Tutorial Problems: Bipolar Junction Transistor (Basic BJT Amplifiers)

Part A. Common-Emitter Amplifier

1. For the circuit shown in Figure 1, the transistor parameters are $\beta = 100$ and $V_A = \infty$. Design the circuit such that $I_{CQ} = 0.25$ mA and $V_{CEQ} = 3$ V. Find the small-signal voltage gain $A_v = v_o / v_s$. Find the input resistance seen by the signal source $v_s$.

![Figure 1](image)

Solution:

For dc analysis, the capacitors $C_C$ and $C_E$ both act as open circuit.

Given the desired operating point $I_{CQ} = 0.25$ mA and $V_{CEQ} = 3$ V, we have:

$$0 - V^- = I_{bE} R_B + V_{BE(on)} + I_{EQ} (R_S + R_E)$$

$$= \frac{I_{CQ}}{\beta} R_B + V_{BE(on)} + \left(\frac{1 + \beta}{\beta}\right) I_{CQ} (R_S + R_E)$$

$$0 - (-5) = \left(\frac{0.25}{100}\right) (50) + 0.7 + \left(\frac{101}{100}\right) (0.25) (0.1 + R_E)$$

$$\Rightarrow R_E = 16.43 \text{ (k}\Omega\text{)}$$

$$V^+ - V^- = I_{CQ} R_C + V_{CEQ} + I_{EQ} (R_S + R_E)$$

$$5 - (-5) = (0.25) R_C + 3 + \left(\frac{101}{100}\right) (0.25) (0.1 + 16.43)$$

$$\Rightarrow R_C = 11.30 \text{ (k}\Omega\text{)}$$
The small-signal parameters are:

\[
\begin{align*}
    r_n &= \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.25} = 10.4 \text{ (kΩ)} \\
    g_m &= \frac{I_{CQ}}{V_T} = \frac{0.25}{0.026} = 9.6154 \text{ (mA/V)} \\
    r_o &= \frac{V_A}{I_{CQ}} = \infty
\end{align*}
\]

For small-signal ac analysis, all dc voltages and capacitors act as *short circuit*. The following expressions are obtained:

\[
\begin{align*}
    v_o &= -\beta i_b R_C \\
    v_s &= i_b r_n + (1 + \beta) i_b R_S \\
    A_v &= \frac{v_o}{v_s} = -\frac{\beta R_C}{r_n + (1 + \beta)R_S} \\
    &= -\frac{(100)(11.30)}{10.4 + (101)(0.1)} \\
    &= -55.12
\end{align*}
\]

The input resistance \( R_i \) seen by the signal source \( v_s \) is:

\[
R_i = R_b || R_b \\
= R_b || (r_n + (1 + \beta)R_S) \\
= 50 || 20.5 \\
= 14.54 \text{ (kΩ)}
\]

2. Consider the circuit shown in Figure 2. The transistor parameters are \( \beta = 100 \) and \( V_A = 100 \text{ V} \). Determine \( R_i, A_v = v_o / v_s \) and \( A_i = i_o / i_s \). 

![Figure 2](image)
Solution:

A dc analysis is performed to determine the dc operating point by treating all capacitors as *open circuit*.

\[
V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC} = \left( \frac{15}{15 + 27} \right) (9) = 3.2143 \text{ (V)}
\]

\[
R_{TH} = R_1 || R_2 = \frac{(15)(27)}{15 + 27} = 9.6429 \text{ (kΩ)}
\]

\[
I_{bQ} = \frac{V_{TH} - V_{BE(on)}}{R_{TH}} = \frac{3.2143 - 0.7}{9.6429 + (101)(1.2)} = 19.22 \text{ (μA)}
\]

\[
I_{CQ} = \beta I_{bQ} = 1.922 \text{ (mA)}
\]

\[
I_{EQ} = (1 + \beta) I_{bQ} = 1.941 \text{ (mA)}
\]

\[
V_{CEQ} = V_{CC} - I_{CQ} R_C - I_{EQ} R_E = 9 - (1.922)(2.2) - (1.941)(1.2) = 2.44 \text{ (V)}
\]

The small-signal parameters are:

\[
r_e = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.922} = 1.353 \text{ (kΩ)}
\]

\[
g_m = \frac{I_{CQ}}{V_T} = \frac{1.922}{0.026} = 73.923 \text{ (mA/V)}
\]

\[
r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.922} = 52.029 \text{ (kΩ)}
\]

For small-signal ac analysis, all dc voltages and capacitors act as *short circuit*. The following small-signal ac equivalent circuit is obtained:

![Small-signal model of transistor circuit (*g_m V_π = β i_b*)](image)

\[
v_o = -\beta i_b \left( R_L || R_C || r_o \right)
\]

\[
V_\pi = \frac{R_{TH} || r_\pi}{R_S + R_{TH} || r_\pi} \quad v_s = i_v r_\pi
\]

\[
A_v = \frac{v_o}{v_s} = -\left( \frac{\beta \left( R_L || R_C || r_o \right)}{r_\pi} \right) \left( \frac{R_{TH} || r_\pi}{R_S + R_{TH} || r_\pi} \right) = -\frac{(100)(1.027)}{1.353} \left( \frac{1.187}{10 + 1.187} \right) = -8.05
\]
The input resistance $R_i$ is:

$$R_i = R_{TH} \parallel r_\pi$$

$$= (9.6429) \parallel (1.353) = 1.187 \text{ (kΩ)}$$

3. The parameters of the transistor in Figure 3 are $β = 100$ and $V_A = 100$ V.
   (a) Find the dc voltages at the base and emitter terminals.
   (b) Find $R_C$ such that $V_{CEQ} = 3.5$ V.
   (c) Assuming $C_C$ and $C_E$ act as short circuits, determine the small-signal voltage gain $A_v = v_o / v_s$.
   (d) Repeat part (c) if a 500 Ω source resistor is in series with the $v_s$ signal source.

![Figure 3](image-url)
Solution:

(a)

A dc analysis is performed to determine the dc operating point by treating all capacitors as open circuit.

\[ I_{CQ} = \frac{\beta}{1+\beta} I_{EQ} = \left( \frac{100}{1+100} \right)(0.35) = 0.347 \, \text{mA} \]

\[ I_{BQ} = \frac{I_{EQ}}{1+\beta} = \frac{0.35}{1+100} = 3.47 \, \mu\text{A} \]

\[ V_B = 0 - I_{BQ}R_B = -\left(3.47 \times 10^{-3}\right)(10) = -0.0347 \, \text{V} \]

\[ V_E = V_B - V_{BE(\text{on})} = -0.737 \, \text{V} \]

(b)

Given \( V_{CEQ} \) is desired to be 3.5 V, hence:

\[ V^* = V_E + V_{CEQ} + I_{CQ}R_c \]

\[ R_c = \frac{V^* - V_E - V_{CEQ}}{I_{CQ}} \]

\[ = \frac{5 - (-0.737) - 3.5}{0.347} = 6.45 \, \text{k}\Omega \]

(c)

The small-signal parameters are:

\[ r_n = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.347} = 7.493 \, \text{k}\Omega \]

\[ g_m = \frac{I_{CQ}}{V_T} = \frac{0.347}{0.026} = 13.346 \, \text{mA/V} \]

\[ r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.347} = 288.184 \, \text{k}\Omega \]

Using the small-signal ac equivalent circuit, the following expressions are obtained:

\[ v_o = -\beta i_o \left( R_c \parallel r_o \right) \]

\[ V_{\pi} = -\frac{R_B \parallel r_n}{R_s + R_B \parallel r_n} v_s = i_s r_{\pi} \]

\[ A_v = \frac{v_o}{v_s} = -\frac{\beta \left( R_c \parallel r_o \right)}{r_n} \left( \frac{R_B \parallel r_n}{R_s + R_B \parallel r_n} \right) \]

\[ = -\frac{(100)(6.309)}{7.493} \left( \frac{4.283}{0.1 + 4.283} \right) = -82.28 \]
(d)

If the source resistor is changed to 500 \( \Omega \), the new value of \( A_v \) is:

\[
A_v = \frac{v_o}{v_s} = \frac{\beta (R_c \parallel r_s)}{r_s} \left( \frac{R_s \parallel r_s}{R_s + R_B \parallel r_s} \right)
\]

\[
= \frac{(100)(6.309)}{7.493} \left( \frac{4.283}{0.5 + 4.283} \right) = -75.40
\]

Therefore the voltage gain \( A_v \) decreases as the source resistance \( R_S \) increases due to a larger voltage drop across the source resistor.

4. The transistor in the circuit in Figure 4 has a dc current gain of \( \beta = 100 \).
   (a) Determine the small-signal voltage gain \( A_v = v_o / v_s \).
   (b) Find the input and output resistances \( R_i \) and \( R_o \).

![Figure 4](image)

**Solution:**

(a)

A dc analysis is performed to determine the dc operating point by treating all capacitors as open circuit.

\[
V^+ = I_{EQ} R_E + V_{EB(on)} + I_{BQ} R_B
\]

\[
I_{BQ} = \frac{V^+ - V_{EB(on)}}{R_B + (1 + \beta) R_E} = \frac{4 - 0.7}{5 + (101)(5)} = 6.47 \text{ (\mu A)}
\]

\[
I_{CQ} = \beta I_{BQ} = (100)(6.47 \times 10^{-3}) = 0.647 \text{ (mA)}
\]

\[
I_{EQ} = (1 + \beta) I_{BQ} = 0.654 \text{ (mA)}
\]

\[
V_{SECQ} = V^+ - V^- - I_{EQ} R_E - I_{CQ} R_C = 4 - (-6) - (0.654)(5) - (0.647)(4) = 4.14 \text{ (V)}
\]
The small-signal parameters are:

\[
    r_\pi = \frac{BV_T}{I_{CQ}} = \frac{(100)(0.026)}{0.647} = 4.019 \text{ (k}\Omega\text{)}
\]

\[
    g_m = \frac{I_{CQ}}{V_T} = \frac{0.647}{0.026} = 24.885 \text{ (mA/V)}
\]

\[
    r_o = \frac{V_A}{I_{CQ}} = \infty
\]

Using the small-signal ac equivalent circuit, the following expressions are obtained:

\[
    v_o = -\beta i_b \left( R_L \parallel R_C \parallel r_o \right)
\]

\[
    V_\pi = \frac{R_B \parallel r_\pi}{R_s + R_B \parallel r_\pi} v_s = i_b r_\pi
\]

\[
    A_v = \frac{v_o}{v_s} = -\beta \left( R_L \parallel R_C \parallel r_o \right) \left( \frac{R_B \parallel r_\pi}{R_s + R_B \parallel r_\pi} \right)
\]

\[
    = -\frac{(100)(2)}{4.019} \left( \frac{2.228}{1+2.228} \right) = -34.35
\]

(b)

The input resistance \( R_i \) is:

\[
    R_i = R_B \parallel r_\pi
\]

\[
    = (5)\parallel (4.019) = 2.228 \text{ (k}\Omega\text{)}
\]

To calculate the output resistance \( R_o \), the signal source \( v_s \) is short-circuited and this gives \( i_b = 0 \). The following equation can be written by KCL at node \( v_o \):

\[
    -i_o + \frac{v_o}{R_C} = 0
\]

\[
    \frac{v_o}{i_o} = R_o = R_C
\]

\[
    = 4 \text{ (k}\Omega\text{)}
\]
Part B. Common-Collector Amplifier (Emitter Follower)

5. The transistor parameters for the circuit in Figure 5 are $\beta = 180$ and $V_A = \infty$.
   (a) Find $I_CQ$ and $V_{CEQ}$.
   (b) Plot the dc and ac load lines.
   (c) Calculate the small-signal voltage gain.
   (d) Determine the input and output resistances $R_{ib}$ and $R_o$.

![Figure 5](image)

Solution:

(a)

For dc analysis, the capacitors $C_{C1}$ and $C_{C2}$ act as open circuit.

\[
V_{TH} = \frac{R_2}{R_1 + R_2} (V^+ - V^-) + V^- = \left(\frac{10}{10 + 10}\right)(18) + (-9) = 0 \text{ (V)}
\]

\[
R_{TH} = R_1 \parallel R_2 = \frac{(10)(10)}{10 + 10} = 5.0 \text{ (kΩ)}
\]

\[
I_{BQ} = \frac{V_{TH} - V_{BE(oo)}}{R_{TH} + (1 + \beta)R_E} = \frac{0 - 0.7}{5 + (181)(0.5)} = 86.91 \text{ (μA)}
\]

\[
I_{CQ} = \beta I_{BQ} = 15.644 \text{ (mA)}
\]

\[
I_{EQ} = (1 + \beta)I_{BQ} = 15.731 \text{ (mA)}
\]

\[
V_{CEQ} = V^+ - V^- - I_{EQ}R_E = 9 - (-9) - (15.731)(0.5) = 10.13 \text{ (V)}
\]

(b)

The dc load line is given by:

\[
V^+ - V^- = V_{CE} + I_E R_E \approx V_{CE} + I_C R_E \text{ (for large } \beta \text{)}
\]

\[
I_C = \frac{V_{CE}}{R_E} + \frac{V^+ - V^-}{R_E} = -2V_{CE} + 36
\]
The ac load line is given by:

\[ v_{ce} + i_c \left( R_E \parallel R_L \right) = 0 \]

\[ i_c \approx i_c = -\frac{v_{ce}}{R_E \parallel R_L} = -5.333v_{ce} \]

(c)

The small-signal parameters are:

\[ r_e = \frac{B V_T}{I_{CQ}} = \frac{(180)(0.026)}{15.644} = 0.299 \text{ (k}\Omega\text{)} \]

\[ g_m = \frac{I_{CQ}}{V_T} = \frac{15.644}{0.026} = 601.692 \text{ (mA/V)} \]

\[ r_o = \frac{V_o}{I_{CQ}} = \infty \]

The small-signal ac equivalent circuit becomes:


\[ v_o = (1 + \beta) i_b \left( R_L \parallel R_E \right) \]

\[ \Rightarrow \frac{v_o}{i_b} = (1 + \beta) \left( R_L \parallel R_E \right) = (181)(0.1875) = 33.9375 \]

\[ v_b = V_x + v_o = i_b r_x + (1 + \beta) i_b \left( R_L \parallel R_E \right) \]

\[ \Rightarrow \frac{v_b}{i_b} = R_{ib} = r_x + (1 + \beta) \left( R_L \parallel R_E \right) = 0.299 + 33.9375 = 34.2365 \]

\[ v_b = \frac{R_{ib} \parallel R_1 \parallel R_2}{R_s + R_{ib} \parallel R_1 \parallel R_2} v_s \]

\[ \Rightarrow \frac{v_b}{v_s} = \frac{R_{ib} \parallel R_1 \parallel R_2}{R_s + R_{ib} \parallel R_1 \parallel R_2} = \frac{4.3628}{1 + 4.3628} = 0.8135 \]

\[ \frac{v_o}{v_s} = \frac{v_o}{i_b} \times \frac{i_b}{v_b} \times \frac{v_b}{v_s} = \frac{(1 + \beta)(R_E \parallel R_L)}{R_{ib}} \times \frac{R_{ib} \parallel R_1 \parallel R_2}{R_s + R_{ib} \parallel R_1 \parallel R_2} \]

\[ = \frac{(1 + \beta)(R_E \parallel R_L)}{R_{ib}} \left( \frac{R_{ib} \parallel R_1 \parallel R_2}{R_s + R_{ib} \parallel R_1 \parallel R_2} \right) \]

\[ = (33.9375) \left( \frac{1}{34.2365} \right)(0.8135) = 0.8064 \]

(d)

The input resistance \( R_{ib} \) is:

\[ R_{ib} = r_x + (1 + \beta)(R_E \parallel R_L) \]

\[ = 0.299 + 33.9375 = 34.24 \text{ (k}\Omega\text{)} \]

To calculate the output resistance \( R_o \), the signal source \( v_s \) is short-circuited and the following equations can be written by KCL at node \( v_o \) and node \( v_b \):

\[ v_b = v_o + r_x i_b \]

\[ \frac{v_b}{R_s \parallel R_1 \parallel R_2} + i_b = 0 \text{ (KCL at node } v_b) \]

\[ \frac{v_o + r_x i_b}{R_s \parallel R_1 \parallel R_2} + i_b = 0 \Rightarrow \frac{v_o}{i_b} = -(r_x + R_s \parallel R_1 \parallel R_2) \]

\[ i_o + (1 + \beta) i_b = \frac{v_o}{R_E} \text{ (KCL at node } v_o) \]

\[ i_o - (1 + \beta) \left( \frac{v_o}{r_x + R_s \parallel R_1 \parallel R_2} \right) = \frac{v_o}{R_E} \]

\[ \Rightarrow \frac{v_o}{i_o} = R_o = R_E \left( \frac{r_x + R_s \parallel R_1 \parallel R_2}{1 + \beta} \right) = 6.18 \text{ (}\Omega\text{)} \]
6. For the circuit shown in Figure 6, let $V_{CC} = 5 \text{ V}$, $R_L = 4 \text{ k}\Omega$, $R_E = 3 \text{ k}\Omega$, $R_1 = 60 \text{ k}\Omega$, and $R_2 = 40 \text{ k}\Omega$. The transistor parameters are $\beta = 50$ and $V_A = 80 \text{ V}$.
   (a) Determine $I_{CQ}$ and $V_{ECQ}$.
   (b) Plot the dc and ac load lines.
   (c) Determine $A_v = v_o / v_s$ and $A_i = i_o / i_s$.
   (d) Determine $R_{ib}$ and $R_o$.

\[ V_{CC} = 5 \text{ V}, \quad R_L = 4 \text{ k}\Omega, \quad R_E = 3 \text{ k}\Omega, \quad R_1 = 60 \text{ k}\Omega, \quad R_2 = 40 \text{ k}\Omega \]

\[ \beta = 50, \quad V_A = 80 \text{ V} \]

\[ I_{CQ}, \quad V_{ECQ} \]

\[ A_v = v_o / v_s, \quad A_i = i_o / i_s \]

\[ R_{ib}, \quad R_o \]

![Figure 6](image_url)

**Solution:**

(a) For dc analysis, the capacitors $C_{C1}$ and $C_{C2}$ act as open circuit.

\[ V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{40}{60 + 40} (5) = 2.0 \text{ (V)} \]

\[ R_{TH} = R_1 \parallel R_2 = \frac{(60)(40)}{60 + 40} = 24.0 \text{ (k}\Omega) \]

\[ I_{BQ} = \frac{V_{CC} - V_{EB(0o)}}{R_{TH} + (1 + \beta)R_E} = \frac{5 - 0.7 - 2}{24 + (51)(3)} = 12.99 \text{ (µA)} \]

\[ I_{CQ} = \beta I_{BQ} = 0.650 \text{ (mA)} \]

\[ I_{EQ} = (1 + \beta)I_{BQ} = 0.663 \text{ (mA)} \]

\[ V_{ECQ} = V_{CC} - I_{EQ}R_E = 5 - (0.663)(3) = 3.01 \text{ (V)} \]

(b) The dc load line is given by:

\[ V_{CC} = I_e R_E + V_{EC} = \left( \frac{1 + \beta}{\beta} \right) I_c R_E + V_{EC} \]

\[ I_c = -\frac{V_{EC}}{\left( \frac{1 + \beta}{\beta} \right) R_E} + \frac{V_{CC}}{\left( \frac{1 + \beta}{\beta} \right) R_E} = -0.3268V_{EC} + 1.6340 \]
The ac load line is given by:

\[ v_{ce} + i_c \left( R_E \parallel R_L \right) = 0 \]

\[ v_{ce} + i_c \left( \frac{1 + \beta}{\beta} \right) \left( R_E \parallel R_L \right) = 0 \Rightarrow i_c = -\frac{v_{ce}}{\left( \frac{1 + \beta}{\beta} \right) \left( R_E \parallel R_L \right)} = -0.5719 v_{ce} \]

(c)

The small-signal parameters are:

\[ r_s = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.650} = 2.0 \text{ (kΩ)} \]

\[ g_m = \frac{I_{CQ}}{V_T} = \frac{0.650}{0.026} = 25.0 \text{ (mA/V)} \]

\[ r_o = \frac{V_A}{I_{CQ}} = \frac{80}{0.650} = 123.077 \text{ (kΩ)} \]

The small-signal ac equivalent circuit becomes:
\[ v_o = \left[ i_b + \beta i_b + \frac{-v_o}{r_o} \right] (R_E \parallel R_L) \]

\[ \Rightarrow \frac{v_o}{i_b} = \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} = \frac{(51)(1.7143)}{1+1.7143/123.077} = 86.2283 \]

\[ v_b = V_x + v_o = i_b r_o + \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} \]

\[ \Rightarrow \frac{v_b}{i_b} = R_{ib} = r_o + \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} = 2.0 + 86.2283 = 88.2283 \]

\[ v_b = v_s \]

\[ \Rightarrow \frac{v_b}{v_s} = 1 \]

\[ \frac{v_o}{v_s} = \frac{v_o}{i_b} \times \frac{i_b}{v_b} \times \frac{v_b}{v_s} = \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} \times \frac{1}{R_{ib}} \times 1 \]

\[ = \frac{1}{R_{ib}} \left[ \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} \right] \]

\[ = \frac{86.2283}{88.2283} \times \frac{1}{0.9773} = 0.9773 \]

\[ v_o = i_o R_L \]

\[ \Rightarrow \frac{v_o}{i_o} = R_L = 4 \]

\[ v_o = \left[ i_b + \beta i_b + \frac{-v_o}{r_o} \right] (R_E \parallel R_L) \]

\[ \Rightarrow \frac{v_o}{i_b} = \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} = \frac{(51)(1.7143)}{1+1.7143/123.077} = 86.2283 \]

\[ i_b = \frac{R_L \parallel R_L}{R_{ib} \parallel R_2} \]

\[ \Rightarrow \frac{i_b}{i_s} = \frac{R_L \parallel R_2}{R_{ib} \parallel R_2 + R_1 \parallel R_2} = \frac{24}{88.2283 + 24} = 0.2138 \]

\[ \frac{i_o}{i_s} = \frac{v_o \times i_b}{v_s \times i_b} \]

\[ = \frac{1}{R_L} \times \frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} \times \frac{R_L \parallel R_2}{R_{ib} \parallel R_2} \]

\[ = \left( \frac{1}{4} \right)(86.2283)(0.2138) = 4.61 \]
(d)

The input resistance $R_{ib}$ is:

$$R_{ib} = r_i + \left[ \frac{(1 + \beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} \right] = 2.0 + 86.2283 = 88.23 \text{ (k}\Omega\text{)}$$

To calculate the output resistance $R_o$, the signal source $v_s$ is short-circuited and the following equations can be written by KCL at node $v_o$:

$$i_b = -\frac{v_o}{r_i}$$

$$i_o + \beta i_b + i_b = \frac{v_o}{r_o \parallel R_E}$$

$$i_o + (1 + \beta) \left( -\frac{v_o}{r_i} \right) = \frac{v_o}{r_o \parallel R_E}$$

$$\frac{v_o}{i_o} = R_o = \frac{1}{\frac{1 + \beta}{r_i} + \frac{1}{r_o \parallel R_E}}$$

$$= r_o \parallel R_E \parallel \frac{r_i}{1 + \beta}$$

$$= 38.70 \text{ (}\Omega\text{)}$$

7. For the transistor in Figure 7, the parameters are $\beta = 100$ and $V_A = \infty$.

(a) Design the circuit such that $I_{EQ} = 1 \text{ mA}$ and the $Q$-point is in the center of the dc load line.

(b) If the peak-to-peak sinusoidal output voltage is 4 V, determine the peak-to-peak sinusoidal signals at the base of the transistor and the peak-to-peak value of $v_s$.

(c) If the load resistor $R_L = 1 \text{ k}\Omega$ is connected to the output through a coupling capacitor, determine the peak-to-peak value in the output voltage, assuming $v_s$ is equal to the value determined in part (b).
Solution:

(a)

For dc analysis, the capacitor $C_C$ acts as open circuit.

\[
V_{CC} = I_{BE}R_S + V_{BE(on)} + I_{EQ}R_E = \left(\frac{R_S}{1+\beta} + R_E\right)I_{EQ} + V_{BE(on)}
\]

\[
\frac{R_B}{101} + R_E = \frac{V_{CC} - V_{BE(on)}}{I_{EQ}} = \frac{10 - 0.7}{1} = 9.3 \text{ (kΩ)} \quad \text{(1)}
\]

\[
V_{CC} = V_{CEQ} + I_{EQ}R_E \quad (V_{CEQ} = \frac{V_{CC}}{2} \text{ for } Q\text{-point is in the center of the dc load line})
\]

\[
10 = 5 + (1)R_E
\]

\[
R_E = 5 \text{ (kΩ)} \quad \text{(2)}
\]

\[
\Rightarrow R_B = (101)(9.3 - R_E) = 434.3 \text{ (kΩ)}
\]

\[
I_{CQ} = \left(\frac{\beta}{1+\beta}\right)I_{EQ} = \left(\frac{100}{101}\right)(1) = 0.990 \text{ (mA)}
\]

\[
r_n = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.990} = 2.6263 \text{ (kΩ)}
\]

\[
g_m = \frac{I_{CQ}}{V_T} = \frac{0.990}{0.026} = 38.0769 \text{ (mA/V)}
\]

\[
r_o = \frac{V_A}{I_{CQ}} = \infty
\]

(b)

The small-signal ac equivalent circuit is given by:

\[r_o = \infty\]
\[ v_o = (1 + \beta)R_E i_b \]

\[ \Rightarrow \frac{v_o}{i_b} = (1 + \beta)R_E = (101)(5) = 505 \]

\[ v_b = V_s + v_o = i_b r_o + (1 + \beta)R_E i_b \]

\[ \Rightarrow \frac{v_b}{i_b} = R_o = r_o + (1 + \beta)R_E = 2.6263 + 505 = 507.6263 \]

\[ v_b = \frac{R_B \parallel R_o}{R_s + R_B \parallel R_o} v_s \]

\[ \Rightarrow \frac{v_b}{v_s} = \frac{R_B \parallel R_o}{R_s + R_B \parallel R_o} = \frac{234.0545}{0.7 + 234.0545} = 0.9970 \]

\[ \frac{v_b}{v_o} = \left( \frac{v_b}{i_b} \times \frac{i_b}{v_o} \right) = \frac{507.6263}{505} = 1.0052 \quad \ldots (3) \]

\[ \frac{v_o}{v_s} = \left( \frac{v_o}{i_b} \times \frac{i_b}{v_s} \times \frac{v_b}{v_o} \right) = \frac{(1 + \beta)R_E \times \frac{1}{R_o} \times \frac{R_B \parallel R_o}{R_s + R_B \parallel R_o}}{505} \times 0.9970 = 0.9918 \quad \ldots (4) \]

If the peak-to-peak output voltage \( v_o(\text{peak-peak}) \) is 4 V,

Eq. (3) \( \Rightarrow v_b(\text{peak-peak}) = 1.0052v_o(\text{peak-peak}) = 4.021 \) (V)

Eq. (4) \( \Rightarrow v_o(\text{peak-peak}) = \frac{v_o(\text{peak-peak})}{0.9918} = 4.033 \) (V)

(c)

If the load resistor \( R_L = 1 \) k\( \Omega \) is added in parallel to \( R_E \), Eq. (4) must be modified accordingly:

\[ \frac{v_o}{v_s} = \left( \frac{(1 + \beta)(R_E \parallel R_L)}{R_o + (1 + \beta)(R_E \parallel R_L)} \right) \left( \frac{R_B \parallel R_o}{R_s + R_B \parallel R_o} \right) \]

\[ = \frac{(101)(0.8333)}{2.6263 + (101)(0.8333)}(0.9970) = 0.9668 \]

\[ \Rightarrow v_o(\text{peak-peak}) = 0.9668v_o(\text{peak-peak}) = (0.9668)(4.033) = 3.90 \) (V)

Therefore \( v_o(\text{peak-peak}) \) becomes smaller due to the loading effect by \( R_L \).
8. An emitter-follower amplifier, with the configuration shown in Figure 8, is to be designed such that an audio signal given by \( v_s = 5 \sin(3000t) \) V but with a source resistance of \( R_S = 10 \, \Omega \) can drive a small speaker. Assume the supply voltages are \( V^+ = +12 \, V \) and \( V^- = -12 \, V \) and \( \beta = 50 \). The load, representing the speaker, is \( R_L = 12 \, \Omega \). The amplifier should be capable of delivering approximately 1 W of average power to the load. What is the signal power gain of your amplifier?

**Solution:**

To deliver 1 W of average power to the load, the peak-to-peak output voltage should be:

\[
\frac{V_{o(peak)}^2}{R_L} = \frac{V_{o(peak)}^2}{2R_L} = 1
\]

\[
\Rightarrow V_{o(peak)} = 4.899 \, (V)
\]

\[
\Rightarrow I_{o(peak)} = \frac{4.899}{12} = 0.408 \, (A)
\]

\[
\Rightarrow V_{o(peak-peak)} = 9.798 \, (V)
\]

The required voltage gain \( A_v \) is:

\[
A_v = \frac{V_{o(peak)}}{V_{o(peak)}} = \frac{4.899}{5.0} = 0.9798
\]

Choose \( I_{EQ} = 0.8 \, A \) and \( V_{CEQ} = 12 \, V \),

\[
R_E = \frac{V^+ - V^- - V_{CEQ}}{I_{EQ}} = \frac{12 - (-12) - 12}{0.8} = 15 \, (\Omega)
\]

\[
R_{TH} = \frac{1}{10} (1 + \beta) R_E = \left( \frac{51}{10} \right) (15) = 76.5 \, (\Omega) \text{ (for bias-stable circuit)}
\]
The small-signal ac equivalent circuit is given by:

![Circuit Diagram]

Choosing $I_{EQ} = 0.5$ A gives:

$$I_{CQ} = \frac{\beta}{1+\beta} I_{EQ} = \left(\frac{50}{51}\right)(0.8) = 0.784 \text{ (A)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{50}{0.784} = 1.658 \text{ (A)}$$

The small-signal voltage gain is taken from Q.7 with some modifications:

$$\frac{v_o}{v_s} = \frac{(1+\beta)(R_E \parallel R_l)}{r_\pi + (1+\beta)(R_E \parallel R_L)} \left(\frac{R_1 \parallel R_2 \parallel R_b}{R_3 + R_1 \parallel R_2 \parallel R_b}\right)$$

$$= \frac{51(6.667)}{1.658 + 51(6.667)} \left(\frac{62.5116}{10 + 62.5116}\right)$$

$$= 0.8579$$

Due to the presence of the source resistance $R_S$ (loading effect) the required voltage gain of $A_v = 0.9798$ cannot be achieved. Note that $A_v = 0.9951$ if $R_S = 0$.

Therefore the maximum achievable peak output voltage is:

$$\frac{v_{o(peak)}}{v_{s(peak)}} = 0.8579 \Rightarrow v_{o(peak)} = 4.290 \text{ (V)}$$
Hence the output power delivered to the load $R_L$ is:

$$P_L = \frac{v_{o(peak)}^2}{2R_L} = 0.767 \text{ (W)}$$

The input power delivered by the signal source $v_s$ is:

$$P_S = v_{s(\text{rms})}i_{s(\text{rms})}$$

$$i_{s(\text{rms})} = \frac{v_{s(\text{rms})}}{R_i} = \frac{v_{s(\text{rms})}}{R_S + R_i || R_2 || R_{bb}} = \frac{5\sqrt{2}}{10 + 62.5116} = 48.758 \text{ (mA)}$$

$$\Rightarrow P_S = v_{s(\text{rms})}i_{s(\text{rms})} = \left( \frac{5}{\sqrt{2}} \right) (48.758) = 172.386 \text{ (mW)}$$

Hence the signal power gain of the amplifier is:

$$G_{\text{power}} = \frac{P_L}{P_S} = \frac{0.767}{172.386 \times 10^{-3}} = 4.45$$

**Part C. AC Load Line Analysis / Maximum Symmetrical Swing**

9. For the circuit in Figure 9, the transistor parameters are $\beta = 100$ and $V_A = 100 \text{ V}$. The values of $R_C$, $R_E$ and $R_L$ are as shown in the figure. Design a bias-stable circuit to achieve the maximum undistorted swing in the output voltage if the total instantaneous C-E voltage is to remain in the range $1 \leq v_{CE} \leq 8 \text{ V}$ and the minimum collector current is to be $i_C(\text{min}) = 0.1 \text{ mA}$. 

![Figure 9](image_url)
Solution:

To obtain a bias-stable circuit, let:

\[ R_{TH} = R_1 \parallel R_2 \]
\[ = \frac{1}{10} (1 + \beta) R_E = \left( \frac{101}{10} \right)(1.2) = 12.12 \, \text{(kΩ)} \]

The dc load line of the circuit is given by:

\[
V_{CC} = I_C R_C + V_{CE} + I_e R_E \\
= I_C R_C + V_{CE} + \left( \frac{1+\beta}{\beta} \right) I_C R_E
\]

\[
I_C = -\frac{V_{CE}}{R_C + \left( \frac{1+\beta}{\beta} \right) R_E} + \frac{V_{CC}}{R_C + \left( \frac{1+\beta}{\beta} \right) R_E} = -0.2931V_{CE} + 2.6377 \quad \text{...(1)}
\]

The ac load line of the circuit is given by:

\[
v_{ce} + i_c \left( R_c \parallel R_L \right) = 0 \\
i_c = -\frac{v_{ce}}{R_c \parallel R_L} = -0.9545v_{ce} \quad \text{...(2)}
\]

Given \( v_{CE(min)} = 1 \, \text{V} \) and \( i_{C(min)} = 0.1 \, \text{mA} \), the maximum swing of \( v_{CE} \) and \( i_C \) from the \( Q \)-point \( (I_{CQ}, V_{CEQ}) \) would be:

\[
|\Delta v_{CE(max)}| = V_{CEQ} - v_{CE(min)} = V_{CEQ} - 1 \\
|\Delta i_{C(max)}| = I_{CQ} - i_{C(min)} = I_{CQ} - 0.1
\]

Since \( |\Delta v_{CE(max)}| \) and \( |\Delta i_{C(max)}| \) are related by the ac load line,

\[
|\Delta i_C| = 0.9545|\Delta v_{CE}| \\
I_{CQ} - 0.1 = 0.9545(V_{CEQ} - 1) \\
I_{CQ} = 0.9545V_{CEQ} - 0.8545 \quad \text{...(3)}
\]

Solving (1) and (3) at the \( Q \)-point \( (I_{CQ}, V_{CEQ}) \):

\[
0.9545V_{CEQ} - 0.8545 = -0.2931V_{CEQ} + 2.6377 \\
1.2476V_{CEQ} = 3.4922 \\
V_{CEQ} = 2.80 \, \text{(V)} \\
I_{CQ} = 1.817 \, \text{(mA)}
\]
To decide the value for $V_{TH}$:

$$V_{TH} = I_{EQ}R_{TH} + V_{BE(on)} + I_{EQ}R_E$$

$$= \left(\frac{1.817}{100}\right)(12.12) + 0.7 + \left(\frac{101}{100}\right)(1.817)(1.2)$$

$$= 3.122 \text{ (V)}$$

$$= \frac{R_2}{R_1 + R_2}V_{CC} = \frac{1}{R_1} (R_1 || R_2)V_{CC}$$

$$\Rightarrow R_1 = 34.93 \text{ (k}\Omega) \quad R_2 = 18.56 \text{ (k}\Omega)$$

10. In the circuit in Figure 10 with transistor parameters $\beta = 180$ and $V_A = \infty$, design the bias resistors $R_1$ and $R_2$ to achieve maximum symmetrical swing in the output voltage and to maintain a bias-stable circuit. The total instantaneous C-E voltage is to remain in the range $0.5 \leq v_{CE} \leq 4.5 \text{ V}$ and the total instantaneous collector current is to be $i_C \geq 0.25 \text{ mA}$. 

![Figure 10](image-url)
Solution:

To obtain a bias-stable circuit, let:

\[ R_{TH} = R_1 \parallel R_2 \]
\[ = \frac{1}{10} (1 + \beta) R_E = \left( \frac{181}{10} \right)(0.1) = 1.81 \text{ (kΩ)} \]

The dc load line of the circuit is given by:

\[ V_{CC} = I_C R_C + V_{CE} + I_E R_E \]
\[ = I_C R_C + V_{CE} + \left( \frac{1 + \beta}{\beta} \right) I_C R_E \]
\[ I_C = \frac{V_{CE}}{R_C + \left( \frac{1 + \beta}{\beta} \right) R_E} + \frac{V_{CC}}{R_C + \left( \frac{1 + \beta}{\beta} \right) R_E} = -0.9086V_{CE} + 4.5432 \quad \text{...(1)} \]

The ac load line of the circuit is given by:

\[ v_{ce} + i_c \left( R_c \parallel R_L \right) = 0 \]
\[ i_c = -\frac{v_{ce}}{R_c \parallel R_L} = -1.8333v_{ce} \quad \text{...(2)} \]

Given \( v_{CE\text{(min)}} = 0.5 \text{ V} \) and \( i_{C\text{(min)}} = 0.25 \text{ mA} \), the maximum swing of \( v_{CE} \) and \( i_C \) from the \( Q \)-point (\( I_{CQ}, V_{CEQ} \)) would be:

\[ |\Delta v_{CE\text{(max)}}| = V_{CEQ} - v_{CE\text{(min)}} = V_{CEQ} - 0.5 \]
\[ |\Delta i_{C\text{(max)}}| = I_{CQ} - i_{C\text{(min)}} = I_{CQ} - 0.25 \]

Since \( |\Delta v_{CE\text{(max)}}| \) and \( |\Delta i_{C\text{(max)}}| \) are related by the ac load line,

\[ |\Delta i_c| = 1.8333 |\Delta v_{ce}| \]
\[ I_{CQ} - 0.25 = 1.8333 \left( V_{CEQ} - 0.5 \right) \]
\[ I_{CQ} = 1.8333V_{CEQ} - 0.6667 \quad \text{...(3)} \]

Solving (1) and (3) at the \( Q \)-point (\( I_{CQ}, V_{CEQ} \)):

\[ 1.8333V_{CEQ} - 0.6667 = -0.9086V_{CEQ} + 4.5432 \]
\[ 2.7419V_{CEQ} = 5.2099 \]
\[ V_{CEQ} = 1.90 \text{ (V)} \]
\[ I_{CQ} = 2.817 \text{ (mA)} \]
To decide the value for $V_{TH}$:

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(on)} + I_{EQ}R_E$$

$$= \left(\frac{2.817}{180}\right)(1.81) + 0.7 + \left(\frac{181}{180}\right)(2.817)(0.1)$$

$$= 1.012 \text{ (V)}$$

$$= \frac{R_2}{R_1 + R_2}V_{CC} = \frac{1}{R_1 \parallel R_2}V_{CC}$$

$$\Rightarrow R_1 = 8.95 \text{ (kΩ)} \quad R_2 = 2.27 \text{ (kΩ)}$$