EIE 441 Advanced Digital Communication

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References

Textbook:

References:

Assessment

Assessment:

Examination 60%

Continuous work
Test 15%
Practical & Assignments 25%

Syllabus

Week 1-2 Information Theory
Week 1 Introduction
Information Contents
Entropy

Week 2 Mutual Information
Channel Capacity
Syllabus

Week 3-5  Source Coding and Channel Coding
Week 3  Huffman Coding
        Lempel-Ziv Coding
Week 4  Linear Block Codes
Week 5  Cyclic Codes
        Convolutional Codes

Syllabus

Week 6-10  Baseband Transmission
Week 6  Matched filter
        Error rate of PAM systems
Week 7  Chung Yeung Festival
Week 8  Intersymbol Interference
        Nyquist’s Criterion
Week 9  Corrective-level Coding
        M-ary schemes
Syllabus

Week 10  Eye Patterns
         Digital Subscriber Lines
         Channel Equalization

Syllabus

Week 11-14  Passband Transmission
Week 11    Coherent PSK

Week 12    Hybrid Amplitude/Phase Modulations
           Coherent FSK

Week 13    Noncoherent FSK
           Differential PSK

Week 14    Comparison of Digital Modulation Schemes
Other arrangements

Tutorial
– followed by each topic
– week 2, 3, 5, 9 and 14

Laboratory
– week 9 to week 14

Test
– week 8

Assignment
– after each lecture

General communication system

The fundamental problem of communication is that of reproducing at one point (at the destination) either exactly or approximately a message selected at another point (source).

The actual message is one selected from a set of possible message. (Example: Select “1” or “0” in a binary system)
### Components

**Information source**
produces a message (or a sequence of messages) to be transmitted to the destination

**Transmitter**
operates on the message to produce a signal **suitable** for transmission over the channel

**Channel**
medium used to transmit the signal from transmitter to the receiver. Noise may be added.

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**Components**

**Receiver**
performs the reverse function of the transmitter

**Destination**
the person or device for which the message is intended
A. INFORMATION THEORY

Introduction
The purpose of a communication system is to carry information-bearing baseband signals from one place to another over a communication channel.

Information Theory deals with mathematical modeling and analysis of a communication system rather than with physical sources and physical channels.

Introduction
Examples:
- Baseband signals: Audio signals and video signals
- Channel: Optical fiber and free space

Questions:
- Information-bearing baseband signals: ?
- Signals containing no information: ?
The amount of information about an event is closely related to its probability of occurrence.

Messages containing knowledge of high probability of occurrence convey relatively little information.

Those messages containing knowledge of a low probability of occurrence convey relatively large amount of information.

\[ P \uparrow I \downarrow \quad P \downarrow I \uparrow \]

Let the probability of producing \( S_i \) be

\[ P(S_i) = P_i \quad \text{for} \quad P_i \geq 0 \quad \sum_i P_i = 1 \]
Example 1-2

Question

If a receiver receives the symbol $S_1$, how much information is received?

Case I: Consider $n = 1$ (only 1 symbol), $P_i = 1$. Answer: $S_1$ is transmitted for sure. Therefore, no information.

Case II: Consider $n > 1$, $P_1 = 1$ and $P_2 = P_3 = ... P_n = 0$. Answer: ?

Conclusion

Information is related to the probability of transmission of a symbol.

Probability = 1 $\Rightarrow$ No information
Example 3

Information from 2 independent source will be the sum of the information from each separately.

Example:
Consider \( n \geq 2 \), if a receiver receives a symbol \( S_1 \) and then a symbol \( S_2 \), the received information is

\[
I(S_1S_2) = I(S_1) + I(S_2)
\]

Example 3

Conclusion

Information of a message of \( M \) symbols = sum of information of each of the \( M \) symbols.

Question
Which function satisfies the above criteria (A.20 & A.18)?

a. first order polynomial: \( ax+b \)
b. sinusoidal function: cos or sin
c. logarithmic function: log
d. exponential function: \( e \)
Definition of Information

Consider a set of $M$ symbols (messages) $m_1, m_2, ..., m_M$ with probabilities of occurrence $P_1, P_2, ..., P_M$, respectively, and $P_1 + P_2 + ... + P_M = 1$

The amount of information or information content in the $k^{th}$ symbols

$$I(m_k) = -\log_2 P_k$$

Unit: bit

Example: $I(m_k) \to 0$ if $P_k \to 1$ (most likely event)
$I(m_k) \to \infty$ if $P_k \to 0$ (highly unlikely event)

Sum of information

Information of symbol $m_1$ + Information of $m_2$ + Information of symbol $m_3$ + ...

$$= I(m_1) + I(m_2) + ... + I(m_M)$$
$$= -\log P_1 - \log P_2 - ... - \log P_M$$
$$= \log \frac{1}{P_1} + \log \frac{1}{P_2} + ... + \log \frac{1}{P_M}$$
$$= \log \frac{1}{P_1 P_2 \cdots P_M}$$
$$= I(m_1 m_2 \cdots m_M)$$

Note: $A \log B = \log B^A$
Sum of Information

Therefore the information in $M$ one-symbol message is equal to the information in one $M$-symbol message

$I(m_k) :$ self information associated with symbol $m_k$
amount of information associated with symbol $m_k$
information content associated with symbol $m_k$

Example

A binary source $m_1 \& m_2$; $P(m_1) = \frac{1}{2}$; $P(m_2) = \frac{1}{2}$

$I(m_1) = I(m_2) = -\log_2 (1/2) = 1$ bit

Note: $\log_2 B = \frac{\log_{10} B}{\log_{10} 2}$
Example 5-6

A binary source $m_1 \& m_2$ ; $P(m_1) = \frac{1}{4}$ ; $P(m_2) = \frac{3}{4}$

$I(m_1) = -\log_2(1/4) = 2 \text{ bits}$

$I(m_2) = -\log_2(3/4) = 0.415 \text{ bits}$

Source alphabet = \{A,B,C,..,Y,Z\}

If each symbol is produced independently, with equal probability, the information content of each symbol is

$$-\log_2\left(\frac{1}{26}\right) = 4.7 \text{ bits}$$

Average Information Content (Entropy)

In practice, a source may not produce symbols with equal probabilities. Then each symbol will carry different amount of information.

Need to know: On average how much information is transmitted per symbol in a $M$ - symbols message? Or, the average information content of symbols in a long message.

Entropy
Entropy

Consider a source emits one of $M$ possible symbols $S_1, S_2, \ldots, S_M$ in a statistically independent manner with probabilities $P_1, P_2, \ldots, P_M$, respectively. (i.e. occurrence of a symbol at any given time does not influence the symbol emitted any other time.)

Assume that a total of $N$ symbols were emitted, self information of the $i^{th}$ possible symbol is

$$I(S_i) = -\log_2(P_i) \text{ bits}$$

$S_i$ will occur, on average, $P_i N$ times for $N \to \infty$.

Therefore, total information of the $N$-symbol message is

$$I_t = -\sum_{i=1}^{M} NP_i \log_2(P_i) \text{ bits}$$

The average information per symbol is $I_t / N = H$, and

$$H = -\sum_{i=1}^{M} P_i \log_2(P_i) \text{ bits/symbol}$$

$H$ is called the source entropy (or entropy of source).
### Maximum Entropy

The entropy is maximum when $P_1 = P_2 = \ldots = P_M$, i.e., it is equally likely for the next emitted symbol be any one of $\{S_i\}$.

The ratio $\frac{H}{H_{\text{max}}}$ is called the source efficiency or relatively entropy.

### Example

Source with 2 symbols, with probabilities $P_1 = \frac{6}{8}$, $P_2 = \frac{2}{8}$, $N=8$

Total information: $I_t = \sum_{i=1}^{2} NP_iI(S_i)$

$$= 8 \left( \frac{6}{8} \right) \log_2 \left( \frac{8}{6} \right) + 8 \left( \frac{2}{8} \right) \log_2 \left( \frac{8}{2} \right)$$

$$= 2.49 + 4$$

$$= 6.49 \text{ bits}$$

$$H = \frac{I_t}{N} = 0.811 \text{ bits/symbol}$$
Example

Source with symbols '0' and '1' and probabilities \( P_1 = \frac{1}{4}, \ P_2 = \frac{3}{4} \)

\[
H = \sum_{i=1}^{2} P_i I(S_i)
\]

\[
= -\left(\frac{1}{4}\right) \log_2 \left(\frac{1}{4}\right) - \left(\frac{3}{4}\right) \log_2 \left(\frac{3}{4}\right)
\]

\[
= 0.811 \text{ bits/symbol}
\]

Note: If \( P_1 = P_2 = 1/2 \), \( H = 1 \) bit/symbol (= \( H_{\text{max}} \))

Example

Source produces symbols from source \( \{S_1, S_2, S_3, S_4\} \)

with the following probabilities:

\[
P_1 = \frac{1}{8}, \ P_2 = \frac{3}{8}, \ P_3 = \frac{3}{8}, \ P_4 = \frac{1}{8}
\]

\[
H = -\sum_{i=1}^{4} P_i \log_2 (P_i)
\]

\[
= 1.8 \text{ bits/symbol}
\]

Note that if \( P_1 = P_2 = P_3 = P_4 = \frac{1}{4} \),

\[
H = H_{\text{max}} = \sum_{i=1}^{4} \frac{1}{4} \log_2 (4) = 2 \text{ bits/symbol}
\]
Example

Consider source alphabet \{0,1\}

Probability of producing symbol ‘0’ = \( p \), \( 0 < p < 1 \)

Probability of producing symbol ‘1’ = \( 1-p \)

\[
H = -P(0) \log_2 (P(0)) - P(1) \log_2 (P(1))
\]

\[
= p \log_2 \frac{1}{p} + (1-p) \log_2 \frac{1}{1-p}
\]

Example

\( H \) against \( p \) is

\( H \) reaches maximum when \( \frac{dH}{dp} = 0 \)

and we get \( p = \frac{1}{2} \); \( H_{\text{max}} = 1 \) bit/symbol
Example

In general, when there are $M$ symbols in the source alphabet, maximum entropy occurs when all symbols are equally probable, i.e. $P_1 = P_2 = \ldots = P_M$

As $P_1 + P_2 + \ldots + P_M = 1 \Rightarrow P_1 = P_2 = \ldots = P_M = \frac{1}{M}$

$$H_{\text{max}} = \sum_i P_i \log_2 \frac{1}{P_i}$$

$$= \sum_i \frac{1}{M} \log_2 M$$

$$= \log_2 M \text{ bits/symbol}$$

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Average information rate

When the symbols are emitted at a rate of $r_s$ symbols/sec, the average source information rate is

$$R = r_s H \quad \text{bits/sec.}$$
Example

A source produces one symbol out of 256 at a rate of 8000 symbols/sec. Determine the information rate.

\[ H = \sum_{256} P_i \log_2 \frac{1}{P_i} \]

\[ = \sum_{i} \frac{1}{256} \log_2 256 \]

\[ = \log_2 256 \]

\[ = 8 \text{ bits/symbol} \]

\[ R = 8 \times 8000 = 64 \text{ kbit/s (kbps)} \]

Example

An analog signal is bandlimited to \( B \) Hz, sampled at the Nyquist rate, quantized into 4 levels. The quantization levels \( Q_1, Q_2, Q_3, Q_4 \) are independent and occur with probabilities

\[ P_1 = P_2 = \frac{1}{8} \quad P_3 = P_4 = \frac{3}{8} \]

\[ H = \sum_{i} P_i \log_2 \frac{1}{P_i} = 1.8 \text{ bits/symbol} \]

\[ r_s = 2B \quad \text{symbols/sec (Nyquist rate).} \]

\[ R = r_s H = 3.6B \text{ bits/s} \]

If all quantified samples are equally probable,

\[ H = 2 \text{ bits/symbol and } R = r_s H = 4B \text{ bits/sec} \]
### Summary

Information content of a message:

\[ I(m_k) = -\log_2 P_k \text{ bit} \]

Entropy of a source:

\[ H = -\sum_{i=1}^{M} P_i \log_2 (P_i) \text{ bits/symbol} \]

Information rate of a source:

\[ R = r_s H \text{ bit/second} \]

### Application

If there is a source with \( M \) symbols, the simplest coding method is using constant codeword length \( L \) where

\[ \log_2 M \leq L \leq 1 + \log_2 M \]

Example: \( M = 9 \)

\( \log_2 9 = 3.2 \) and choose \( L = 4 \)

\[ S_0 : 0000 \quad S_1 : 0001 \quad \cdots \quad S_8 : 1000 \]

However, if \( P(S_i) \) are not all equal, the entropy \( H < 3.2 \)

We might find a coding method with average codeword length \( L \) where

\[ L \approx H \]