Highly coherent supercontinuum generation with picosecond pulses by using self-similar compression

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Abstract: The low coherence of the supercontinuum (SC) generated using picosecond pump pulses is a major drawback of such SC generation scheme. In this paper, we propose to first self-similarly compress a high power picosecond pump pulse by injecting it into a nonlinearity increasing fiber. The compressed pulse is then injected into a non-zero dispersion-shifted fiber (NZ-DSF) for SC generation. The nonlinearity increasing fiber can be obtained by tapering a large mode area photonic crystal fiber. The fiber nonlinearity is varied by varying the pitch sizes of the air holes. By using the generalized nonlinear Schrödinger equation, we show that a 1 ps pump pulse with random noise can be compressed self-similarly down to a pulse width of 53.6 fs with negligible pedestal. The noise level of the compressed pulse is reduced at the same time. The 53.6 fs pulse can then be used to generate highly coherent SC in an NZ-DSF. By using the proposed scheme, the tolerance of noise level for highly coherent SC generation with picosecond pump pulses can be improved by 5 order of magnitude.

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References and links

1. Introduction

Supercontinuum (SC) generation has attracted much attention in the last two decades since the invention of photonic crystal fiber (PCF) [1–4]. The ultrabroad band SC has found many important applications in frequency metrology [5, 6], communications [7], and biomedical sensing [8], etc. In SC generation, the coherence of the spectra from different pump shots is a crucial parameter to estimate the quality of the SC output [6, 9–12]. The coherence of the generated SC depends on the pulse width and noise level of the pump pulses [9–12]. For pump pulses shorter than 100 fs, soliton fission is the dominant mechanism to generate the SC which has high coherence between shots. For pump pulses longer than 100 fs, the noise sensitive modulation instability (MI) is the dominate mechanism to generate the SC. The output spectrum varies significantly from shot to shot. Thus the coherence of the SC spectra generated from long pulses (> 100 fs) is much lower than that generated from short pulses (< 100 fs) [10–12]. However, SC generation with long pump pulses are more attractive than that with short pump pulses because high power long pump pulses are readily available. Picosecond pulses can be obtained directly from active or passive mode locked fiber lasers without the bulky chirp compensation grating pair which is unavoidable to obtain less than 100 fs pulses [13–16]. Using all-fiber lasers as the pump source will greatly reduce the weight and size to enhance the mobility and applicability of the SC source. It is therefore attractive if picosecond pulses can be used directly to generate highly coherent SC.

Much effort has been made to enhance the coherence of the SC generated with picosecond pulses [17–21]. Pulse compression in fibers with varying dispersion profile has been proven effective to enhance the coherence in SC generation [3, 20, 21]. Compared to the pulse compression with separate spectral broadening and linear chirp compensation [22, 23], adiabatic pulse compression in dispersion decreasing fibers (DDF) can provide pulse compression with much lower pedestal without the chirp compensation elements [24, 25]. But adiabatic compression requires long length of fiber [26–29]. Self-similar compression can avoid the drawbacks in the above two pulse compression schemes and provide pedestal-free pulse compression with high compression factor in a short segment of fiber [30–34]. Self-similar compression of chirped solitons in dispersion exponentially decreasing fibers (DEDFs) has been discussed [21, 32, 33]. The fiber nonlinearity is assumed to be constant when the dispersion varies. PCF tapers are typically proposed to realize the DEDFs. In PCFs, when the core size is reduced to comparable to the optical wavelength, the fiber dispersion will be sensitive to the variation of the core size. Thus the dispersion at a given wavelength can be engineered by tapering the PCFs [29]. However, it is not possible to engineer the dispersion in PCF tapers without changing the nonlinearity [27]. To date, no practical designs of dispersion decreasing DEDFs supporting self-similar compression have been reported.

A major drawback of using DEDFs for self-similar compression is that they limit the power of the injected pulse. It is because PCFs with small core have high (γ ~10−100 W−1/km) nonlinearity. For a given pulse width, a higher nonlinearity or higher power will require a higher dispersion in order to satisfy the soliton condition. For example, for a 1 ps pulse with a kilowatt peak power, the dispersion should be ~10,000 ps²/km. Although dispersion can be enhanced to a very high value in specially designed fibers, such high dispersion values can be attained only at very narrow bandwidth and most of them show normal dispersion which does not support soliton propagation [35–37]. If the dispersion is limited to a relative low level, another way to realize self-similar compression for high power picosecond pulse using DEDFs is to reduce the nonlinearity. Large mode area (LMA) PCFs with low nonlinearity have been broadly used in propagation and even pulse compression for high peak power pulses [22, 23, 38, 39]. However since the waveguide dispersion of LMA PCFs is very weak and the total dispersion varies slowly with the variation of fiber parameters, it is not possible to utilize LMA PCFs to realize dispersion decreasing self-similar compression either.

In this paper, we propose a self-similar compression scheme using nonlinearity increasing fiber. Since the relative variation of dispersion is much smaller than that of nonlinearity when
core size of LMA PCF varies, we propose to engineer the nonlinearity of LMA PCF by varying the core size of the fiber to support self-similar compression of high power picosecond pulses. We show that the compressed pulses can be used to generate highly coherent SC in a non-zero dispersion-shifted fiber (NZ-DSF) even in the presence of high level of noises. The rest of the paper is organized as follows. In Section 2, we will introduce the principle and theoretical model of self-similar compression in optical fibers. Section 3 gives the designs of the LMA PCF tapers using both a comprehensive model and a simplified model in which the fiber dispersion is assumed to be constant. Section 4 simulated the self-similar pulse compression in the tapers designed in Section 3 using both a simplified propagation model and the full propagation model. In Section 5, we compare the coherence of the SC generated using picosecond pump pulses and the self-similarly compressed femtosecond pump pulses with noise. Section 6 concludes the paper.

2. Theoretical model and principle

We investigate the propagation of high power ultra-short pulses in an LMA PCF taper and an NZ-DSF by using a general nonlinear Schrödinger equation (GNLSE) which includes high order dispersions, self-steepening, and Raman scattering [2, 12]

\[
\frac{\partial A}{\partial z} + \frac{\alpha A}{2} - \sum_{k=2}^{k=k} \beta_k(z) \frac{\partial^k A}{\partial t^k} = i\gamma(z) \left( 1 + it \tau_s \frac{\partial}{\partial t} \right) \left( \int_{-\infty}^{t} R(t') A(z, t-t') dt' \right),
\]

where \(\alpha\) is the loss coefficient and \(\beta_k(z)\) is the \(k\)-th order dispersion coefficient at distance \(z\). The right-hand side of Eq. (1) models the nonlinear effects in the fiber which includes self-phase modulation, self-steepening, and Raman scattering. The nonlinear coefficient \(\gamma(z)\) can vary along the fiber length and \(\tau_s\) models the dispersion of the nonlinear coefficient. The response function \(R(t)\) is given by

\[
R(t) = (1 - f_R) \delta(t) + f_s \tau_s (\tau_s^{-2} + \tau_s^{-2}) \exp(-t/\tau_s) \sin(t/\tau_s) \Theta(t),
\]

where the first term on the R.H.S. is the instantaneous nonlinear response, and the second term is the delayed Raman response. The functions \(\delta(t)\) and \(\Theta(t)\) are the Dirac delta function and Heaviside step function respectively. The parameters \(f_R = 0.18, \tau_s = 12.2\) fs and \(\tau_s = 32\) fs are used to model the delayed Raman response.

In an optical fiber with varying coefficients along the fiber length, there is a special class of solutions in which the pulse width and amplitude vary during propagation in a way that the pulse profile retains the same functional form, i.e. self-similar propagation. If we neglect the fiber loss, high order dispersions and high order nonlinearities, Eq. (1) is reduced to the nonlinear Schrödinger equation (NLS) which is given by

\[
\frac{i \partial A}{\partial z} - \frac{\beta_2(z)}{2} \frac{\partial^2 A}{\partial t^2} + \gamma(z) |A|^2 A = 0.
\]

Self-similar solutions of Eq. (3) is given by [31]

\[
A(t, z) = \left( \frac{P_0}{1 - \xi D(z)} \right)^{1/2} \text{sech} \left( \frac{t-t_0}{\tau_0(1-\xi D(z))} \right) \exp \left( \frac{i \xi (t-t_0)^2}{2(1-\xi D(z))} \right),
\]

where \(\xi\) is the chirp factor of the initial pulse, \(T_0\) and \(P_0\) are the initial pulse width and peak power which satisfy \(\beta_2(0) = -\gamma(0) P_0 T_0^2\). The fiber parameters satisfy the self-similar condition

\[
\rho(z) = \rho(0)[1 - \xi D(z)].
\]

The functions \(\rho(z)\) and \(D(z)\) are defined as
A special case of Eq. (5) in which the nonlinearity is assumed to be constant, i.e. \( \gamma(z) = \gamma(0) \), has been well studied [21, 33, 40]. From Eqs. (5) and (6), the fiber dispersion is anomalous and exponentially decreasing along the fiber length as \( \beta_2(z) = \beta_2(0) \exp(-\sigma z) \) where \( \sigma = \beta_2(0) \xi \).

The self-similar solution can then be written as

\[
A(t, z) = \left( \frac{P_0 e^{\sigma z}}{T_0} \right)^{1/2} \text{sech} \left( \frac{i(t-t_0) e^{\sigma z}}{T_0} \right) \exp \left( \frac{i \xi (t-t_0)^2 e^{\sigma z}}{2(1-\sigma z)} \right).
\]

From Eq. (7), the pulse will be compressed exponentially along the propagation direction in the DEDF. DEDF has been proposed to realize high degree pulse compression but it requires high dispersion for high power picosecond pulse compression. If we input a hyperbolic secant pulse with pulse duration \( T_0 = 1 \) ps and peak power \( P_0 = 1 \) kW into a fiber with typical value of \( \gamma = 2W^{-1}/\text{km} \), then at the input port, the fiber should have \( \beta_2(0) = -2000 \) ps²/km to realize the self-similar compression. Attaining such a high anomalous dispersion value is challenging even in a narrow bandwidth [35–37]. Realizing such a high anomalous dispersion in the whole spectrum of the ultrashort pulse is not feasible based on current fiber design techniques.

Another route to satisfy the condition in Eq. (5) with high power picosecond pulses without high anomalous dispersion is by reducing the nonlinear coefficient \( \gamma \) and varying it along the fiber. If the dispersion remains unchanged in such a nonlinearity varying fiber, i.e. \( \beta_2(z) = \beta_2(0) \), self-similar pulse compression can be supported if the fiber nonlinearity varies along the fiber with the profile [30, 31]

\[
\gamma(z) = \frac{\gamma(0)}{1-\sigma z},
\]

where \( \sigma z < 1 \) and \( \sigma \) is a constant along the fiber. The pulse evolution can then be described as

\[
A(t, z) = \left( \frac{P_0}{1-\sigma z} \right)^{1/2} \text{sech} \left( \frac{i(t-t_0)}{T_0(1-\sigma z)} \right) \exp \left( \frac{i \xi (t-t_0)^2}{2(1-\sigma z)} \right),
\]

where the pulse width and peak power vary along the fiber length in the forms

\[
T(z) = T_0(1-\sigma z), \quad \text{and} \quad P(z) = \frac{P_0}{1-\sigma z}.
\]

3. LMA PCF taper design for self-similar high power picosecond pulse compression

Although self-similar compression of chirped solitons has been widely discussed particularly in DEDFs [21, 30–33], no practical designs of dispersion decreasing PCF tapers supporting self-similar compression have been reported since the dispersion cannot be independently engineered without affecting the nonlinearity of the PCF tapers. As discussed in Section 2, because of the limitation on the achievable anomalous dispersion values in a wide frequency range, the nonlinearity of the PCF tapers suitable for dispersion management is too high to support self-similar propagation of high power picosecond pulses. The nonlinearity of LMA PCF is low, but the dispersion of LMA PCF is not sensitive to the core size. To overcome the above problems, we propose to realize self-similar compression of high power picosecond pulses using the nonlinearity increasing scheme described in Eqs. (8)-(10). LMA PCF is used because the nonlinearity is low and the nonlinearity can be controlled by varying the effective mode area of the fiber. Another advantage of LMA PCF is the weak dependence of the dispersion on the fiber core size owing to the weak confinement of the light.
In this Section, we will design LMA PCF tapers which are suitable for engineering the fiber nonlinearity profile $\gamma(z)$ and dispersion profile $\beta_2(z)$ along the fiber to satisfy the self-similar compression condition Eq. (5) for high power picosecond pulses. To engineer $\gamma(z)$ and $\beta_2(z)$, we first determine the dependence of $\gamma$ and $\beta_2$ on the fiber parameters such as the air hole diameter and air hole pitch size. Then the required variations $\gamma(z)$ and $\beta_2(z)$ along the fiber can be achieved by varying the fiber parameters along the fiber in a specific way. As the dispersion of LMA PCF is not sensitive to the fiber parameters, we will first neglect the dispersion variation in the LMA PCF taper design in this Section.

3.1 LMA PCF design

We proposed to use an LMA PCF design with the cross section shown in Fig. 1(a). The PCF has a solid core which is surrounded by 5 layers of air holes in a triangular lattice. The whole fiber except the air holes is fabricated with silica glass. To ensure single mode propagation with large core size in the whole spectral range, the air hole diameter $d$ is chosen to be $0.3\Lambda$, where $\Lambda$ is the air hole pitch size. The propagation mode and properties of the LMA PCF are then evaluated by finite element method. Figure 1(b) shows a typical electrical field profile of the propagation mode in the LMA fiber with $\Lambda = 20\ \mu m$. In the following we will determine the dependence of the nonlinear coefficient $\gamma$ and dispersion $\beta_2$ on the air hole pitch size $\Lambda$.

![Fig. 1.](image)

The dependence of the nonlinear coefficient $\gamma$ of the LMA PCF on the air hole pitch size $\Lambda$ at the center frequency $\omega_0$ can be obtained by calculating the effective mode area of the propagation mode as [2]

$$\gamma(\Lambda) = \frac{\omega_0 n_2}{c A_{\text{eff}}(\omega_0, \Lambda)}.$$  (11)
Figure 2(a) shows the variation of $A_{\text{eff}}$ with the air hole pitch size $\Lambda$ at wavelength 1550 nm determined from the LMA PCF shown in Fig. 1 using finite element analysis (circles). Figure 2(b) gives the corresponding nonlinear coefficient $\gamma$ (diamonds) using Eq. (11) and the Kerr coefficient $n_2 = 2.6 \times 10^{-20}$ m$^2$/W. The variation of $A_{\text{eff}}$ versus $\Lambda$ is fitted by a quadratic function $A_{\text{eff}} = a\Lambda^2 + b\Lambda + A_0$ with $a = 1.6$, $b = 1.36$ $\mu$m and $A_0 = 2.62$ $\mu$m$^2$. The fitted $A_{\text{eff}}$ values is shown by the solid curve in Fig. 2(a). The solid curve of dispersion does not vary significantly with the design of the taper profile.

For self-similar compression, the nonlinearity and dispersion profiles, $\gamma$ and $\beta_2$, should satisfy condition Eqs. (5) and (6). Determining a taper profile $\Lambda(z)$ which will give a $\gamma(z)$ and $\beta_2(z)$ pair that satisfy condition Eqs. (5) and (6) is not straight forward. However, if the dispersion $\beta_2(z)$ remains unchanged or approximately unchanged along the fiber, i.e. if the dispersion does not vary significantly with $\Lambda$, then condition Eq. (8) can be used. In this case, the required taper profile $\Lambda(z)$ can be determined easily from $\gamma(z)$ using Fig. 2(a). From the nonlinearity and dispersion curves in Figs. 2 and 3, we note that the nonlinearity varies significantly while the dispersion varies relatively slowly when the air hole pitch size $\Lambda$ changes. In this Section, as an approximation to simplify the fiber design, we assume that the dispersion $\beta_2$ is only about 30% which is much smaller than that of $\gamma$ which increases by more than 25 times.

Besides the fiber dispersion, we also obtained the effective index of the fundamental propagation modes and the effective mode areas $A_{\text{eff}}$ at different optical frequencies which can be used to calculate the dispersion of the nonlinear coefficient. As the shock time $\tau_s$, that describes the dispersion of the nonlinear coefficient varies only slightly with $\Lambda$, we will use an effective value $\tau_s = 0.85$ fs in the following simulations. Figures 2 and 3 form the basis of the design of the taper profile $\Lambda(z)$ to achieve self-similar compression in the nonlinearity increasing fiber.

### 3.2 Simplified LMA PCF taper design

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dispersion $\beta_2$ is constant. Combining Eqs. (8) and (11), and applying the fitted $A_{\text{eff}}$ function of $\Lambda$, the taper profile can be obtained by solving

$$a\Lambda^2 + b\Lambda + A_0 - \frac{\omega_0 n_2 (1 - \sigma z)}{c\gamma(0)} = 0.$$  \hspace{1cm} (12)

However, since the dispersion does vary with $\Lambda$ albeit slowly, the dispersion profile $\beta_2(z)$ will be evaluated from the $\Lambda(z)$ obtained by Eq. (12).

$$\gamma(0) = 0.05 \text{ W}^{-1}/\text{km} \text{ at the input point. At the output point of the LMA PCF taper where } \sigma z = 0.95, \gamma(z) \text{ increases to } 1.0 \text{ W}^{-1}/\text{km which is } 20 \text{ times the value at input point. Substitution of } \gamma(0) \text{ and the } \sigma z \text{ values in Eq. (12), the } \Lambda(z) \text{ function can be obtained and is shown in Fig. 4(b). The air hole pitch size } \Lambda(0) \text{ at the input point is } 35.9 \mu m \text{ and is reduced to } 7.6 \mu m \text{ at the output point. Thus self-similar compression of the input pulse at a compression factor of } 20 \text{ can be achieved if the LMA PCF taper is fabricated with the profile shown in Fig. 4(b). The corresponding dispersion profile } \beta_2(z) \text{ is shown in Fig. 4(a) by the blue dashed curve.}}$$

### 4. Self-similar high power pulse compression

#### 4.1 Simplified propagation model

To study the self-similar compression of high power picosecond pulse in the LMA PCF taper designed in Section 3.2, we model the pulse propagation with the GNLSE in Eq. (1). We use the $\gamma(z)$ profile from Fig. 4(a) but for simplicity we assume $\beta_2(z) = \beta_2^{\text{eff}} = \frac{1}{\ell} \int_0^\ell \beta_2(z) dz$ which is a constant and $\beta_2(z)$ is obtained from Fig. 4(a). We will discuss the impact of the variation of dispersion in Section 4.3. The higher order dispersion coefficients at the position $z$ where $\beta_2(z) = \beta_2^{\text{eff}}$ are used to represent the effective higher order dispersions at other wavelengths in the LMA PCF taper. In this fiber design, this position is found to be $\sigma z = 0.682$ which corresponds to $\Lambda = 20 \mu m$. At wavelength 1550 nm, the up to 9-th order of dispersion coefficients $\beta_k$’s, $k = 2$ to 9 used in simulations are $-29.6 \text{ ps}^2/\text{km}$, $0.155 \text{ ps}^3/\text{km}$, $-5.04 \times 10^{-4} \text{ ps}^4/\text{km}$, $2.31 \times 10^{-6} \text{ ps}^5/\text{km}$, $-1.40 \times 10^{-8} \text{ ps}^6/\text{km}$, $1.10 \times 10^{-10} \text{ ps}^7/\text{km}$, $-7.44 \times 10^{-13} \text{ ps}^8/\text{km}$, and $2.57 \times 10^{-15} \text{ ps}^9/\text{km}$, respectively. Taking into account the tight confinement of the mode field in LMA PCF, the fiber loss $\alpha$ is assumed to be zero in the following simulations.

The FWHM of the input hyperbolic secant pulse defined in Eq. (9) is 1 ps corresponding to $T_0 = 567.3$ fs, and the peak power is 1.84 kW to satisfy the soliton condition. The chirp factor $\zeta$ is $-5 \text{ ps}^{-2}$ so the parameter $\sigma = 0.148 \text{ m}^{-1}$. The LMA PCF taper length $L$ is 6.42 m which corresponds to $\sigma L = 0.95$. It should be noted that for self-similar compression in...
nonlinearity increasing fibers, the fiber length should always be less than the chirp length $L_C = 1/\sigma$.

Figure 5 shows the evolutions of the temporal profile and spectrum of the pulse in the LMA PCF taper. The pulse width and peak power vary slowly when $z < 4$ m and rapidly when $z > 4$ m in agreement with the analytical form shown in Eq. (10). The output pulse has a peak power of 34.2 kW which is 18.6 times of the input peak power and the FWHM is 53.6 fs. The compression factor 18.6 is slightly lower than the theoretical compression factor of 20. Figures 5(c) and 5(d) show the temporal profile and the spectrum of the compressed pulse (blue solid curves) for comparison. The output pulse shape shows slightly smaller peak power than the ideal output and the pulse is delayed by ~50 fs with a slight asymmetry on the waveform. The reason for the difference can be seen in the spectrum. The output power at the input wavelength 1550 nm is slightly lower than the ideal case because part of the energy is transferred to 1562.3 nm because of the inclusion of the self-steepening, delayed Raman scattering and high order dispersions. And this wavelength shift manifests as a time delay of the pulse in the time domain by the dispersion. The compression factor 18.6 is not an optimized result and it can be increased by using longer LMA PCF taper but as the pulse compresses further, the high order effects will eventually become important and self-similar compression will breakdown. From Fig. 5, the pulse evolution of self-similar compression using the simplified propagation model can be approximated by the ideal case in most part of the propagation.

![Fig. 5. Evolutions of the (a) temporal profile and (b) spectrum of the pulse in the LMA PCF taper. The output (c) temporal profile and (d) spectrum of the self-similar compressed pulse from the LMA PCF taper is plotted in blue solid curves. The ideal temporal profile and spectrum of the self-similarly compressed pulse without any high order dispersion and high order nonlinearity in the fiber are also plotted in red solid curves for comparison.](image)

Fig. 6. Comparison of the evolutions of the (a) FWHM and (b) peak power of the pulse in the LMA PCF taper (blue curves) to ideal case without high order dispersion and high order nonlinearity (red curves). The insets are the corresponding quantitative differences. The evolutions of (c) dispersion length $L_D$ (blue solid curves), nonlinear length $L_N$ (red solid curves), and chirp length $L_C$ (black dashed curves) during self-similar pulse compression.
Figures 6(a) and 6(b) show the comparison of the evolutions of the FWHM and the peak power of the pulse in the LMA PCF taper designed in Section 3.2 with the ideal case. The insets in the figures show the corresponding quantitative differences. We observed that the differences increase only when the pulse is close to the output point. To better understand the contribution of the dispersion and nonlinearity in the self-similar compression, Fig. 6(c) plots the evolutions of the dispersion length $L_D$ (blue solid curves), nonlinear length $L_N$ (red solid curves), and chirp length $L_C$ (black dashed curves). We note that $L_C = 6.757$ m is a constant. In the propagation, the relationship $L_D = L_N$ is well satisfied to maintain the soliton order $N_{\text{soliton}} = 1$ when they decrease from 10.87 m at $z = 0$ to 0.03 m at $z = L$. In the region that $L_N$ is comparable to $L_C$, the pulse compression is contributed by both the dispersion and nonlinearity. In the region close to the output point of the LMA PCF taper, where $L_N$ is much smaller than $L_C$, the nonlinear effect dominates in the compression. Thus high order nonlinear effects, accompanying the high order dispersions, lead to the increase in the deviations from the ideal case as shown in Figs. 6(a) and 6(b).

4.2 Impact of noise on self-similar compression

From Figs. 5 and 6, the designed LMA PCF taper can successfully self-similar compress a 1 ps high power optical pulse to nearly 50 fs which can then be used in the generation of highly coherent SC. In SC generation with picosecond pulses, the unavoidable noise on the input pulse is the intrinsic noise source that degrades the coherence of the resulting SC. To investigate the effect of noise on the proposed high power self-similar compression scheme, we inject the following optical pulse with shot noise into the LMA PCF taper [2]:

$$A(t) = P_0^{|2|} \text{sech} \left( \frac{t}{T_0} \right) \left[ \exp \left( \frac{i \xi t^2}{2} \right) + \eta \hat{N} \exp(i2\pi \hat{U}) \right],$$

where $\hat{N}$ is a normally distributed random variable with mean value 0 and standard deviation 1, and $\hat{U}$ is a uniformly distributed random variable between 0 and 1. The noise factor $\eta$ indicates the amplitude of the noise relative to the pulse amplitude. As an example, Fig. 7 shows the input signal with black solid curves and output signal with blue solid curves. In this simulation, the noise amplitude $\eta = 0.05$ and other parameters are the same as that used.
previously. The red dashed curves show the output signal with noiseless input which is the same as the blue curve in Fig. 5(c). Figure 7(a) shows the waveforms in logarithmic scale. The 1 ps input pulse is compressed to about 53 fs with a noise floor in the whole time window. The pedestal is 40 dB lower than the pulse peak. The main part of the pulse above the pedestal is almost the same as the noiseless pulse shown in the red dashed curve. Figures 7(b) and 7(c) show the pulses in linear scale. We observed that the noise fluctuations on the input pulse has been greatly reduced and the pulse compression is unaffected by the presence of noise. The output waveform shown in Fig. 7(c) is almost identical to that from the noiseless input pulse. Because of the large bandwidth of the noise signals, the shot noise power in the initial pulse is distributed to the whole time window. It should be pointed out that the induced noise floor is dependent on the simulation time window chosen which can be related to the pulse repetition rate of the laser source.

In summary, using the nonlinearity increasing LMA PCF taper designed in Section 3.2, we showed that a 1 ps input pulse can be self-similarly compressed down to a pulse width of 53.6 fs using a simplified propagation model. We have also showed that the self-similar compression can largely reduce the shot noise on the input pulse and the compression process is unaffected by the presence of noise. The highly compressed high quality pulse is expected to show good performance in SC generation which will be investigated in Section 5.

4.3 The impact of dispersion variation

In the simplified taper design (STD) model and the simplified propagation model (SPM) used so far, the variation of fiber dispersion is neglected in the study of self-similar compression by the LMA PCF taper. It is therefore important to verify that such simplifications will not significantly change the design of the LMA PCF taper and the self-similar compression in the designed LMA PCF taper. We will first repeat the simulation of the pulse propagation in the LMA PCF taper designed with the simplification of Eq. (8) in Fig. 4 but will replace the constant effective dispersion by the varying dispersion profile shown in Fig. 4(a). Clearly, the varying dispersion will affect the self-similar compression scheme because the varying dispersion will violate the self-similar condition even if high order dispersion and high order nonlinearity effect are not included in the propagation model, i.e. Equation (3).

![Fig. 8. (a) The LMA PCF taper design, (b) dispersion and (c) nonlinearity with full taper design (FTD) model (red solid curves) and simplified taper design (STD) model (blue dashed curves). The (d) output waveforms and (e) evolutions of pulse width in both the STD and FTD models simulated with the simplified propagation model (SPM) and full propagation model (FPM).](image-url)

Next, for the fiber taper design, instead of using the simplification Eq. (8) which assumes a constant dispersion in the determination of $\gamma(z)$, we use Eqs. (5) and (6) directly to design
the fiber taper profile \( \Lambda(z) \). From Eqs. (5) and (6), if both the dispersion and nonlinearity vary along the fiber taper, the nonlinearity and dispersion profiles of the LMA PCF taper should follow the condition [31]

\[
\gamma(z) = \gamma(0) \frac{\beta_z(z)}{\beta_z(0)} \left[ 1 - \xi \int_0^z \beta_z(z') dz' \right]. \tag{14}
\]

Since the \( \gamma(z) \) and \( \beta_z(z) \) cannot be integrated analytically as in the case of Eq. (8), the required \( \Lambda(z) \) profile that satisfies Eq. (14) can only be obtained numerically through iterations based on the fitted \( \gamma(\Lambda) \) and \( \beta_z(\Lambda) \) values shown in Figs. 2 and 3. Here we use the same input pulse as that used in Figs. 5 and 6. Figure 8(a) shows the \( \Lambda(z) \) profile of the LMA PCF taper designed with the full taper design (FTD) model Eq. (14) (red solid curves). The taper profile with STD model Eq. (12) is also plotted (blue dashed curves) for comparison. The new profile starts with \( \Lambda = 36.6 \) \( \mu \)m and ends with \( \Lambda = 6.5 \) \( \mu \)m. The total taper length is 6.44 m. The corresponding dispersion and nonlinearity profiles \( \beta_z(z) \) and \( \gamma(z) \) are plotted in Figs. 8(b) and 8(c) in order to compare with those from the STD model.

For the self-similar compression, Figs. 8(d) and 8(e) show the output waveforms and the evolutions of pulse width in both STD and FTD simulated with the full propagation model (FPM) which includes the dispersion variation \( \beta_z(z) \) and the simplified propagation model (SPM) in which the dispersion variation \( \beta_z(z) \) is assumed to be constant. The red solid curves represent the taper designed with FTD and simulated with FPM. The blue dashed curves represent the results from the taper designed with STD, i.e. from Eq. (8), and simulated with SPM. The black solid curves represent the results from the taper designed with STD but simulated with FPM. In both Figs. 8(d) and 8(e), the results from STD + SPM and FTD + FPM are similar to each other. As for the propagation with STD + FPM, the pulse at the output is compressed to 66.4 fs with a slightly larger pedestal when compared to that from the other two models. From Fig. 8, the STD model using Eq. (8) and the SPM assuming \( \beta_z(z) = \beta_z^{\text{eff}} = \frac{1}{t} \int_0^t \beta_z(z) dz \) can predict the FWHM of the compressed pulse very well. Thus the simplified fiber design and propagation models are very handy in the design of the LMA PCF taper.

5. Highly coherent supercontinuum generation

From the above studies of self-similar pulse compression in the LMA PCF taper design, a 1 ps input pulse with large noise component can be compressed to ~53.6 fs with very low pedestal and the noise level is also reduced. We note that SC generated with < 50 fs pulses have much better coherence than that generated by > 1 ps pulses [2, 10, 12]. In this Section, we will characterize the pulse evolution and the coherence of the SC pumped by the 1 ps pulse and the self-similarly compressed 53.6 fs pulse.

To generate SC with the high power pump pulse at 1550 nm, we used an NZ-DSF with a nonlinear coefficient \( \gamma \) of 3 \( W^{-1}/km \) with \( \tau_s = 0.85 \) fs. The zero dispersion wavelength of the NZ-DSF is ~1500 nm and the up to 8-th order dispersion coefficients \( \beta_{k} \)'s, \( k = 2 \) to 9 at 1550 nm used in simulations are \(-5 \) ps\(^2\)/km, \(0.129 \) ps\(^2\)/km, \(-4.13 \times 10^{-4} \) ps\(^2\)/km, \(2.01 \times 10^{-6} \) ps\(^2\)/km, \(-1.32 \times 10^{-8} \) ps\(^3\)/km, \(1.11 \times 10^{-10} \) ps\(^3\)/km, \(-7.83 \times 10^{-13} \) ps\(^3\)/km, and \(2.70 \times 10^{-15} \) ps\(^3\)/km. The dispersion parameters are obtained from shifted dispersion curve of SMF-28 fiber by adjusting \( \beta_z \) according to commercially available non-zero dispersion-shifted fibers.

Figure 9 shows the evolutions of typical pulse shapes and spectra of the SC generated in the NZ-DSF. The results shown in Figs. 9(a) and 9(b) are pumped by the 1 ps pulse and the compressed pulse with 53.6 fs duration respectively at noise level \( \eta = 0.01 \). The top and bottom figures show the spectra and pulse shapes of the input and output pulses respectively. The middle figures show the evolutions of the spectra and pulse shapes which are normalized to the maximum power value in respective figure.
With the 1 ps pump pulse, an NZ-DSF with 6 m length generates a SC with a spectrum ranging from 1250 to 1850 nm. From Fig. 9(a), the waveform of the input pulse experiences very complex dynamics in the propagation. After propagating about 1 m in the NZ-DSF, fluctuation on the pulse shape arises quickly because of modulation instability, which greatly extend the bandwidth of the spectrum within a very short distance. The fluctuation eventually evolves to multiple solitons at about $z \sim 2$ m. The solitons then separate from the main pulse and form the complex details on the spectrum. We note that the intensities and positions of the multiple solitons, and the details of the spectrum depend on the noise seed and vary significantly from shot to shot.

Figure 9(b) shows the SC generated by the self-similarly compressed 53.6 fs pulse. The fiber length used for SC generation is 0.5 m which is much shorter than that used for the 1 ps pulse. From the output spectrum shown in the bottom figure, the generated SC covers more than 800 nm from 1150 to 1950 nm. The output spectrum is flatter and smoother than that obtained by the 1 ps pump pulse. The reason of such improvement can be seen in the evolution of the pulse shape, only one soliton is emitted from the main pulse in the soliton fission process. More important, the evolution of the pulse shape and spectrum is not sensitive to the noise seed which is therefore more coherent than that in Fig. 9(a).

![Fig. 9. The evolutions of the pulse shapes and spectra of supercontinuum generated by (a) the 1 ps pulse and (b) the compressed 53.6 fs pulse. The top and bottom figures show the spectra and pulse shapes of the input and output pulses respectively. The power shown in the evolution figures in the middle are normalized to the maximum power value in each figure respectively.](image)

To quantify the enhancement in coherence of the self-similar compression in the LMA PCF taper, the degree of coherence is used to characterize the output spectra of the SC which is defined as [10–12]:

$$\rho_{12}^{(1)}(\omega) = \frac{\left\langle \tilde{A}^*(\omega) \tilde{A}_s(\omega) \right\rangle_{\omega}}{\left\langle \tilde{A}(\omega)^2 \right\rangle}$$

$$\left(15\right)$$

where $\tilde{A}(\omega)$ is the amplitude of the signal in frequency domain. The degree of coherence in Eq. (15) measures the correlation of the signals from shot to shot at frequency $\omega$. The weighted degree of coherence can be used to estimate the averaged coherence in the whole spectral range of the SC which is defined as [11]

$$R = \frac{\int_0^\infty \left|\rho_{12}^{(1)}(\omega)\right| P(\omega) d\omega}{\int_0^\infty P(\omega) d\omega}$$

$$\left(16\right)$$
where \( \mathcal{P}(\omega) = \langle | \tilde{A}(\omega) |^2 \rangle \) is the ensemble average of the spectra with different noise seeds.

We first study the coherence of the SC generated with the 1 ps pump pulse. In the evaluation of \( |g_{12}^{(1)}| \), for each value of the noise amplitude \( \eta \), 50 instances of the initial 1 ps pulse with different random noise seeds are used. Figure 10 shows the calculated \( |g_{12}^{(1)}| \) curves and the corresponding spectra of the SC generated by the 1 ps pump pulse. The grey plots in Figs. 10(a) and 10(c) are the overlapped spectra of the 50 shots with noise level \( \eta = 1 \times 10^{-3} \) and \( \eta = 1 \times 10^{-6} \) respectively. The averaged spectra of each 50 shots are shown by the blue curves. The degree of coherence of the spectra in Figs. 10(a) and 10(c) are shown in Figs. 10(b) and 10(d) respectively. At noise level \( \eta = 1 \times 10^{-3} \), the SC spectra varies significantly from shot to shot in the whole frequency range and the coherence are almost zero in most spectral range except at a narrow band around the pump wavelength 1550 nm with the maximum \( |g_{12}^{(1)}| = 0.81 \). When the noise level \( \eta \) decreases to \( 1 \times 10^{-6} \), the spectra fluctuation decreases in most frequencies but there are still significant fluctuations at some wavelengths as shown in Fig. 10(c). The degree of coherence \( |g_{12}^{(1)}| \) has improved to \( > 0.5 \) in most part of the spectrum but \( |g_{12}^{(1)}| \) still varies greatly with wavelength. Thus for SC generation with 1 ps pump pulse, even very weak noise level will greatly degrade the coherence of the SC generated.

![Fig. 10. (a), (c) The spectra and (b), (d) degree of coherence of the SC generated by 1 ps pump pulse with noise level (a), (b) \( \eta = 1 \times 10^{-3} \) and (c), (d) \( \eta = 1 \times 10^{-6} \). The grey plots in (a) and (c) are the overlapped spectra of 50 shots in each figure. The blue curves are the averaged spectrum of the 50 shots. The red curves in (b) and (d) are the degree of coherence of the spectra in (a) and (c) respectively.](image)

![Fig. 11. (a), (c) The spectra and (b), (d) degree of coherence of the SC generated by the compressed 53.6 fs pump pulse with noise level at (a), (b) \( \eta = 0.1 \) and (c), (d) \( \eta = 1 \times 10^{-3} \). The grey plots in (a) and (c) are the overlapped spectra of 50 shots in each figure. The blue curves are the averaged spectrum of the 50 shots. The red curves in (b) and (d) are the degree of coherence of the spectra in (a) and (c) respectively.](image)
Fig. 12. The weighted degree of coherence (a) $R$ and (b) $K = \log(1 - R)$ measured for the SC generated by the 1 ps pulse (red curves with stars) and the compressed 53.6 fs pulse (blue curves with circles).

After the self-similar compression of the 1 ps pulse with noise in the LMA PCF taper, the compressed 53.6 fs pulse is then injected into an NZ-DSF with a shorter length 0.5 m, and the coherence of the generated SC is significantly enhanced as shown in Fig. 11. The plots in Fig. 11 are similar to that in Fig. 10 but the noise levels $\eta$ are 0.1 for Figs. 11(a) and 11(b), and $1 \times 10^{-3}$ in Figs. 11(c) and 11(d). Figs. 11(c) and 11(d) can be compared directly with Figs. 10(a) and 10(b) as they are at the same noise level $1 \times 10^{-3}$. The spectra shown in Fig. 11(c) show little variations from shot to shot where $|g_{12}^{(1)}|$ is almost 1 in the whole spectrum of the SC generated. Even at very large noise level $\eta = 0.1$, the SC generated with the compressed pulse still show very high level of coherence with $|g_{12}^{(1)}| > 0.8$ in most range of the spectrum except in a few narrow gaps.

To quantitatively compare the coherence of the SC generated with the pico- and compressed femto-second pulses, the weighted degree $R$ is measured for the spectra with different noise levels $\eta$ as shown in Fig. 12(a). For the 1 ps pump pulse, $R$ is close to 1 for $\log(\eta) < -7$ and drops quickly from 1 to < 0.3 when $\log(\eta)$ increases to > -5. For the compressed 53.6 fs pump pulse, $R$ is close to 1 until $\log(\eta)$ increases to > -2, then $R$ begins to drop and decreases to lower than 0.52 when $\eta > 0.3$ ($\log(\eta) > -0.52$). Since $R$ are very close to 1 for most of the $\eta$ values with the compressed 53.6 fs pump pulse, we plot $K = \log(1 - R)$ in Fig. 12(b). From Fig. 12, $\log(\eta)$ differs by about 5 for the pico- and compressed femtosecond pulses to generate SC for the same level of coherence. Thus self-similar compression of the 1 ps pulse has greatly enhanced the immunity of the coherence of the SC generated to noise by 5 order of magnitude.

6. Conclusion

In this paper, we proposed to use an LMA PCF taper to realize self-similar compression of high power picosecond pulses. We designed an LMA PCF the nonlinearity of which varies significantly with the air hole pitch size. Based on the nonlinearity profile and using an effective dispersion to represent the slightly varying dispersion profile, we proposed a simplified model to design the taper profile. Self-similar pulse compression a 1 ps pulse to 53.6 fs in a 6.4 m length LMA PCF taper is demonstrated by simulation using a GNLSE and realistic fiber parameters. The compressed pulse shows low pedestal. We also showed that large noise fluctuations on the input pulse are significantly reduced by the self-similar compression. We then compared the simplified fiber taper design method with that from the full design model taking into account the dispersion variation and found that they are in good agreement. Then both the 1 ps pulse and the compressed femtosecond pulse are injected into an NZ-DSF for supercontinuum generation. By evaluating the degree of coherence, we found that the compressed pulse can greatly enhance the coherence of the generated supercontinuum when compared to that generated by the 1 ps pulse. We found that self-similar compression can improve the tolerance of the SC generated to noise level $\eta$ by 5 order of magnitude.
Compared to using dispersion decreasing fiber for self-similar compression which is difficult to be realized, the proposed nonlinearity increasing fiber is based on mature and widely adopted PCF fabrication techniques. No special or challenging fiber design is required. Thus highly coherent supercontinuum generation by using 1 ps pump pulses can be realized in experiments as well as practical applications.

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