Fast-Scale Instability in a PFC Boost Converter Under Average Current Mode Control *

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Abstract

This paper describes the fast-scale instability in a power-factor-correction (PFC) boost converter under a conventional average current mode control. The converter is operated in continuous mode. Computer simulations and theoretical analysis are performed to study the effects of the time-varying input voltage under the variation of some chosen parameters on the qualitative behaviour of the system. It is found that fast-scale instability may occur during a line cycle. The results provide useful information for the design of PFC boost converters for stable operation.

Keywords — Power-factor-correction boost converter, average current-mode control, bifurcation, fast-scale instability.

1 Introduction

Power-factor-correction (PFC) pre-regulators are mandatorily used in power supplies for medium to high power applications [1]–[4]. In practice, the boost converter has been a favourable and popular choice for PFC. The usual technique in the control of PFC pre-regulators is to directly program the input current (inductor current), for example, using current-mode control, so that its waveform closely tracks the input voltage’s. In this paper, we study the conventional average current-mode controlled boost PFC pre-regulator in which the envelope of the average input current is forced to vary sinusoidally [2] and a feedback circuit adjusts the amplitude of the envelope according to the load power demand. Such kinds of control methods have been widely used in industry [5]. The major motivation of our work is the problem of subtle distortion of the line current found in some intervals of the line cycle. In this paper, we focus our attention on the bifurcation phenomena related to fast-scale instability which can cause distortion to the line current [6].

The paper is organized as follows. In Section 2, we outline the control method of the system and derive the state equations that describe the system dynamics. In Section 3, we study the bifurcation phenomena related to fast-scale instabilities by simulation. Then, detailed analysis is given in Section 4, where a discrete-time map, its Jacobian and the characteristic multipliers are derived. Finally, we give the conclusion in Section 5.

2 System Description

A typical PFC boost converter is shown in Fig. 1. The closed-loop system has an outer voltage loop and an inner current loop. The voltage loop provides the reference for the inner current loop. In

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most practical implementations, the action of the current-mode control forces the average of the input current (inductor current) to follow cycle-by-cycle a pre-defined waveform, which is proportional to the input voltage. The amplitude of the pre-defined waveform is set to give the required input power. Let $e(t)$ be the input voltage. If $e(t) = E_p | \sin 2\pi f_m t | = E_p | \sin \theta |$, where $E_p$ is the peak input voltage, $f_m$ is the line frequency and $\theta$ is the phase angle, then the pre-defined waveform for the input current (reference current), $i_{\text{ref}}$, should be

$$i_{\text{ref}} = g E_p | \sin \theta | = I_p | \sin \theta |,$$

where $g$ is the input conductance controlled via feedback, $I_p$ is the peak value of the reference current. The average of the input current (inductor current) will always follow the pre-set envelope, $i_{\text{ref}}$. Therefore, a near unity power factor is maintained. Usually, the switching frequency is much higher than the line frequency. This condition is assumed throughout the paper.

![Block diagram of a PFC boost pre-regulator.](image)

Figure 1: Block diagram of a PFC boost pre-regulator.

### 2.1 Overview of Control

The amplitude of the input current is controlled via feedback, which employs a typical proportional and integral (PI) control method [7]-[8]. The resulting input conductance $g$ is set at

$$g = k_1 (V_{\text{ref}} - v) + k_2 \int (V_{\text{ref}} - v) \, dt,$$

where $k_1$ is the proportional feedback gain, $k_2$ is the integral feedback gain, $V_{\text{ref}}$ is the reference voltage and $v$ is the output voltage (after filtering). During normal operation, the output voltage $v$ should approach the desired reference voltage $V_{\text{ref}}$.

As our aim is to force the average of $i_L$ to follow $i_{\text{ref}}$, a control voltage signal can be described as

$$v_{\text{con}} = k_3 (i_{\text{ref}} - i_L) + k_4 \int (i_{\text{ref}} - i_L) \, dt,$$

where $i_{\text{ref}}$ is governed by equation (1), $k_3$ is the proportional feedback gain and $k_4$ is the integral feedback gain. This control voltage signal $v_{\text{con}}$ is compared with a sawtooth signal to generate a PWM signal that drives the switch, as shown in Fig. 1. The sawtooth signal is represented by

$$v_{\text{ramp}} = V_L + (V_U - V_L) \frac{t \mod T}{T},$$

where $V_L$ and $V_U$ are the lower and upper voltage limits of the ramp, and $T$ is the switching period. The PWM output is “high” when the control voltage is greater than $v_{\text{ramp}}$, and is “low” otherwise.
2.2 State Equations for Simulation

The boost converter is a second-order circuit comprising an inductor, a diode, a switch and a load resistance connected in parallel with a capacitor. When the converter is operating in continuous mode, only two switch states are possible during a switching cycle, namely, (i) $S$ is on and $D$ is off; (ii) $S$ is off and $D$ is on. When it is operating in discontinuous mode, an additional switch state is possible, i.e., (iii) $S$ is off and $D$ is off. The state equations corresponding to these switch states are generally given by

\[
\begin{align*}
\dot{x} &= A_1 x + B_1 e & \text{for } S \text{ on and } D \text{ off} \\
\dot{x} &= A_2 x + B_2 e & \text{for } S \text{ off and } D \text{ on} \\
\dot{x} &= A_3 x + B_3 e & \text{for } S \text{ off and } D \text{ off}
\end{align*}
\]

where $e$ is the input voltage, $x$ is the state vector defined as $x = [v \ i_L]^T$, and the $A$'s and $B$'s are given by

\[
A_1 = \begin{bmatrix} -\frac{1}{C(R + r_C)} & 0 \\ 0 & -\frac{1}{L} \frac{r_L}{R} \end{bmatrix}, A_2 = \begin{bmatrix} -\frac{1}{C(R + r_C)} & \frac{R}{L} & \frac{R}{r_C(1 + r_C + r_L)} \\ \frac{R}{L} & -\frac{1}{L} \frac{r_L}{R} + \frac{1}{L} \frac{r_C}{R + r_C} \\ \frac{1}{L} \frac{r_C}{R + r_C} & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} -\frac{1}{C(R + r_C)} & 0 \\ 0 & 0 \end{bmatrix}.
\]

(6)

\[
B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \frac{L}{L} \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

(7)

The above state equations, in conjunction with the control equations discussed earlier, will be used in the simulation study. In our study, the converter is intended to work in continuous mode, but the simulation should cater for possible entry into discontinuous mode.

3 Fast-scale Instability and Bifurcation Phenomena by Computer Simulations

Since we are primarily concerned with system stability in relation to the time varying input voltage, we focus our attention on the dynamical behaviour of the system subject to input voltage variation. Our simulation is based on the state equations derived in Section 2. Essentially, for each set of parameter values, time-domain cycle-by-cycle waveforms are generated by solving the appropriate linear equation in a sub-interval of time, according to the state of the switch $S$. After the transient period, we capture the steady-state time-domain waveforms of the inductor current and output voltage. The circuit parameters used in our simulations are shown in Table 1.

<table>
<thead>
<tr>
<th>Circuit Components</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching Period $T$</td>
<td>12.5 $\mu$s</td>
</tr>
<tr>
<td>Line Period $T_{in}$</td>
<td>0.02 s</td>
</tr>
<tr>
<td>Input Voltage $e$</td>
<td>110 V rms</td>
</tr>
<tr>
<td>Output Voltage $v$</td>
<td>280 V</td>
</tr>
<tr>
<td>Reference Voltage $V_{ref}$</td>
<td>280 V</td>
</tr>
<tr>
<td>Inductance $L, ESR r_L$</td>
<td>1.0 mH, 0.05 $\Omega$</td>
</tr>
<tr>
<td>Capacitance $C, ESR r_C$</td>
<td>200 $\mu$F, 0.01 $\Omega$</td>
</tr>
<tr>
<td>Load Resistance $R$</td>
<td>200 $\Omega$</td>
</tr>
</tbody>
</table>

Table 1: Component values and steady-state voltages used in simulations.

A large number of time-domain waveforms and trajectories have been generated. In the following, only representative figures are shown, which serve to exemplify the main findings concerning the dynamical behaviour of the system.
In normal operation, the input current tracks the reference current \( i_{\text{ref}} \) very well, and a near unity power factor can be achieved. In order to observe the change in dynamical behaviour clearly, we collect the sampled state variables at \( t = nT \) in the steady state. This is essentially the stroboscopic map at the switching frequency.

For a certain range of parameters, a fast-scale instability may occur within a line cycle. Such instability manifests itself as a period-doubling bifurcation at the switching frequency. Figure 2 shows various scenarios with different values of the proportional feedback gain \( k_3 \). Figure 2(a) shows the bifurcation-free operation. Figure 2(b) shows the operation with fast-scale instability in some intervals of the line cycle. Figure 2(c) shows the operation with fast-scale instability in the whole line cycle. In Fig. 2(b), fast-scale instabilities are observed clearly in some intervals of the line cycle. Figures 2(d)-(f) show the corresponding stroboscopic map of Figs. 2(a)-(c). Figures 3(a)-(c) show the corresponding phase portraits of sampled inductor current versus output voltage. When \( k_3 \) is larger than a certain value, we observe period-doubling bifurcations at the switching frequency. There are two bifurcation points in a line cycle. We denote the two values of \( \theta \) at which bifurcations occur as critical angles, \( \theta_{\text{A}} \) and \( \theta_{\text{B}} \). When \( k_3 \) becomes even larger, we observe repeated period-doubling bifurcations. When \( v/E_p \) is set at a relatively large value or \( L/R \) is set at a relatively small value, we observe a very similar scenario.

The previous figures show the scenarios in particular instants. In order to summarize the results, we record the critical angles \( \theta_{\text{A}} \) and \( \theta_{\text{B}} \) (i.e., the phase angles at which period-doubling bifurcation occurs) under a range of values of \( v/E_p, L/R \) and \( k_3 \), as shown in Fig. 4. In Figs. 4(a)-(c), the solid line represents \( \theta_{\text{A}} \) (the critical angle in the first quarter line cycle) while the dotted line represents \( \theta_{\text{B}} \) (the critical angle in the second quarter line cycle). Figure 5 shows an alternative view of the critical angles under variations of \( v/E_p \) and \( L/R \).

![Time-domain waveforms of \( i_L \)](image)

Figure 2: Time-domain waveforms of \( i_L \). (a) Bifurcation-free operation; (b) operation with fast-scale instability in some intervals of the line cycle; (c) operation with fast-scale instability in the whole line cycle; (d)-(f) Corresponding sampled time-domain waveforms of (a)-(c). From (a) to (c), value of \( k_3 \) increases.
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4 Analysis

From the foregoing simulation study, we have identified period-doubling bifurcation in certain parameter ranges. In this section, we try to analyze the bifurcation in term of a suitable discrete-time map [9]–[11]. In normal operation, the input voltage $e(t)$ of the system is time varying in a periodic fashion. For ease of analysis, we assume that the input voltage $e(t)$ is a constant with a switching period. We first derive its discrete-time map. Then we study the dynamical behaviour of the system using this map.

4.1 Derivation of Discrete-time Map

We let $x$ be the state vector, and $d$ be the duty cycle of the converter. The discrete-time map that we aim to find takes the following form:

$$x_{n+1} = f(x_n, d_n, e_n)$$

where subscript $n$ denotes the value at the beginning of the $n$th cycle, i.e., $x_n = x(nT)$. For the closed-loop system, we also need to find the feedback equation that relates $d_n$ to $x_n$ and $e_n$.

The state equations are given in (5) for different switch states. Assuming the system is operating in continuous mode, we have two switch states. They are as follows.

1. For $nT < t \leq nT + d_nT$, $S$ is turned on.
2. For $nT + d_nT < t \leq (n+1)T$, $S$ is turned off.
Figure 5: Summary of the critical angles under different values of \( v/E_p \) and \( L/R \).

In each switch state, the state equation is \( \dot{x} = A_j x + B_j e \), where \( j = 1, 2 \). For each state equation, we can derive the solution by stacking up the solutions, i.e.,

\[
x_{n+1} = \Phi_2((1-d_n)T) \Phi_1(d_n T)x_n + \Phi_2((1-d_n)T)(\Phi_1(d_n T) - 1) A_1^{-1} B_1 e_n + (\Phi_2((1-d_n)T) - 1) A_2^{-1} B_2 e_n
\]

where \( 1 \) is the unit matrix, and \( \Phi_j(\xi) \) is the transition matrix corresponding to \( A_j \) and is given by

\[
\Phi_j(\xi) = e^{A_j \xi} = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} A_j^k \xi^k, \quad \text{for} \quad j = 1, 2.
\]

For the boost converter, we have \( B = B_1 = B_2 \). Hence, (9) can be rewritten as

\[
x_{n+1} = \Phi_2((1-d_n)T) \Phi_1(d_n T)x_n + \Phi_2((1-d_n)T)(\Phi_1(d_n T) - 1) A_1^{-1} B e_n + (\Phi_2((1-d_n)T) - 1) A_2^{-1} B e_n.
\]

Our next step is to find the feedback relation that connects the duty cycle and the state variables. Assume the output voltage is well regulated and \( q \) is a constant approximately. Now, the control voltage \( v_{\text{con}} \), as given before in (3), can be rewritten as

\[
v_{\text{con}} = U + k_3 i_{\text{ef}} + \kappa^T x \quad (12)
\]

where \( U \) is a constant and the gain vector \( \kappa \) is

\[
\kappa^T = [0 \quad -k_3].
\]

With a sufficiently high switching frequency, \( i_{\text{ef}} \) within a switching period can be approximated by a linear approximation

\[
i_{\text{ef}} = I_p \sin \theta + m_c t, \quad (14)
\]

where \( 0 < \theta \leq \pi \), \( I_p \sin \theta \) is a constant and \( m_c (=2\pi f_m I_p \cos \theta) \) is the slope of the reference current, which can be a positive or negative value. The situation is thus equivalent to the ramp compensation in the typical current-mode control. The idea is illustrated in Fig. 7. The ramp function can also be rewritten as

\[
v_{\text{ramp}} = \alpha + \beta (t \mod T) \quad (15)
\]

where \( \alpha \) and \( \beta \) are constants. To find the defining equation for the duty cycle, we first note that the switch is turned off when \( v_{\text{con}} = v_{\text{ramp}} \). Now, we define \( s(x_n, d_n) \) as

\[
s(x_n, d_n) \overset{\text{def}}{=} v_{\text{con}} - v_{\text{ramp}} = U + k_3 i_{\text{ef}} + \kappa^T x(d_n T) - (\alpha + \beta d_n T)
\]

\[
= U + k_3 i_{\text{ef}} + \kappa^T [\Phi(d_n T)x_n + (\Phi(d_n T) - 1) A_1^{-1} B e_n] - (\alpha + \beta d_n T). 
\]

(16)
Thus, $S$ is turned off when
\[ s(x_n, d_n) = 0. \]  
Solving (17), $d_n$ can be obtained. Combining with (11), we have the discrete-time iterative map for the closed-loop system.

### 4.2 Derivation of the Jacobian

The Jacobian plays an important role in the study of dynamical systems [12–13]. The essence of using a Jacobian in the analysis of dynamical systems lies in the capture of the dynamics in the small neighbourhood of an equilibrium point or orbit (stable or unstable). We will make use of this conventional method to examine the bifurcation phenomena observed in Section 3. But before we move on, we need to find the necessary expressions that enable the Jacobian to be computed.

Suppose the equilibrium point is given by $x(nT) = X_Q$. The Jacobian of the discrete-time map evaluated at the equilibrium point can be written as follows [9–11]:
\[ J(X_Q) = \left. \frac{\partial f}{\partial x_n} - \frac{\partial f}{\partial d_n} \left( \frac{\partial s}{\partial d_n} \right)^{-1} \right|_{x_n = X_Q} \]  
\[ \frac{\partial f}{\partial x_n} = \Phi_2((1 - d_n)T)\Phi_1(d_nT). \]  
(18)

Using (11) and (16), we can find all the derivatives in (18). First, $\partial f / \partial x_n$ can be found from (11), i.e.,
\[ \frac{\partial f}{\partial x_n} = \Phi_2((1 - d_n)T)\Phi_1(d_nT). \]  
(19)

Also, direct differentiation gives $\partial f / \partial d_n$ as
\[ T\Phi_2((1 - d_n)T) \left[ \Phi_1(d_nT)(A_1x_n + Be_n) - A_2(\Phi_1(d_nT)x_n + (\Phi_1(d_nT) - 1)A_1^{-1}Be_n) - Be_n \right]. \]  
(20)

From (16), we obtain $\partial s / \partial x_n$ as
\[ \frac{\partial s}{\partial x_n} = \kappa^T\Phi_1(d_nT) \]  
(21)

and $\partial s / \partial d_n$ as
\[ \frac{\partial s}{\partial d_n} = T\kappa^T\Phi_1(d_nT)(A_1x_n + Be_n) + k_m T - \beta T. \]  
(22)

Now, putting all the derivatives into (18) gives an expression of $J(X_Q)$ that can be computed numerically. Numerical algorithms can now be developed for computing $J(X_Q)$ and hence the characteristic multipliers.

### 4.3 Characteristic Multipliers and Period-doubling Bifurcation

The Jacobian derived in the foregoing subsection can be used to evaluate the dynamics of the system. Here, we study the loci of the characteristic multipliers (also called eigenvalues), the aim being to identify possible bifurcation scenarios as the input voltage varies with time. To find the characteristic multipliers, we solve the following polynomial equation in $\lambda$, whose roots actually give the characteristic multipliers:
\[ \det[\lambda I - J(X_Q)] = 0 \]  
(23)

We will pay special attention to the movement of the characteristic multipliers as we vary the phase angle $\theta$. Any crossing from the interior of the unit circle to the exterior indicates a bifurcation. In particular, if a real characteristic multiplier goes through $-1$ as it moves out of the unit circle, a period-doubling bifurcation occurs [12–13].

Using (18), we can study the loci of characteristic multipliers numerically. Here, we notice that one of the characteristic multipliers crosses $-1$ as we vary $\theta$. This implies a period-doubling bifurcation, as our simulation reveals earlier in Section 3.
Figure 6: Comparison of critical angles found analytically with those from simulation \((L/R=5 \times 10^{-6} \text{s})\).

Figure 7: Inductor current and reference current (a) when \(m_c\) is zero; (b) when \(m_c\) is negative; (c) when \(m_c\) is positive.

4.4 Comparison of Critical Angles Found Analytically with Those from Simulation

In this section, we will make use of the Jacobian and loci of characteristic multipliers found previously. In a line cycle, the input voltage is \(E_p|\sin \theta|\), where \(0 < \theta \leq \pi\). For a range of values of \(v/E_p\), we vary the phase angle \(\theta\) and record the values of \(\theta\) when one of the characteristic multipliers reaches \(-1\). In fact, these are the critical angles where period-doubling occurs. We compare this result with those obtained by computer simulations \((L/R=5 \times 10^{-6} \text{s})\), as shown in Fig. 6. Clearly, our analysis matches well with the simulation results.

5 Conclusion

A popular choice for PFC applications, the boost converter is worth studying to a greater depth. This paper reports a fast-scale instability in a PFC boost pre-regulator under average current mode control. We have observed and analyzed the period-doubling bifurcation in the system with sinusoidally varying input voltage. The study facilitates convenient selection of parameter values to guarantee stable operation. The results are useful for practical design of PFC regulators to ensure bifurcation-free operation.
References


