.
\( \pi/4 \)-shifted QPSK

- Can be differentially encoded.
- **Example:**
  - \( \pi/4 \)-Differential Quaternary Phase Shift Keying (DQPSK) is used in IS-95 (standard for CDMA mobile communication)
Similar to other passband data transmission system, the function model of FSK is
Binary FSK

Symbols 1 and 0 are distinguished from each other by transmitting one of \(two \text{ sinusoidal waves}\) that differ in frequency.

\[
s_i(t) = \begin{cases} 
\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\
0 & \text{elsewhere}
\end{cases}
\]

where \(f_i = \frac{n+i}{T_b}\) for some fixed integer and \(i = 1,2\)

Note: the two sinusoidal waves must be orthogonal
It is a continuous-phase signal as phase continuity is always maintained, including the inter-bit switching times.

Called continuous-phase frequency-shift keying (CPFSK)

Example:

If $T_b = 1 \mu s$, $n = 9$ then $f_1 = 10 \text{MHz}$ and $f_1 = 11 \text{MHz}$.

For sending a symbol 1, the sinusoidal wave with $f_1$ completes ten cycles.
The transmitted signal can be written as

\[ s_1(t) = \sqrt{E_b} \phi_1(t) \] and

\[ s_2(t) = \sqrt{E_b} \phi_2(t) \]

where \( \phi_i(t) = \frac{2}{T_b} \cos(2\pi f_i t) \quad 0 \leq t < T_b \)
Therefore, the constellation of binary FSK is

\[ \phi_2(t) \]

\[ \phi_1(t) \]

Decision boundary

\[ \sqrt{E_b} \]

\[ \sqrt{2E_b} \]

\[ \sqrt{E_b} \]

Constellation of binary PSK

Decision boundary

\[ -\sqrt{E_b} \]

\[ \sqrt{E_b} \]

PB.70
The binary data sequence is first applied to an on-off level encoder, at the output of which symbol 1 is represented by a constant amplitude of $\sqrt{E_b}$ and symbol 0 is represented by zero volts.
Detection

The detector consists of two correlators with a common input, which are supplied with locally generated coherent signals. The correlator outputs are then subtracted.

\[ x(t) \times \phi_1(t) \rightarrow \int_0^T x_1 \]

\[ x(t) \times \phi_2(t) \rightarrow \int_0^T x_2 \]

\[ y = x_1 - x_2 \]

Decision device:

- 1 if \( y > 0 \)
- 0 if \( y < 0 \)
As the decision boundary is $\phi_1(t) = \phi_2(t)$,

choose 1 if $y > 0$
choose 0 if $y < 0$
Error Probability

The received signal is
\[ y = x_1 - x_2 \]
\[ = \int_0^{T_b} x(t)\phi_1(t)dt - \int_0^{T_b} x(t)\phi_2(t)dt \]

The mean of \( y \) is
\[ \bar{y} = \begin{cases} \sqrt{E_b} & \text{Symbol 1 was sent} \\ -\sqrt{E_b} & \text{Symbol 0 was sent} \end{cases} \]

The variance of \( y \) is
\[ \sigma^2 = \text{var}(x_1) + \text{var}(x_2) \]
\[ = \frac{N_o}{2} + \frac{N_o}{2} \]
\[ = N_o \]
Therefore, if symbol 0 was sent,

\[
f(y | 0) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(y - \bar{y})^2}{2\sigma^2}\right]
\]

\[
= \frac{1}{\sqrt{2\pi N_o}} \exp\left[-\frac{(y + \sqrt{E_b})^2}{2N_o}\right]
\]

and the error probability is

\[
p_{10} = P(y > 0|\text{symbol 0 was sent})
\]

\[
= \int_0^\infty \frac{1}{\sqrt{2\pi N_o}} \exp\left[-\frac{(y + \sqrt{E_b})^2}{2N_o}\right] dy
\]
Let \( z = \frac{y + \sqrt{E_b}}{\sqrt{2N_o}} \),

\[
p_{10} = \frac{2}{\sqrt{\pi}} \int_{\sqrt{E_b/2N_o}}^{\infty} \exp[-z^2] \, dz
\]

\[
= \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{2N_o}}\right)
\]

As \( p_{01} = p_{10} \),

\[
P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{2N_o}}\right)
\]

The error probability of binary PSK is

\[
p_e = \frac{1}{2} \text{erfc}\left(\frac{\sqrt{E_b}}{\sqrt{N_o}}\right).
\]

i.e., we have to double the bit energy-to-noise density ratio, \( E_b / N_o \), to maintain the same bit error rate as in a binary PSK system.
The general form of binary FSK is

\[ s_i(t) = \begin{cases} 
\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\
0 & \text{elsewhere}
\end{cases} \]

Consider \( f_1 - f_2 = 1/T_b \) and their arithmetic mean equals to \( f_c \)

\[ s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t \pm \pi t / T_b) + \text{ for symbol 1; - for symbol 0} \]

\[ = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \cos(\pm \pi t / T_b) - \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_c t) \sin(\pm \pi t / T_b) \]

\[ = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \cos(\pi t / T_b) \pm \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_c t) \sin(\pi t / T_b) \]
\[ s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \cos(\pi t / T_b) \pm \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_c t) \sin(\pi t / T_b) \]

- The in-phase component is independent of the input binary wave. The power spectral density of this component consists of two delta functions. (one delta function if baseband spectrum is considered)

- The quadrature component is related to the input binary wave. The power spectral density is

\[
\frac{8E_b \cos^2(\pi T_b f)}{\pi^2 (4T_b^2 f^2 - 1)}
\]
\[ s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \cos(\pi t / T_b) \pm \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_c t) \sin(\pi t / T_b) \]

- The average powers of the delta function adding up to one-half the total power of the binary FSK signal.

- The presence of these two discrete frequency components provides a means of synchronizing the receiver with the transmitter.
Normalized power spectral density, $S_B(f)/2E_b$

Binary PSK

Delta function (part of FSK spectrum)

Binary FSK

Normalized frequency, $fT_b$
M-ary Quadrature Amplitude Modulation (QAM)

Introduction

– PSK is usually limited to BPSK, QPSK and 8-PSK.
– For further reducing the transmission bandwidth, QAM is used.

– Example:

  Consider a voice signal with 3kHz bandwidth,
  • If analog amplitude modulation (AM) is used, the transmission bandwidth is 2x3kHz = 6kHz
  • If analog single sideband modulation (SSB) is used, the transmission bandwidth is 3kHz.
If digital method is used, the minimum sampling rate is $2 \times 3 \text{kHz} = 6 \text{kHz}$. If there are 256 levels for encoding, the data rate is 48kbps.

Referring to the diagram of PB.70, the transmission bandwidth for BPSK is

$$2/T = 2R_b = 96 \text{kHz}$$

Transmission bandwidth for 8-PSK is

$$2/T = 2R_b / \log_2 M = 32 \text{kHz}$$

Transmission bandwidth for 1024-QAM is

$$2/T = 2R_b / \log_2 M = 9.6 \text{kHz}$$
Generation

- **2-PAM Modulator**
  - Input: I, Q
  - Modulated Signal: QPSK

- **4-PAM Modulator**
  - Input: I, Q
  - Modulated Signal: 16-QAM

- **32-PAM Modulator**
  - Input: I, Q
  - Modulated Signal: 1024-QAM
The M-ary QAM signal is defined by

\[ s_k(t) = \sqrt{\frac{2E_b}{T}} a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T}} b_k \sin(2\pi f_c t) \quad 0 \leq t \leq T \]

\[ k = 0, \pm 1, \pm 2, ... \]

\[ = [s_1 \quad s_2] \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} \]

where

\[ \phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad 0 \leq t \leq T \]

\[ \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad 0 \leq t \leq T \]

\[ [s_{k1} \quad s_{k2}] = [a_k \sqrt{E_o} \quad b_k \sqrt{E_o}] \]
If there are $M$ symbols and $L = \sqrt{M}$, the $M$-ary square constellation can always be viewed as the Cartesian product a one-dimensional $L$-ary PAM constellation with itself.

Therefore, we have

$$\{a_i, b_i\} = \begin{bmatrix}
(-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\
(-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\
\vdots & \vdots & \cdots & \vdots \\
(-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1)
\end{bmatrix}$$
Example: Consider a 16-QAM, \( L=4 \)

\[
\{a_i \quad b_i\} = \begin{bmatrix}
(-L+1, L - 1) & (-L + 3, L - 1) & \cdots & (L - 1, L - 1) \\
(-L + 1, L - 3) & (-L + 3, L - 3) & \cdots & (L - 1, L - 3) \\
\vdots & \vdots & \ddots & \vdots \\
(-L + 1, -L + 1) & (-L + 3, -L + 1) & \cdots & (L - 1, -L + 1)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(-3,3) & (-1,3) & (1,3) & (3,3) \\
(-3,1) & (-1,1) & (1,1) & (3,1) \\
(-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\
(-3,-3) & (-1,-3) & (1,-3) & (3,-3)
\end{bmatrix}
\]
The signal constellation is
The encoding of the message is as follows:

- Two of the four bits, namely, the left-most two bits, specify the quadrant in the constellation plane in which a message point lies. Thus, starting from the first quadrant and proceeding counterclockwise, the four quadrant are represented by 11, 10, 00, and 01.

- The remaining two bits are used to represent one of the four possible lying within each quadrant of the plane.
Error Probability

The probability of correct detection for $M$-ary QAM is

\[ P_c = (1 - P_e')^2 \]

where $P_e'$ is the probability of symbol error for the corresponding $L$-ary PAM.

The probability of symbol error for $M$-ary QAM is

\[ P_e = 1 - P_c \]

\[ = 1 - (1 - P_e')^2 \]

\[ \approx 2P_e' \]

\[ = 2\left(1 - \frac{1}{\sqrt{M}}\right)\text{erfc}\left(\sqrt{\frac{E_o}{N_o}}\right) \]
As the transmitted energy in $M$-ary QAM depends on the particular symbol transmitted, the probability of symbol error is expressed in terms of the average value of the transmitted energy.

Assuming that the $L$ amplitude level levels of the in-phase or quadrature component are equally likely, we have

$$E_{av} = \frac{2(M-1)E_o}{3}$$

Therefore,

$$P_e = 2\left(1 - \frac{1}{\sqrt{M}}\erfc\left(\sqrt{\frac{3E_{av}}{2(M-1)N_o}}\right)\right)$$
Noncoherent systems

Introduction

– For coherent systems, the receiver is perfectly synchronized to the transmitter, and the only channel impairment is noise.

– In certain situations, the receiver cannot follow the change of the received signal phase.
  • Noncoherent detection is used.
Noncoherent binary FSK

The transmitted signal is

\[ s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \quad 0 \leq t \leq T_b \]

For the noncoherent detection of this frequency-modulated wave, the receiver consists of a pair of matched filters followed by envelope detectors.
Filter matched to $\cos(2\pi f_1 t)$, $0 \leq t \leq T_b$

Filter matched to $\cos(2\pi f_2 t)$, $0 \leq t \leq T_b$

Envelope detector

Sample at $t = T_b$

Comparison device

Sample at $t = T_b$

1 if $l_1 > l_2$

0 if $l_1 < l_2$
For example, if the received signal is
\[ x(t) = \cos(2\pi f_1 t + \theta) \]

The upper matched filter output is
\[
y(t) = \int_0^{T_b} x(\tau) \cos[2\pi f_1 (T - t + \tau)]d\tau
\]
\[
= \cos[2\pi f_1 (T - t)] \int_0^{T_b} x(\tau) \cos[2\pi f_1 \tau]d\tau - \sin[2\pi f_1 (T - t)] \int_0^{T_b} x(\tau) \sin[2\pi f_1 \tau]d\tau
\]

The envelope of the matched filter output is
\[
\left\{ \left[ \int_0^{T_b} x(\tau) \cos[2\pi f_1 \tau]d\tau \right]^2 + \left[ \int_0^{T_b} x(\tau) \sin[2\pi f_1 \tau]d\tau \right]^2 \right\}^{1/2}
\]
For example, the bit error rate for noncoherent binary FSK is

$$P_e = \frac{1}{2} \exp \left( - \frac{E_b}{2N_o} \right)$$
Differential Phase-shift keying

Introduction

– the noncoherent version of PSK
– differential encoding of the input binary wave

• To send symbol 0, advance the phase of the current signal waveform by 180°.
• To send symbol 1, the phase of the current signal waveform is unchanged.

– Provided that the unknown phase $\theta$ contained in the received wave varies slowly, the phase different between waveforms received in two successive bit intervals will be independent of $\theta$
Suppose the transmitted DPSK signal equals to
\[ \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b, \]

the transmitted signal for \( 0 \leq t \leq 2T_b \) is
\[ s_1(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \leq t \leq T_b \\ \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & T_b \leq t \leq 2T_b \end{cases} \]
if symbol 1 was sent

No phase change

Change in phase

\[ s_2(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \leq t \leq T_b \\ \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) & T_b \leq t \leq 2T_b \end{cases} \]
if symbol 0 was sent
The differential encoding process starts with an arbitrary first bit, serving as reference.

If the incoming binary symbol $b_k$ is 1, leave the symbol $d_k$ unchanged with respect to the previous bit.
If the incoming binary symbol $b_k$ is 0, change the symbol $d_k$ with respect to the previous bit.

Example,

\[
\begin{align*}
\{b_k\} & \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \\
\{d_{k-1}\} & \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \\
\{d_k\} & \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 & (d_k = b_k \oplus d_{k-1}) \\
\text{Output} & \quad 0 \quad 0 \quad \pi \quad 0 \quad 0 \quad \pi \quad 0 \quad 0 \quad 0 \quad 0 \\
\text{phase} & \\
\end{align*}
\]
Suppose the carrier phase is unknown and the initial phase is 0, the received signal is
\[ x(t) = A \cos(2\pi f_c t + \theta) \]
The output of the In-phase correlator is
\[
\int_0^{T_b} A \cos(2\pi f_c t + \theta) \cos(2\pi f_c t) dt = \frac{AT_b}{2} \cos \theta = A' \cos \theta
\]

and the output of the Quadrature-phase correlator is
\[
\int_0^{T_b} A \cos(2\pi f_c t + \theta) \sin(2\pi f_c t) dt = \frac{AT_b}{2} \sin \theta = A' \sin \theta .
\]

If the initial phase is 180°, the outputs are \(- A' \cos \theta\) and \(- A' \sin \theta\).
Therefore, the two received signal points are 
\((A'\cos \theta, A'\sin \theta)\) and \((-A'\cos \theta, -A'\sin \theta)\)
The receiver measures the coordinates \((x_{I0}, x_{Q0})\) at time \(t = T_b\) and \((x_{I1}, x_{Q1})\) at time \(t = 2T_b\).

Therefore,
\[
x_{I1}x_{I1} + x_{Q0}x_{Q1} > 0 \quad \text{if symbol } 1 \text{ was sent}
\]
\[
x_{I1}x_{I1} + x_{Q0}x_{Q1} < 0 \quad \text{if symbol } 0 \text{ was sent}
\]

**Error probability**
The bit error rate for DPSK is
\[
P_e = \frac{1}{2} \exp \left( -\frac{E_b}{N_0} \right)
\]
## Comparison of Digital Modulation Schemes

### Probability of Error

<table>
<thead>
<tr>
<th>Signaling Scheme</th>
<th>Bit Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherent BPSK</td>
<td>$\frac{1}{2} \operatorname{erfc}(\sqrt{\frac{E_b}{N_o}})$</td>
</tr>
<tr>
<td>Coherent QPSK</td>
<td>$\frac{1}{2} \operatorname{erfc}(\sqrt{\frac{E_b}{2N_o}})$</td>
</tr>
<tr>
<td>Coherent MSK</td>
<td>$\frac{1}{2} \exp(-\frac{E_b}{N_o})$</td>
</tr>
<tr>
<td>Coherent FSK</td>
<td>$\frac{1}{2} \exp(-\frac{E_b}{2N_o})$</td>
</tr>
<tr>
<td>DPSK</td>
<td>$\frac{1}{2} \exp(-\frac{E_b}{N_o})$</td>
</tr>
<tr>
<td>Noncoherent binary FSK</td>
<td>$\frac{1}{2} \exp(-\frac{E_b}{2N_o})$</td>
</tr>
</tbody>
</table>
Comparison of Digital Modulation Schemes

The bit error rate for all the systems decrease monotonically with increasing value of $E_b / N_0$. 
Bandwidth Efficiency

- Example: M-ary PSK for probability of symbol error $= 0.0001$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$(\text{Bandwidth})_M$</th>
<th>$(\text{Average Power})_M$</th>
<th>$(\text{Bandwidth})_{\text{Binary}}$</th>
<th>$(\text{Average Power})_{\text{Binary}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.34 dB</td>
<td>0.5</td>
<td>0.34 dB</td>
</tr>
<tr>
<td>8</td>
<td>0.333</td>
<td>3.91 dB</td>
<td>0.333</td>
<td>3.91 dB</td>
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<tr>
<td>16</td>
<td>0.25</td>
<td>8.52 dB</td>
<td>0.25</td>
<td>8.52 dB</td>
</tr>
<tr>
<td>32</td>
<td>0.2</td>
<td>13.52 dB</td>
<td>0.2</td>
<td>13.52 dB</td>
</tr>
</tbody>
</table>