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ROLE OF HYSTERESIS ON THE MODE-SHIFT CHARACTERISTICS OF INJECTION LOCKING A LASER DIODE

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Using Lang's equation for the dynamics of injection locking of a laser diode, we show that the hysteresis property of the excess carrier density has direct influence on the mode-shift characteristics, which makes the shift of the slave laser mode to be different from the frequency detune of the external master laser signal within the locking range of the slave laser. The theoretical prediction is experimentally confirmed by the measurement of the mode-shift of an injection locked Fabry-Perot laser diode.

Keywords: Laser diode; injection locking; mode-shift; hysteresis; Fabry-Perot laser diode.

1. Introduction

In 1927, Van der Pol pioneered the well-known phenomenon of injection locking.[1] Injection locking theory has been refined[2,3] and extended to microwave solid-state oscillators,[4] as well as to laser oscillators.[5] Injection locking of a laser diode (LD), with external coherent signal is used for various applications like retiming of signals,[6] amplitude regeneration,[7] wavelength conversion,[8] polarization stabilization[9] and all-optical switching.[10] The injection locking properties of a semiconductor laser was theoretically reported by Lang in 1982.[11] Later, Lang's equation was used for investigating the bistability and hysteresis properties in the output characteristics of injection locked LDs.[12,13] Lang's equation is also very much used in the investigation of large signal analysis and multi-mode injection locking of LD.[8] Recently, the nonlinear dynamics induced by external optical injection in semiconductor lasers has also been theoretically investigated using Lang's equation.[14–16] In general, it is found that Lang's equation can explain many interesting properties of injection locking the LD.

Fundamentally, injection-locking a LD makes the slave laser oscillate at the external master laser signal frequency. With respect to the input intensity of the external signal, there is a definite frequency locking range, during which the LD can be injection locked and can be made to lase exactly at the external master signal frequency. In semiconductor lasers, generally this locking range is asymmetric in nature, favoring the lower frequency side of the free lasing LD (i.e., lasing without any external light injection).[11] Thus the laser mode undergoes a mode-shift with respect to its free lasing frequency (or wavelength) position. All these dynamics are due to the change in the carrier density. The presence of the external light depletes the carrier density, which in turn increases the effective refractive index of the laser medium. This gives a mode-shift to the laser mode. This also indicates that the mode-shift of the laser diode frequency has a direct relationship to the excess carrier density.

In this work, we show that the mode-shift of the laser mode is directly proportional to the excess carrier density with the help of Lang's equation. Hence, the hysteresis property of the excess carrier density will have direct implications on the mode-shift. This results in a difference between the mode-shift and frequency detune of the external master laser signal within the locking range. By injection-locking a Fabry-Perot LD (FPLD), we have also experimentally confirmed the theoretical predictions.

2. Theory Based on Lang's Equation

For the theoretical investigation, we use the following Lang's steady state conditions equation[11]:

$$z_0 = \frac{q^2}{[x^2/4 + (Rx/2 + pd)^2]}, \tag{1a}$$

$$z_u = \frac{-C_{sp}}{x}, \tag{1b}$$

$$-x - (1 + x)(z_0 + z_u) + r = 0, \tag{1c}$$

where z_0 and z_u represent the normalized locked and unlocked modes output intensities, respectively. The terminology locked mode and unlocked mode respectively represent the laser output mode oscillating at master laser frequency and the original slave laser mode. These terms are used in the same meaning as in Ref. 11, and have no direct relation between the locking and unlocking of the slave laser. The parameter R is called the linewidth enhancement factor, and it is introduced to express the carrier-density-dependent refractive index. q^2 represents the normalized external injected light intensity. x and r respectively represent the normalized excess carrier density and the difference between the threshold and injected current densities. p is the ratio between the effective refractive index and the refractive index. C_{sp} is the spontaneous emission factor. Parameter d stands for the normalized "nominal" detuning. Although this Lang's equation is twenty-two years old, theoretical investigations on injection locking of laser diodes using one or more external signals are still based on these equations. One can also note that many changes in the notation of the variables have been taking place while using Lang's equation.[12–16] Except for the variable changes, all the equations that have appeared so far in the literature still use the same basic dynamics described by Eq. (1).

Lang[11] has clearly mentioned that the quantity in the bracket in Eq. (1a) represents the "true" detuning under light injection, which differs from d because of the changes in the carrier density and the oscillation frequency caused by the light injection. From this, it is obvious that the mode-shift of the laser mode is given by the expression $-Rx/(2p)$. To our knowledge, this mode-shift quantity $-Rx/(2p)$ has been calculated only for a two-mode injection locking case in Fig. 4(c) of Ref. 8. It is straightforward that the mode-shift is directly proportional to the excess carrier density x. So any natural characteristics of the excess carrier density will have direct implications on the mode-shift. For the injection current above threshold and for not too high external signal intensity, Lang[11] has derived an approximate analytical expression for the locking range as

$$\frac{-q}{\sqrt{r}}\sqrt{1+R^2} < pd < \frac{q}{\sqrt{r}}. \tag{2}$$

The fourth order polynomial equation for x can be derived by substituting z_0 from Eq. (1a) and z_u from Eq. (1b) in Eq. (1c). On solving the fourth order polynomial equation of x (considering only the real non-degenerative roots) for various nominal detuning d and fixed values of other input parameters (injection current and external injected signal intensity), we can obtain the characteristics of the excess carrier density x and in turn the mode-shift. We use the same set of normalized parametric values as in Ref. 11 for the numerical simulations given by $a = 10^{-3}$, $m = 1$, $C_{sp} = 10^{-5}$, $q^2 = 10^{-4}$, $R = -4$, $r = 0.1$ and $p = 1.25$. The inset in Fig. 1 shows the normalized excess carrier density versus normalized frequency detune. Hysteresis characteristic is quite evident from the results. The mode-shift, which is straightforwardly calculated as $-Rx/(2p)$, is shown in Fig. 1. The dashed straight line is the reference line representing the frequency detunes equal to the mode-shift. As the mode-shift is proportional to the excess carrier density, the hysteresis characteristics of the excess carrier density is directly reflected on the mode-shift characteristics. The shaded region represents the locking range. Note that the locking range also includes the hysteresis part. In a muti-mode laser like

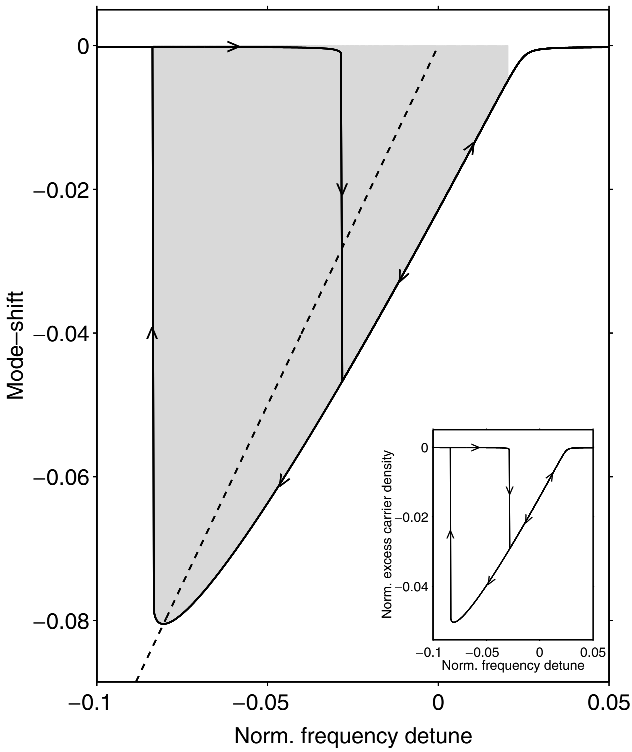
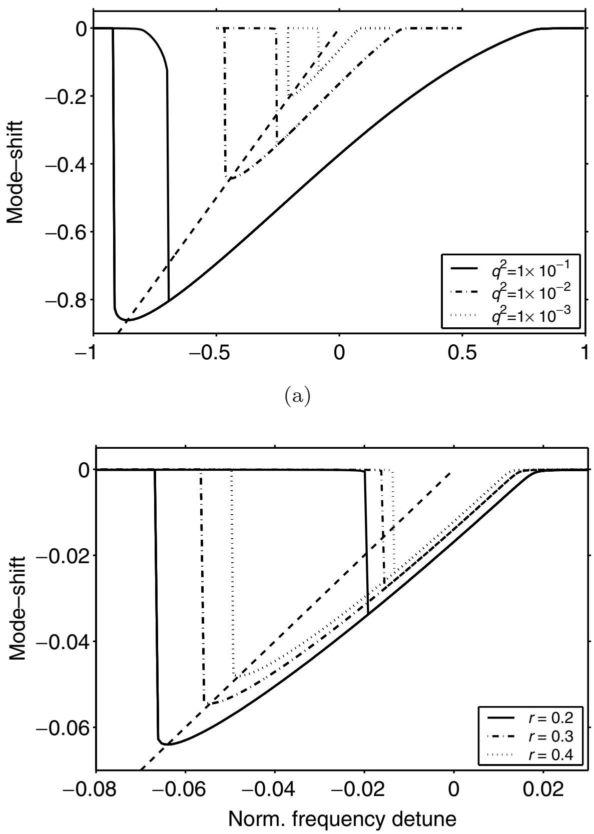


Fig. 1. Plot showing the mode-shift versus normalized frequency detune of the external signal for $q^2 = 10^{-4}$ and $r = 0.1$. Dashed straight line is the reference line for detune equal to the mode-shift. Shaded region is the locking range. Inset: Normalized excess carrier density versus normalized frequency detune.

FPLD, this red-shift of the mode-comb caused by the presence of the first external signal in the blue-side of one laser mode will greatly influence the injection locking properties of the second external signal in another laser mode. Hence, one needs to carry out a very careful investigation on the mode-shift characteristics in the case of multi-mode injection locking.[8–10]

From Eq. (2), it is quite obvious that the locking range is directly proportional to the intensity of the external injected signal and inversely proportional to the square root of the injected current. This can also be verified numerically. Figure 2(a) shows the mode-shift characteristics for various external injected signal power and for constant excitation level $r = 0.1$. Solid, dot-dashed and dotted curves show the mode-shift characteristics for the normalized input power 10^{-1}, 10^{-2} and 10^{-3}, respectively. All other numerical parameters used in Fig. 2 remain the same as in Fig. 1. The higher the external injected light power, the wider the locking range. Here, also, the dashed reference line which represents the mode-shift values equal to the detune intersects the mode-shift curves at the locations very near the red-side of the locking range, immediately after which the LD gets unlocked. Similar



(b)

Fig. 2. Plots showing the mode-shift versus normalized frequency detune of the external signal for (a) various q^2 and $r = 0.1$ (b) various r and $q^2 = 10^{-4}$.

characteristics can be seen from Fig. 2(b), where the mode-shift characteristics are derived for various excitation levels and for constant external injected signal power $q^2 = 10^{-4}$. Solid, dot-dashed and dotted curves show the mode-shift versus nominal detuning for the normalized injection currents 0.2, 0.3 and 0.4, respectively. It can be seen that the locking range reduces as the excitation level increases, as predicted by Eq. (2). As expected, the dotted reference line is intersecting the family of curves at the same locations.

3. Experimental Method

In practice, after injection locking a LD, one cannot observe the suppressed unlocked mode, as it is either too close to the locked mode to be experimentally resolved separately or its power level is in the noise level. So all the features of the mode-shift characteristics explained above will be extremely difficult to observe experimentally while injection locking a distributed feedback laser or any other single-mode semiconductor lasers. In the case of a FPLD, the mode-shift of the locked laser mode can be indirectly measured from the wavelength locations of the side-modes. As per the property of a Fabry-Perot filter, for a given set of input parametric conditions, the wavelength spacings between adjacent modes has to be equal. Based on this fundamental property, after injection locking, the locked laser mode has to be lasing at the mid-point of the two adjacent modes on either side, if it is lasing at the same frequency of the unlocked laser mode. Indirectly, we can say that the wavelength location of the suppressed unlocked mode is at the mid-point of the two adjacent modes on either side of the locked laser mode. However, we observed that the wavelength spacings between the locked laser mode and the two adjacent modes on either side are not the same after injection locking the FPLD. Figure 3 shows the results from the optical spectrum analyzer (set for a resolution of 0.01 nm) which shows the FPLD locked mode output power with two adjacent suppressed side modes. The dashed line shows the mid-point of the locations of the two side modes which indirectly show the location of the suppressed unlocked mode. Obviously, the wavelength location of the locked laser output is not at the mid-point of the two adjacent side modes. Figure 4(a) shows the experimental measurement of the mode-shift of a FPLD injection locked by an external signal of input powers 0 dBm (solid curves), −5 dBm (dot-dashed curves) and −10 dBm (dotted curves), respectively. The mode-shift is measured as the frequency difference between the center of the locked-mode and the mid-point of the two modes adjacent to the locked injection-mode. Again, the dashed reference line represents the mode-shift values equal to the wavelength detune. These mode-shift measurements have been made on one of the side-mode of the FPLD, which was driven by 1.02 times the threshold current (I_{th}), and the temperature was maintained at 298 K. Similar characteristics have also been measured and shown in Fig. 4(b), where the mode-shift characteristics are derived for various injection currents and for constant master laser signal power

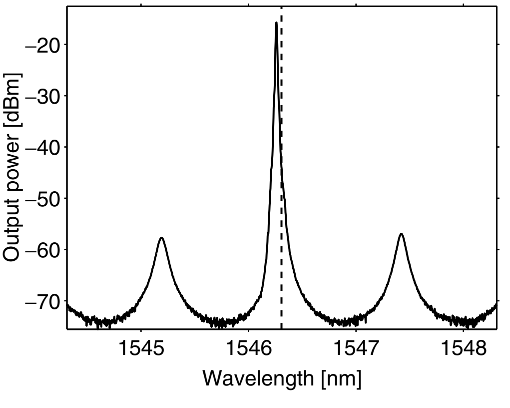
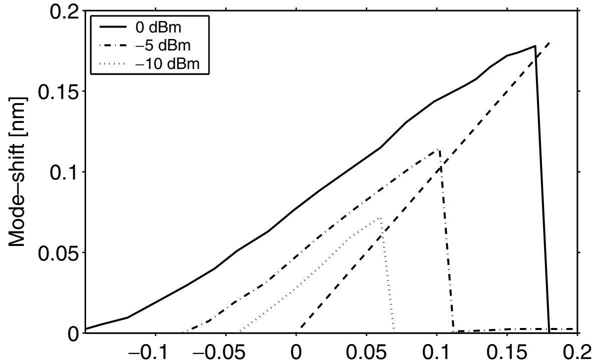
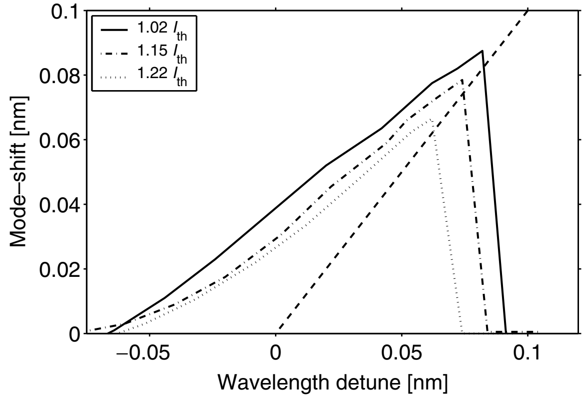


Fig. 3. Output spectrum of FPLD showing the locked mode and two adjacent suppressed modes. Dashed line shows the mid-point of the wavelength locations of the side modes.



(a)



(b)

Fig. 4. Plot showing the experimental measurement of the mode-shift versus wavelength detune of a FPLD injection locked by an external signal for (a) various input power and $1.02 \times I_{\rm th}$ (b) various injection currents and -7 dBm input power. The mode-shift is measured as the frequency difference between the center of the locked-mode and the mid-point of the two modes adjacent to the locked injection-mode.

of -7 dBm. Solid, dot-dashed and dotted curves show the mode-shift versus wavelength detune for the injection currents of $1.02I_{\rm th}$, $1.15I_{\rm th}$ and $1.22I_{\rm th}$, respectively. It is quite evident that the experimental measurement of the mode-shift characteristics agrees with the theoretical predictions shown in Fig. 1. In the experimental characteristics reported in Fig. 4, we have not shown the hysteresis part because of the experimental limitations to measure the small range of hysteresis. If we extend the mode-shift characteristics to meet the dashed reference line, it can be noticed that the hysteresis width is less than 8 pm. From the theoretical calculations, the mode-shift is given by the quantity $-Rx/(2p)$. The linewidth enhancement factor R is an experimentally measurable quantity.[17,18] With the knowledge of R and the mode-shift value $-Rx/(2p)$, one can experimentally measure the excess carrier density (x), which is not a directly measurable quantity.

4. Conclusion

In conclusion, we have reported the mode-shift characteristics from the process of injection locking a LD. We have also experimentally confirmed this phenomenon by injection locking a FPLD. This kind of mode-shift measurements in lasers will help in calculating the excess carrier density. It has been found that this fundamental behavior is due to the hysteresis property of a LD. As the injection locking principle is a vastly observed and utilized phenomenon in various systems, we believe that a similar property can also be observed in any system showing hysteresis property during injection locking.

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