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MAXIMUM GRAY-SCALE IMAGE ENTROPY SEGMENTATION

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ABSTRACT

In recent years, image segmentation has been studied from the information point of view [2-5]. Similar to the design of artificial neural networks [6], information about the grayscale image and the true scene is usually utilized in image segmentation. Two common types of information are the gray-level values and some a priori knowledge about the scene or the grayscale image. To illustrate this point, consider thresholding where the image pixels having a gray-level value greater than a certain threshold value are classified into a specific class; otherwise they are classified into another [7]. In this approach, the gray-level values of the image are utilized explicitly to determine the threshold value and to perform actual classification for each pixel. The implicit a priori knowledge about the image is that the gray-level values of a class of pixels usually have statistics (mean or standard deviation) sufficiently different from those of the other class. The success of a thresholding method usually depends on the validity of such a priori information.

In this paper, the Gray-scale Image Entropy (GIE) is defined and utilized to measure the amount of information in a grayscale image, which is a representation of an underlying true scene. An iterative segmentation algorithm produces a segmented image in each step. The segmented image is employed as a postulated true scene to estimate the GIE. When the estimated GIE is maximized, the segmented image approximates the true scene optimally in an entropy sense. Two image segmentation algorithms are proposed and described. The first algorithm thresholds the image and then applies two-dimensional filtering to the thresholded image. The second algorithm is a template matching method where a series of templates are taken as the estimated true scene to calculate the GIE until the optimum template is obtained. The feasibility of the maximum GIE principle and these segmentation algorithms are illustrated by simulation results.

INTRODUCTION

In image segmentation problems, a true scene, which consists of some objects of interest in a background, is probed by an imaging system to form a grayscale image. When the imaging process is done digitally, the grayscale image is usually a rectangular grid of picture elements (pixels) each of which has a quantized gray-level value. An image segmentation process operates on the gray-scale image to determine to which class (object or background) an image pixel belongs. Image segmentation has been an important image processing step for many higher level image processing operations such as pattern recognition and object identification [1]. The relationship between the true scene, the grayscale image, and the segmented image, is illustrated in Figure 1.

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Consider a scene consisting of \( J \) different objects in a background. There are \( J \) different classes of pixels. A grayscale image is captured to represent this scene. The mission of image segmentation is to identify to which class each pixel of the image belongs.

Assume that the size of the \( j^{th} \) class pixels relative to the entire image is \( \alpha_j, 0<\alpha_j<1 \) for \( j=0, 1, \ldots, J-1 \). If a pixel \( X \) is selected randomly from this image, the probability that it belongs to the \( j^{th} \) class is \( P(X=x_j) = \alpha_j \). The probabilities for all classes collectively describe a probability distribution \( (\alpha_0, \alpha_1, \ldots, \alpha_{J-1}) \). According to Shannon's definition for entropy [8], the average

\[ H(X) = -\sum_{j=0}^{J-1} \alpha_j \log \alpha_j \]
uncertainty about which class an image pixel belongs to is given by the entropy:

\[ H(X) = -\sum_{j=0}^{L-1} \alpha_j \log \alpha_j \text{ bits/pixel} \quad (1) \]

where the \( \log(.) \) function is assumed to be of base 2. \( H(X) \) as defined in Eq.(1) is denoted as the scene entropy.

3. GRAY-SCALE IMAGE ENTROPY

When a pixel X in a gray-scale image is observed to have a gray-level value of g, where \( g \in [0,1,\ldots,L-1] \), the a posteriori probability that this pixel belongs to class j is given by \( f_j(g) = P(X=j \mid g) \), \( j=0,1,\ldots,L-1 \). The uncertainty about which class an image pixel belongs to after its gray-level value has been observed to be g is given by:

\[ H(X \mid g) = -\sum_{j=0}^{L-1} f_j(g) \log f_j(g) \quad (2) \]

The average uncertainty about which class an image pixel belongs to after its gray-level value has been observed is given by the weighted sum:

\[ H(X \mid G) = \sum_{g=0}^{L-1} p(g) H(X \mid g) \quad (3) \]

where \( p(g) \) is the probability that a pixel has a gray-level value of g, i.e., the \( p(.) \) function is probability density function (p.d.f.) of the gray-scale image.

If the gray-scale image is observed, the uncertainty about the scene must have been reduced when compared to the case that no observation is made. The reduction in uncertainty is equivalent to the information supplied by the gray-scale image. Based on standard results of information theory, it can be proved mathematically that \( H(X \mid G) \) is always smaller than, or at most equal to, \( H(X) \) \[9\]. We denote such a reduction in uncertainty as \( I(X ; G) \), the gray-scale image entropy (GIE):

\[ I(X ; G) = H(X) - H(X \mid G) \quad (4) \]

Defined in this way, the GIE is a quantitative measure of the amount of information contained in a gray-scale image. When the gray-scale image is a good representation of the true scene, the GIE will be large; otherwise it will be small.

Both the true scene and the gray-scale image need to be known to calculate the GIE for an image according to Eq.(4). The term \( H(X) \) is calculated from the knowledge of the true scene according to Eq.(1). The term \( H(X \mid G) \) is calculated according to Eqs.(2) and (3), when the p.d.f. of the gray-scale image and the terms \( f_j(g) \) are known. The gray-scale image p.d.f. can be obtained in a standard way by scanning the gray-scale image. The value of \( f_j(g) \) can be calculated as the ratio of the number of class j pixels that have a gray-level value of g to the number of pixels that have a gray-level value of g.

4. MAXIMUM GRAY-SCALE IMAGE ENTROPY SEGMENTATION

To calculate the GIE, both the true scene and the gray-scale image must be known. In most image segmentation applications, the true scene is usually unknown and hence the GIE cannot be calculated. Therefore the GIE of a gray-scale image cannot be used directly in image segmentation. However, if an approximated scene which approximates the true scene is obtained by some means, its GIE with respect to the gray-scale image can be calculated and can be taken as an estimation of the true GIE. If the approximated scene resembles closely the true scene, the gray-scale image will contain a large amount of information and hence the estimated GIE will be large. If the approximated scene does not resemble the true scene very well, the gray-scale image will not have much correlation with the approximated scene and hence the estimated GIE will not be very large. Based on the estimated GIE, it is possible to judge whether the approximated scene is a good approximation to the true scene or not.

If several segmented images are obtained in a segmentation process, the best one may be identified as the one that results in the largest estimated GIE when the segmented images are taken to be the approximated scenes. From this reasoning, a block diagram for the maximum gray-scale image entropy (MGIE) segmentation principle can be shown in Figure 2.

![Fig.2 MGIE segmentation principle](image)

Based on this principle, different image segmentation algorithms can be designed using different methods in estimating the true scene and incorporating various a priori information.

4.1 Maximum Gray-scale Image Entropy Segmentation by Thresholding and 2D Filtering (MGIE-TH&2D)

A binary segmentation scheme is considered. The a priori knowledge about the scene is that it is composed of an object in a background. The object pixels are spatially contiguous, so are the background pixels.

The image is firstly thresholded at some threshold value. In the thresholded image, there will be "holes" in the object and background regions. Many of these holes may be removed by performing a two-dimensional (2D-) filtering on the thresholded image. In this algorithm, an eight-neighborhood filter mask as shown in Figure 3 is employed to perform 2D-filtering on the thresholded image. The 2D-filtering rule is that if more than 6 neighborhood pixels are with a different...
classification, the classification of the current pixel will be changed; otherwise its classification will remain unchanged.

- ○: current pixel
- X: neighborhood pixel

Fig. 3 Neighborhood scheme for 2D filtering

After 2D-filtering, the thresholded image would have been "cleaned up" and would approximate the true scene better. The 2D-filtered thresholded image is then taken as an approximated scene and the GIE estimated. This operation is repeated for all possible threshold values until the maximum estimated GIE is obtained. This will result in an optimum, in an entropic sense, segmented image. The block diagram of such an image segmentation algorithm is shown in Figure 4.

The detailed flow-chart of this algorithm is as shown in Figure 5.

**Fig. 4** MGIE-TH&2D algorithm block diagram

**Fig. 5** MGIE-TH&2D flow-chart

4.2 Maximum Gray-scale Image Entropy Segmentation by Template Matching (MGIE-TM)

With more a priori knowledge about the scene or the image, better image segmentation performance could be achieved in general. In many industrial applications, it is not unusual that the size and shape of the object to be located in an image are known a priori, but that the object location remains unknown. In such cases, it is possible to form a "template" to approximate the true scene. Such a template can be used to estimate the GIE. With the template moving around the gray-scale image, different estimated GIE values are obtained. The optimum position of the template will be found when the estimated GIE is maximum. This template matching approach to image segmentation is illustrated in the block diagram as shown in Figure 6.

**Fig. 6** Maximum GIE segmentation by template matching

5. SIMULATION STUDIES

To investigate the characteristics of the maximum gray-scale image entropy principle and the two segmentation algorithms as described in Sections 4.1 and 4.2, a computer simulation study is performed.

First of all, some true scenes are synthesized. Based on certain model of image formation, the corresponding gray-scale images are synthesized. These images are then used in three different studies. The first is to investigate the dependence of the estimated GIE value on the degree of true scene approximation. The second is to investigate the performance of the MGIE-TH&2D segmentation algorithm. Finally, the performance of the GMIE-TM algorithm is investigated.

5.1 GIE Dependence on the Degree of True Scene Approximation

The dependence of the estimated GIE value on the extent of true scene approximation is investigated. To quantify the extent to which a true scene is approximated, random flip-flap noises with a certain error probability are added to the true scene to generate the approximated scene. For instance, when the flip-flap noise error probability is 0.1, each pixel of the true scene is changed in classification with probability 0.1. After an approximated scene is generated, its GIE is calculated and the GIE values are plotted against the flip-flap noise error probability.

In the present study, the true scene consists of a circular object situated in the centre of a square background. The size of the image is 256x256 pixels. The size of the circular object is half that of the total
image. The gray-scale image is generated according to a bi-Gaussian model that has been commonly employed in segmentation studies [10]. The mean value of the gray-level values of the object pixels is 100, while that of the background pixels is 180. The standard deviation of the gray-level values is 20 for both classes of pixels.

As an illustration, the true scene, the gray-scale image, a corrupted image with flip-flap noise error probability of 0.1, and 0.4, respectively, are shown in Figure 7(a-d).

![Fig.7 Flip-flap noise on true scene. (a) true scene, (b) gray-scale image, (c) true scene corrupted by flip-flap noise of error probability 0.1, (d) true scene corrupted by flip-flap noise of error probability 0.4](image)

After the approximated scene is generated, the GIE is calculated with reference to the generated gray-scale image. Figure 8 shows the relationship between the estimated GIE against the flip-flap noise error probability ranging from 0 to 1 in steps of 0.05.

![Fig.8 Flip-flap noise effect on true scene](image)

From these results, it can be noted that the estimated GIE gets smaller as the flip-flap noise error probability gets larger, until the error probability becomes greater than 0.5. When the flip-flap noise error probability is $p$ where $p$ is larger than 0.5, it is similar to the case with flip-flap noise error probability $1-p$, except that the labels for object and background are swapped. Hence the GIE will be similar to the case when the flip-flap noise error probability is $1-p$. This explains why the curve is more or less symmetrical about flip-flap noise error probability of 0.5. From the curve in Figure 8, the largest GIE is obtained when the flip-flap noise error probability is either 0 or 1. When the flip-flap noise error probability is 0, the true scene approximation is exact. When the flip-flap noise error probability is 1, the GIE is also maximum due to the symmetry effect as has been explained.

From other simulation results obtained in this experiment, it is found that good approximation to the true scene will usually be accompanied by a large estimated GIE. Hence the estimated GIE can be used as a reliable parameter to evaluate whether a scene approximation is good or not.

### 5.2 Performance of MGIE-TH&2D Algorithm

In this investigation, the MGIE-TH&2D segmentation algorithm is implemented to segment some synthesized images with Gaussian p.d.f. for the gray-level values of the object and background pixels. For illustration purposes, consider the case when the object is circular with size 0.5 of the total image size. The mean gray-level value for the object pixels is 80 and that for the background pixel is 180. The standard deviation of the gray-level values of both the object pixels and the background pixels is 20. For comparison purposes, the thresholding results by Otsu’s method [11] and Kapur’s method [12] are also obtained, the three results are presented in Table 1 below.

![Table 1](image)

<table>
<thead>
<tr>
<th>Otsu</th>
<th>Kapur</th>
<th>MGIE-TH&amp;2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold value</td>
<td>130</td>
<td>129</td>
</tr>
<tr>
<td>no. of segmentation error pixels</td>
<td>386</td>
<td>381</td>
</tr>
</tbody>
</table>

From these results, and other results in the same study, it is found that very good segmentation results can be obtained by the MGIE-TH&2D method. The small number of segmentation error pixels in this case is attributed to the application of 2D-filtering after thresholding so that a lot of “holes” in the object and background are removed.

### 5.3 Performance of MGIE-TM Algorithm

In this study, a true scene containing an object is synthesized. The gray-scale image is synthesized according to the bi-Gaussian model as discussed in previous sections. As an illustration, consider the case where the object is a square with size 0.5 of that of the total image size. The object pixels and background pixels have Gaussian p.d.f. for their gray-level values. The mean of the gray-level values of the square pixels is 80, and that of the background pixels is 180. The standard deviation of the gray-level values of both the object and background pixels is 50. The true scene and the gray-scale image is shown in Figures 9(a) and 9(b), respectively.

![Fig.9 (a)True scene, (b)gray-scale image](image)
A template, which is a binary image with the same object as that in the true scene but displaced by \( X \) and \( Y \) pixels in the horizontal and vertical directions, respectively, is then formed. The GIE is calculated. The displacement parameters \( X \) and \( Y \) range from -20 to 20, giving a total of 1681 different templates, and hence 1681 GIE values. The GIE values are plotted against \((X,Y)\) in Figure 10.

From the shape of the plot in Figure 10, it can be seen that when the displacement is large, the estimated GIE is small. The largest GIE occurs when the displacement is \((0,0)\), i.e., when the template is aligned exactly. Hence it can be seen that this method is quite effective in determining the correct position of the template.

Other object shapes that have been tested in this experiment include ellipse, circle, and triangle. All these tests have shown that good results can be obtained.

6. CONCLUSION

The principle of maximum GIE has been utilized in designing two different image segmentation algorithms employing different \( a \) priori information. The MGIE-TH&2D algorithm relies on the \( a \) priori information that pixels from the same class tend to cluster spatially. The MGIE-TM algorithm requires the \( a \) priori knowledge of the size and shape of the object in a true scene. It is shown by simulation results that the maximum GIE principle can be put into practical use.

7. REFERENCES