The parity check bits of a (7,4) linear block code are generated by
\[
\begin{align*}
    c_1 &= m_1 + m_2 + m_3 \\
    c_2 &= m_1 + m_2 + m_4 \\
    c_3 &= m_1 + m_3 + m_4 \\
\end{align*}
\]
where \( m_1, m_2, m_3 \) and \( m_4 \) are the message digits and the codeword is \([c_1, c_2, c_3, m_1, m_2, m_3, m_4]\).

(a) Find the generator matrix \( G \) and the parity check matrix \( H \) for this code.

(b) Find the error-correcting and detecting capabilities of this code.

(c) If there is an error in the \( i \)th bit of the received codeword, show that the syndrome is equal to the \( i \)th row of the transpose of the parity check matrix.

\[ \text{Hint: Assume } e = [0 \ \cdots \ e_i \ \cdots \ 0] \text{ and show that } s = i \text{th row of the transpose of the parity check matrix.} \]

(d) If there is one error bit in the received codeword \([1001011]\), find the correct codeword using the result of part (c).

Please submit to your class representative by **10:30am, 7 October 2002**