Control of Bifurcation in Current-Programmed DC/DC Converters:
A Reexamination of Slope Compensation *

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Abstract — This paper reexamines the conventional current-mode control strategy as applied to dc/dc converters in the light of “avoiding bifurcation”. This alternative viewpoint permits convenient selection of parameter values to guarantee stable operation. Slope compensation is viewed as a means to keep the system sufficiently remote from the first bifurcation point. It is shown that excessive bifurcation clearance is accompanied by undesirably slow dynamical response. A variable ramp compensation is proposed to dynamically adjust the slope magnitude such that the system is kept clear of bifurcation yet responding sufficiently fast during transients.

I. INTRODUCTION

Power electronics circuits are designed for stable operations. In most practical situations, the required stable operation is a period-1 operation. Thus, any effective design automatically has to avoid the occurrence of bifurcation for the range of variation of the parameters [1].

Bifurcations and chaos have been observed and analysed for various kinds of power electronics circuits [2]–[4]. For systems that have been shown to bifurcate when a certain parameter is changed, the design problem is, in a sense, addressing the “control of bifurcation”. In this paper we reexamine the current-mode control of dc/dc converters [5]–[6] and the use of compensating ramp from the point of view of bifurcation control.

II. REVIEW OF PERIOD-DOUBLING BIFURCATION IN CURRENT-MODE CONTROLLED DC/DC CONVERTERS

We begin with a brief review of the typical period-doubling bifurcation in the boost converter [2, 4, 7]. Fig. 1 shows the schematic of a boost converter under a typical current-programming control. Let \( I_{set} \) be the reference current level which defines the peak inductor current. During a switching cycle, from \( t = nT \) to \( t = (n + 1)T \), the inductor current ramps up linearly until it reaches \( I_{set} \), and then ramps down [6]. We let \( i_n \) and \( i_{n+1} \) be the inductor current at \( t = nT \) and \( (n + 1)T \) respectively. Denote also the output voltage (voltage across the output capacitor) by \( v \). By inspecting the slopes of the inductor current, we get

\[
\frac{I_{set} - i_{n+1}}{1 - D} = \frac{v - V_{in}}{L} \quad \text{and} \quad \frac{I_{set} - i_n}{DT} = \frac{V_{in}}{L}
\]

where \( D \) is the duty cycle defined in the usual way. Combining the above equations, we have the following iterative function:

\[
i_{n+1} = \left(1 - \frac{v}{V_{in}}\right)i_n + \frac{i_{n+1}v}{V_{in}} + \frac{(v - V_{in})T}{L}
\]

\[
\delta i_{n+1} = \left(\frac{-D}{1 - D}\right)\delta i_n + O(\delta i_n^2)
\]

Clearly, the characteristic multiplier or eigenvalue, \( \lambda \), is given by

\[
\lambda = \frac{-D}{1 - D}
\]

which must fall between -1 and 1 for stable operation. In particular, the first period-doubling occurs when \( \lambda = -1 \) which corresponds to \( D = 0.5 \). Consistent with what is well known in power electronics, current-mode controlled converters must operate with the duty cycle set below 0.5 in order to maintain a stable period-1 operation [6].

In the application of current-mode control, the error signal derived from the output voltage is often used to modify \( I_{set} \) directly (not the duty cycle as in the case of voltage mode PWM control). It is thus helpful to look at the period-doubling bifurcation in terms of the current reference \( I_{set} \). Specifically we can express the “criterion of no bifurcation”, \( D < 0.5 \), in terms of \( I_{set} \) by using the steady-state equation relating \( R \), \( D \) and \( I_{set} \). For the boost converter, the equivalent criterion of no bifurcation is

\[
I_{set} < \left[\frac{V_{in}}{R} \left(\frac{\text{DRT}}{2L} + \frac{1}{1 - D^2}\right)\right]_{D=0.5} = I_{set,c}
\]
Figure 2: Illustration of current-mode control with compensating ramp showing inductor current in a boost converter. $V_{in}$ is input voltage, $v$ is output voltage, $i$ is inductor current and $m_c$ is compensating slope.

which can be derived from the power-balance equation

$$\left(I_{ref} - \frac{\Delta i}{2}\right) V_{in} = \frac{V_{in}^2}{(1-D)^2 R} \tag{6}$$

where $\Delta i = DTV_{in}/L$ and all symbols have their usual meanings. The critical value (upper bound) of $I_{ref}$ for the uncompensated case is thus given by

$$I_{ref,c} = \frac{V_{in}}{R} \left(\frac{RT}{4L} + 4\right) \tag{7}$$

Hence, period-doubling occurs when $I_{ref}$ exceeds the above-stated limit. To prevent period-doubling, we must therefore control $I_{ref,c}$, i.e., the power level. Indeed, the use of a compensating ramp, as we will see, is to raise the upper bound of $I_{ref}$, thereby widening the operating range.

III. CONTROL OF BIFURCATION BY RAMP COMPENSATION

A. Design to Avoid Bifurcation

With compensation, the reference current is first subtracted from an artificial ramp before it is used to compare with the inductor current, as shown in Figure 2. By inspecting the inductor current waveform, we obtain the modified iterative function for the inner loop dynamics as

$$\delta i_{n+1} = \left(\frac{M_c}{1 + M_c} - \frac{D}{(1-D)(1+M_c)}\right) \delta i_n + O(\delta i_n^2) \tag{8}$$

where $M_c = m_c L/V_{in}$ is the normalized compensating slope, and $m_c$ is defined in Figure 2. Now, using (8), we get the eigenvalue or characteristic multiplier, $\lambda$, for the compensated inner loop dynamics as

$$\lambda = \frac{M_c}{1 + M_c} - \frac{D}{(1-D)(1+M_c)} \tag{9}$$

Hence, by putting $\lambda = -1$, the critical duty cycle, at which the first period-doubling occurs, is obtained, i.e.,

$$D_c = \frac{M_c + 0.5}{M_c + 1} \tag{10}$$

Figure 3: (a) Specific boundary curves $I_{ref,c}$ versus $R$ for current-mode controlled boost converter without compensation and with normalized compensating slope $M_c = 0.2$, 0.4, and 0.8; (b) specific boundary curves $I_{ref,c}$ versus $V_{in}$ for current-mode controlled boost converter without compensation and with compensation slope $M_c = 0.2$, 0.4, and 0.8; (c) boundary curves with normalized parameters.
Figure 4: Output voltage v of boost converter under current-mode control, (a) without compensation; (b) with fixed ramp compensation \( M_c = 0.2 \); (c) with fixed ramp compensation \( M_c = 0.4 \); (d) with variable ramp compensation. Input voltage is stepped down from 5V to 3.9V at \( t = 5 \text{ms} \), and back to 5V at \( t = 40 \text{ms} \).

Using (5) and the above expression for \( D_c \), we get the critical value of \( I_{set} \) for the compensated system as

\[
I_{set} < I_{set,c} = \frac{V_{in}}{R} \left[ \frac{RT}{2L} \frac{M_c + 0.5}{M_c + 1} + 4(M_c + 1)^2 \right] \]  

(11)

Note that \( I_{set,c} \) increases monotonically as the compensating slope increases. Hence, it is obvious that compensation effectively provides more margin for the system to operate without running into the bifurcation region. Figure 3(b) shows some plots of the critical value of \( I_{set} \) against \( R \), for a few values of \( M_c \). In a likewise manner, we may consider the input voltage variation and produce a similar set of design curves that provide information on the choice of the compensating slope for ensuring no bifurcation for a range of input voltage. This is shown in Figure 3(b). Also, for a general reference, the boundary curves in terms of normalized parameters are shown in Fig. 3(c).

B. Effects on Dynamical Response

The transient response of a power converter can be compromised if bifurcation is kept too remote in order to give a large safe margin, especially when the operating range required is very wide, since guaranteeing "no bifurcation" for a wide range of parameter values would inevitably make the safe margin excessively large at one extreme end of the range.

It is therefore of interest to study the effect of the presence of compensating ramp on the closed-loop dynamics of the overall system. We will take a simple averaging approach to derive the eigenvalues of the stable closed-loop system, mainly to reveal the transient speed for different values of the compensating slope. Using standard averaging procedure, we can write down the state equations for the boost converter, and its normalized form is as follows:

\[
\begin{align*}
\frac{dx}{d\tau} &= -x + \frac{(1 - d)y}{\gamma} \\
\frac{dy}{d\tau} &= \frac{(1 - d)x}{\zeta} + \frac{E}{\zeta}
\end{align*}
\]

(12)  

(13)

where the normalized variables and parameters are defined by \( x = v/V_{set} \), \( y = i/(V_{set}/R) \), \( E = V_{in}/V_{set} \), \( \tau = t/T \), \( \gamma = CR/T \), and \( \zeta = L/RT \). The closed-loop control can be modelled approximately by (see Figure 2)

\[
i + \frac{\Delta i}{2} \approx \frac{P_o}{V_{in}} + I_{set} - k(v - V_{set}) - m_c\Delta T
\]

(14)

where \( P_o \) is the output power (i.e., \( P_o = V_o^2/R \)) and \( k \) is the voltage feedback gain. This control equation can readily be translated, in terms of the normalized parameters, into

\[
d = 1 - \frac{2\zeta \left( \frac{1}{R} - y - \kappa(x - 1) \right)}{x - E(1 - 2M_c)}
\]

(15)

where \( \kappa = kR \). Hence, putting (15) into the state equations, we get the closed-loop state equations which can then be used to study
the closed-loop dynamics. Specifically, we can obtain the Jacobian matrix $J_p$ as

$$J_p = \begin{bmatrix}
\frac{2\gamma(2\gamma + 1 + 2M_e\gamma)}{E(1 + 2M_e\gamma)} & \frac{2\gamma(2\gamma + 1 + 2M_e\gamma)}{E(1 + 2M_e\gamma)} \\
\frac{2\gamma(2\gamma + 1 + 2M_e\gamma)}{E(1 + 2M_e\gamma)} & \frac{2\gamma(2\gamma + 1 + 2M_e\gamma)}{E(1 + 2M_e\gamma)}
\end{bmatrix}_{x=x, y=y}$$

(16)

Note that in the steady state, $x = X = 1$ and $y = Y = 1/E$. Suppose the eigenvalues of the closed-loop system, $\lambda_i$, are complex. The real part of $\lambda_i$ can be easily found as

$$\text{Re}(\lambda_i) = \frac{-E^2(1 + 2M_e) + 2E(\gamma + M_e) - 2\alpha\gamma}{2E^2(1 + 2M_e)\gamma}$$

(17)

In practice, $E < 1$ and $\gamma \gg 1$. Also, for stable operation, $\alpha$ has to be small enough so that $\text{Re}(\lambda_i) < 0$. Under such condition, we can readily show that $\frac{1}{E\gamma} \text{Re}(\lambda_i) < 0$. In other words, the transient becomes slower as $M_e$ increases.

**Remarks** — It should be reiterated that the overall dynamics is modeled by (12) and (13), while the inner current loop dynamics is described by (8). Inconsistent conclusion may be drawn from studying the two dynamical equations. Specifically, from (8), we observe that increasing $M_e$ will make the inner loop dynamics “faster”. However, the foregoing analysis of the overall system dynamics reveals that for some range of parameter values, the system actually becomes slower as $M_e$ increases. Obviously, (8) is inadequate for the purpose of examining the overall system dynamics.

C. **Computer Simulations**

Suppose the circuit parameters are: $L = 1.5mH$, $C = 20\mu F$, $R = 400\Omega$, $V_i = V_{in} = 8.2V$, and $T = 100\mu s$. The system has an appropriate integral control to adjust the steady-state level of $i_{out}$ in the event of a change in $V_{in}$. Such an arrangement is common in current-mode control of dc/dc converters. Now, from

$$(1 - D)V_i = V_{in},$$

we know that the uncompensated system will walk out of the stable region if $V_{in}$ is reduced to below about 4.1V, since the output voltage is kept at 8.2V by the integral control. We may apply compensation to restore stability.

Our simulation is done with exact piecewise switched model of the boost converter, with the current reference given by a typical proportional-integral control formula, which ensures that the output is regulated at 8.2V under varying input voltage. The results of simulation show that, without compensation (Figure 4 (a)) , the system becomes chaotic when the input voltage falls to 3.9V. Moreover, with compensation, the system remains stable. We further observe, from Figures 4 (b) and (c), that excessive compensation lengthens the response time. Further elaboration will be given in the next subsection.

D. **Variable Ramp Compensation**

In order to keep bifurcation away while maintaining fast response, the control should incorporate a special function that dynamically adjusts the compensating ramp. The aim is to give just enough compensation under all input voltage conditions. Thus, the controller may contain, in addition to a conventional proportional-integral gain, a variable ramp generator providing necessary, but not excessive, compensation. For this simplified scenario (i.e., fixed load and output voltage), the compensating ramp needs only be controlled according to

$$M_e(V_{in}) \geq \frac{V}{2V_{in}} - 1$$

(18)

which is derivable from (10). Figure 4 (d) shows the simulated waveforms for the boost converter under such control.

The series of waveforms shown in Figs. 4 (b) through (d) serve to illustrate the effect of applying ramp compensation to the system dynamics. Specifically, from (18), the value of $M_e$ needed for a 3.5V input is about 0.14, and no compensation is at all needed for a 5V input. Thus, with $M_e = 0.4$ (Fig. 4 (c)), the system is very much over-compensated and hence suffers a much slower transient compared to the case with $M_e = 0.2$ (Fig. 4 (b)). Furthermore, even for $M_e = 0.2$, the system is still over-compensated when the input is 5V. Thus, we can see a much faster response with the variable ramp compensation (Fig. 4 (d)) since it applies just enough compensation for the 3.5V input and none for the 5V input.

IV. **Conclusion**

In this paper the conventional current-mode control strategy as applied to dc/dc converters is reexamined in the light of “avoiding bifurcation”. We have illustrated the function of slope compensation as a means to keep the system sufficiently remote from the first bifurcation point, thereby maintaining stability in the traditional sense. It has also been shown that excessive bifurcation clearance is accompanied by undesirably slow dynamical response. A variable ramp compensation is proposed to dynamically adjust the slope magnitude such that the system is kept clear of bifurcation yet responding sufficiently fast during transients.

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**REFERENCES**


