



Multiresolution discontinuity-preserving surface reconstruction

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Abstract

This paper proposes a single multiresolution framework for both surface fitting and discontinuities identification in the surface reconstruction problem. In particular, the Mallat–Zhong wavelet transform modulus maxima (WTMM) representation is employed for representing the surface which allows the discontinuity detection and surface fitting to be carried out simultaneously in a cooperative manner under the same multiresolution framework. On the one hand, the multiresolution feature analysis inherent in the WTMM representation helps characterize the strength of the different significant features, and thus by updating the WTMM surface in a number of energy-minimization processes, the discontinuities can be detected and tracked across various resolution levels. On the other hand, the detected discontinuities in turn control spatially the suitable degree of smoothness at various image positions when the surface is fitted to the input data in the energy-minimizing processes. In this way, the discontinuity detection and the surface fitting processes mutually assist each other. Experimental results show that a piecewise smooth surface can be reconstructed without discontinuity being over-smoothed. © 2001 Pattern Recognition Society. Published by Elsevier Science Ltd. All rights reserved.

Keywords: Surface reconstruction; Regularization; Discontinuity detection; Discontinuity characterization; Wavelet transform

1. Introduction

Surface reconstruction is a common problem encountered in computer vision, for instance in stereo imaging and visual motion analysis, when a dense depth map of the imaged scene is desired. It refers to a process in which a piecewise smooth surface is reconstructed from a set of noisy measurements. As identifying and locating depth and orientation discontinuities in a scene in many applications are important, the goal of surface reconstruction is not only to reconstruct the surface, but also to identify and preserve these discontinuities in the reconstruction.

In stereo imaging, measurements are often obtained through the feature correspondence between the left and the right images. It thus gives an irregular sampling pattern and the sampling density could be very sparse. It may also happen that some parts in the image have no measurements as there may be no feature detectable in either the left or the right image. The reconstruction problem is therefore ill posed in nature. Some additional constraints are needed in order to make the problem well posed.

A popular approach to solve this ill-posed problem is by the use of the regularization technique [1,2]. It restricts the admissible solution to be a smooth function, denoted as $f(x, y)$. The problem can be formulated as finding a depth function $f(x_k, y_k)$ for all image positions (x_k, y_k) that minimizes the error function

$$E(f) = \sum_k (z_k - f(x_k, y_k))^2 + \lambda S(f). \quad (1)$$

The first term is the data constraint, i.e., the residual error in fitting the surface f to the measurements, z_k . The

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second term, $S(f)$, is the smoothness requirement placed on f , and λ is a regularization constant which controls the tradeoff between the data constraint and the smoothness constraint. A popular choice for the functional $S(f)$ is the following quadratic form:

$$S(f) = \iint [(D_x^2 f)^2 + 2(D_x D_y f)^2 + (D_y^2 f)^2] dx dy, \quad (2)$$

where D_x and D_y are the differential operators with respect to x and y , respectively.

There are two main concerns in the regularization approach. One is the slow convergence rate of the minimization process. Another is the discontinuity-preserving ability. It can be seen that smoothness constraint is applied globally to the entire scene. While ensuring a smooth surface, this constraint is inadequate near discontinuities. As a result, discontinuities are blurred by minimizing E without a proper choice of λ .

One way to improve the convergence behaviour of the regularization approach is to employ multiresolution processing, such as the multigrid method [3], the hierarchical basis methods [4] and the wavelet approaches [5,6]. However, such work aims primarily at speeding up the surface fitting process; how the discontinuities can be identified and preserved during the surface fitting process is not explicitly addressed.

On discontinuity detection, different methods, such as bending moment and the variational continuity control, have been proposed to quantify discontinuities. The discontinuities information is then used to control spatially the amount of smoothness applied in different parts of an image [2,3,7]. However, most of these methods are single resolution based. It was demonstrated in the field of image coding [8] that multiresolution feature analysis is better than single resolution analysis in identifying discontinuities (the significant features in their language of image coding) from noise. It would be desirable to incorporate the multiresolution concept into both surface fitting and discontinuity detection in the surface reconstruction problem. However, results on multiresolution discontinuity identification in image coding cannot be directly borrowed for surface reconstruction, as the two problems are drastically different. While image coding addresses the problem of preserving the discontinuities in an image (or some regularly sampled data) for compression and decompression purposes, surface reconstruction addresses the problem of fitting a dense surface to some irregularly sampled and often sparse data while identifying the discontinuities at the same time.

This paper proposes to combine the surface reconstruction with the multiresolution-based discontinuities detection into a single framework. In particular, the wavelet transform modulus maxima (WTMM) representation [9] is employed for representing the surface,

which allows the two processes: discontinuity detection and surface fitting, to be carried out simultaneously in a cooperative manner under the same multiresolution framework. On the one hand, the multiresolution feature analysis inherent in the WTMM representation helps to characterize the strength of different significant features, and thus by updating the WTMM represented surface in a number of energy minimization processes, the discontinuities can be detected and tracked across various resolution levels. On the other hand, the detected discontinuities in turn control spatially the degree of smoothness at various image positions when the surface is fitted to the input data in the energy-minimizing processes. In this way, the discontinuity detection and surface fitting processes mutually assist each other.

2. Preliminaries on wavelet transform

Wavelet transform (WT) is a mathematical tool for analyzing features in a signal. Singularities and irregular structures often carry important information from an informatic–theoretic point of view. Wavelet analysis provides a kind of mathematical “microscope” to zoom in or zoom out on these interesting structures.

A representation known as the WTMM was proposed by Mallat and Zhong to enable a multiresolution-significant feature analysis [10]. The wavelet bases are similar to the deviative of Gaussian, which induces a near optimal spatial-frequency joint localization. The associated WT thus acts like an edge detector at multiple scales. The WTMM representation then consists of the modulus maxima points of the resulting WT, which correspond to significant features detected at different scales.

One property of this scheme is that the modulus maxima can only stay or disappear but cannot be created as one moves from fine to coarse scales. Therefore, a significant feature, such as a large jump discontinuity will have modulus maxima points across a few scales, while a weak feature such as those induced by noise would exist only in the fine scale. Therefore, significant feature characterization is inherent in the representation through the modulus maxima tracking across scales, as was demonstrated in Ref. [11].

In this paper, the significant features characterized by the modulus maxima was used to define the degree of smoothness spatially and will be explained in more detail later. The filter coefficients are summarized in Table 1. The scheme used is shown in Fig. 1. The discrete filters G_j , H_j , K_j and L_j are obtained by inserting $2^j - 1$ zeros between the successive filter coefficients of each of G , H , K , and L , respectively. \bar{H}_j denotes the time reverse of H_j and j is the level of decomposition/reconstruction in WT.

3. Multiresolution approach in the wavelet framework

3.1. Problem formulation

Discretizing Eqs. (1) and (2) gives the following expression for the surface reconstruction error:

$$E = \sum_k (z_k - f(m_k, n_k))^2 + \sum_{i,j} \lambda(i,j)A^2(i,j) + \sum_{i,j} \lambda(i,j)B^2(i,j) + 2\sum_{i,j} \lambda(i,j)C^2(i,j), \quad (3)$$

where

$$A(i,j) = f(i,j + 1) - 2f(i,j) + f(i,j - 1),$$

$$B(i,j) = f(i + 1,j) - 2f(i,j) + f(i - 1,j),$$

Table 1

Filter coefficients of the finite impulse response filters H , G , K , and L that correspond to the separable two-dimensional quadratic spline wavelets [10]

n	H	G	K	L
-3			0.0078125	0.0078125
-2			0.054685	0.046875
-1	0.125		0.171875	0.1171875
0	0.375	-2.0	-0.171875	0.65625
1	0.375	2.0	-0.054685	0.1171875
2	0.125		-0.0078125	0.046875
3			0.0078125	

$$C(i,j) = f(i + 1,j + 1) - f(i + 1,j) - f(i,j + 1) + f(i,j), \quad (4)$$

and A , B and C are the discrete approximations to the differential operations D_x^2 , D_y^2 and $D_x D_y$, respectively. The image up to a particular resolution can be represented using the quadratic spline wavelets as follows:

$$f_{a-1}(m,n) = \sum_i \sum_j \bar{h}_{a-1}(m,i) s_{a-1}(i,j) \bar{h}_{a-1}(n,j) + \sum_i \sum_j l_{a-1}(m,i) w_{a-1}^1(i,j) k_{a-1}(n,j) + \sum_i \sum_j k_{a-1}(m,i) w_{a-1}^2(i,j) l_{a-1}(n,j), \quad (5)$$

where $\sum_i h_a(m,i) s_a(i,j)$ designates the convolution of rows of $s_a(i,j)$ with 1D filter $h_a(m)$ and $\sum_j s_a(i,j) h_a(n,j)$ designates the convolution of columns of $s_a(i,j)$ with 1D filter $h_a(n)$. Eq. (5) can be written in matrix-vector notation as

$$f_{a-1} = \bar{H}_{a-1} S_{a-1} \bar{H}_{a-1}^T + L_{a-1} W_{a-1}^1 K_{a-1}^T + K_{a-1} W_{a-1}^2 L_{a-1}^T \quad (6)$$

where a is the resolution level, f_{a-1} is the image which consists of information only up to $a - 1$ level, s_a is the scaling coefficient of f at level a , w_a^1 and w_a^2 are the wavelet coefficients at level a in the horizontal and vertical directions, respectively, \bar{h} , l and k are the reconstruction filters for s_a , w_a^1 and w_a^2 and are defined in Fig. 1 and Table 1.

The surface is completely characterized by parameters s_a , w_a^1 and w_a^2 at a particular resolution $a - 1$. A gradient approach can be employed to update these parameters

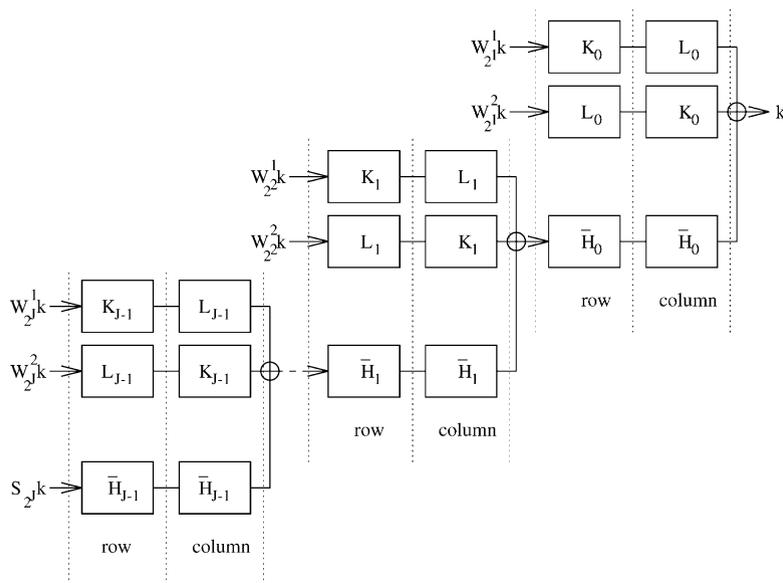


Fig. 1. The j -level 2D inverse discrete dyadic WT.

iteratively. The gradient of E with respect to $s_a(m, n)$ is given by

$$\begin{aligned} \frac{\partial E}{\partial s_a} = & -2 \sum_k (z_k - f(m_k, n_k)) D(m_k, n_k, m, n) \\ & + 2 \sum_{i,j} \lambda(i, j) A(i, j) A'(i, j) \\ & + 2 \sum_{i,j} \lambda(i, j) B(i, j) B'(i, j) \\ & + 4 \sum_{i,j} \lambda(i, j) C(i, j) C'(i, j), \end{aligned} \quad (7)$$

where

$$\begin{aligned} A'(i, j) = & D(i, j + 1, m, n) - 2D(i, j, m, n) \\ & + D(i, j - 1, m, n), \end{aligned} \quad (8)$$

$$\begin{aligned} B'(i, j) = & D(i + 1, j, m, n) - 2D(i, j, m, n) \\ & + D(i - 1, j, m, n), \end{aligned}$$

$$\begin{aligned} C'(i, j) = & D(i + 1, j + 1, m, n) - D(i + 1, j, m, n) \\ & - D(i, j + 1, m, n) + D(i, j, m, n), \end{aligned}$$

$$\begin{aligned} D(i, j, k, l) = & \frac{\partial f(i, j)}{\partial s_a(k, l)} \\ = & \bar{h}_a(i, k) \bar{h}_a(j, l). \end{aligned}$$

The derivative of E with respect to w_a^1 and w_a^2 can be obtained accordingly.

It should be noted that as the surface is resolution dependent, in the sense that it is represented only up to $a - 1$ resolution, the parameters A, B, C, D, A', B', C' and the spatial distribution of λ are also resolution-level dependent. The problem then is to find a systematic way to estimate the unknowns s_a, w_a^1, w_a^2 and λ at different resolutions.

3.2. The general algorithm

Two main parts of our algorithm are the multiple resolution surface reconstruction and the determination of λ spatially based on the reconstructed surface. As WTMM is employed for surface representation, surface reconstruction at a particular resolution reduces to parameter estimation as outlined above. From the WTMM representation, the reconstructed wavelet coefficients, w_a^1 and w_a^2 , give us clues about the locality of significant features. We can apply feature tracking based on the reconstructed w_a^1 and w_a^2 as described in Ref. [10] to test the significance of a particular feature. If large wavelet coefficients are observed for a number of scales, it can be assured that a significant feature exists. In this case, a

weak smoothness constraint should be applied in order to preserve the sharpness of that feature. However, if only small coefficients are observed for a number of scales, it is likely that a significant feature is absent at that segment. In this case, a smoothness constraint should be applied to ensure the smoothness of the reconstructed surface.

Therefore, the reconstructed wavelet coefficients w_a^1 and w_a^2 across a number of scales can be used to provide an indication of the degree of smoothness that should be applied spatially. In other words, λ can be controlled spatially using the multiresolution inherent in the WTMM representation.

In summary, our algorithm can be outlined as follows. If we consider the resolution level to be three,

1. Obtain the scaling coefficient, s_3 , by assuming no discontinuities. It is done through minimizing the error E by gradient approach.
2. Based on s_3 , estimate w_3^1 and w_3^2 by using the gradient search to minimize the error E . A surface, f_2 , with resolution up to level 2 is then constructed which can be used to give an estimate of the discontinuities. The information can then be used to update λ as described below.
3. Based on the above-estimated f_2 (i.e., w_3^1, w_3^2 and s_3 and hence s_2), estimate w_2^1 and w_2^2 again by the gradient search to minimize E . Then extract the modulus maxima of both pairs w_2^1, w_3^1 and w_2^2, w_3^2 and apply feature tracking for feature characterization. This in turn gives an estimate of discontinuities and λ is updated accordingly. Coefficient shaping can also be applied to smooth out some small variations and ensure the causality principle being satisfied. This point will be explained more in detail later.
4. Based on the above-estimated f_1 , estimate w_1^1 and w_1^2 by the similar process. Then apply feature tracking for discontinuities characterization and coefficient shaping as Step (3).

Two important aspects of the algorithm are the feature tracking for feature characterization and coefficient shaping/smoothing. They will be outlined in the following two subsections.

3.3. Feature characterization

Feature tracking is essential to characterize the “significance” of a particular feature which then determines the degree of smoothness constraint to be applied spatially. The individual WTMM represents significant features across scales. These maxima can be tracked in the 1D domain for feature characterization. However, in two or higher dimensions, they are usually not independent features; they refer to points or extended boundaries and belong to certain lines or curves. Hence, the individual

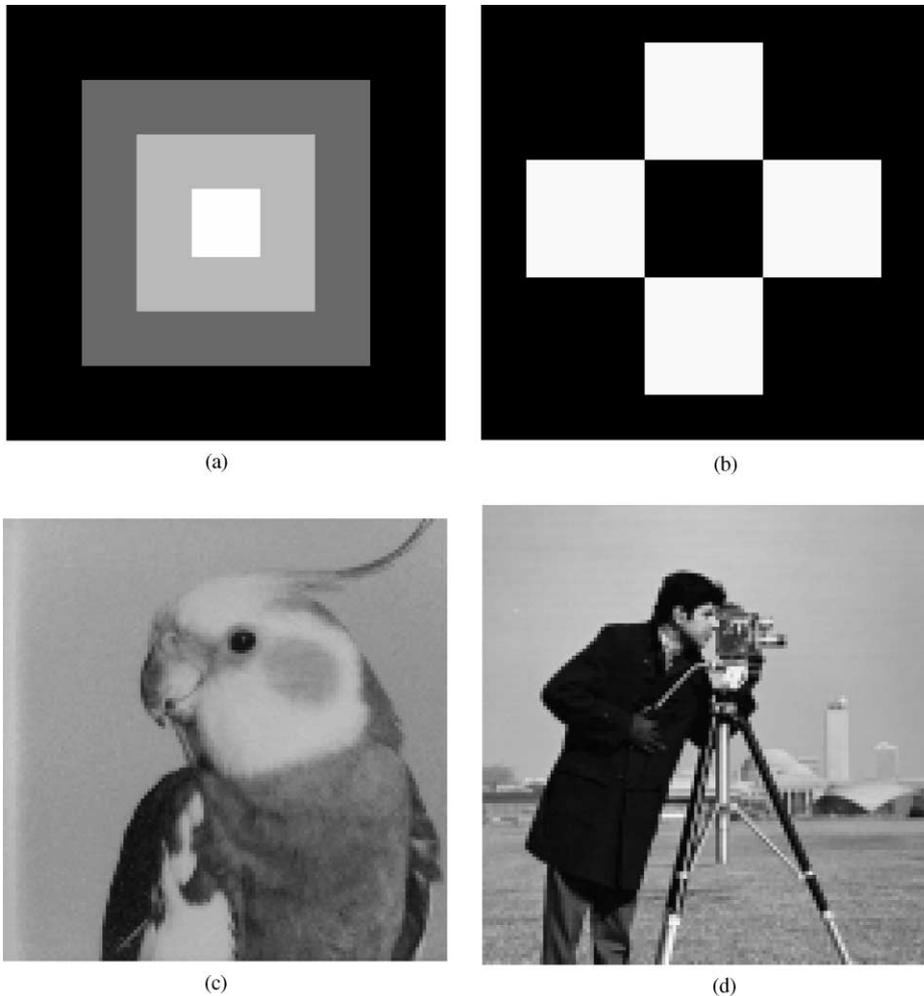


Fig. 2. The images used to test the surface reconstruction algorithm: (a) the wedding cake range image; (b) the checkerboard image; (c) the bird image and (d) the camera man image.

wavelet modulus maxima can be linked together to form contours and can be used as primitives in feature tracking across different scales.

In forming the wavelet modulus maxima contours, it is necessary to discard weak and short contours to facilitate the feature tracking process. The boundaries of important coherent structures often generate long contour curves, whereas contours resulting from noise are short. We thus remove any contour curve whose length is smaller than a given length threshold. Also, contour that has a low average amplitude corresponds to small variation in the image which can be ignored. The length threshold and the amplitude threshold are set to be 30% of their respective mean values of all contours in an image. Here, instead of fixing the threshold values, pre-determined percentages are set which could characterize

different distributions of length/amplitude variations in different images. We keep only those contours that satisfy both the length threshold and the amplitude threshold. Thus, the thresholds for both length and amplitude are set automatically for every test image.

The criteria used for feature tracking are position, sign and amplitude of the contours. The determining factor is the sign. If the sign between two modulus maxima contours at two scales are not the same, then they do not match with each other. If the sign is correct, check the position. The positional tolerance which is determined by the width of the wavelet reproducing kernels is used to decide whether the contours are close or not. If the contours are close together, check the amplitude to see if they are similar. It should be noted that the matching between the fine-scale and the coarse-scale contours can

only be a many-to-one mapping, but not a one-to-many mapping. Since contours instead of individual pixels are matched, the contour is kept if a match is found. As we need to ensure that no “extra” features are created from fine to coarse scales, the whole contour is smoothed for those unmatched contours in the coarse scale.

So from this feature tracing step, significant feature characterization is achieved. The problem then is to set $\lambda(i, j)$ based on this characterization.

A popular choice for $\lambda(i, j)$ is a binary set, i.e., $\{0, 1\}$. If a particular pixel (i, j) is known to be a discontinuous point, then the smoothness constraint is turned off by setting

$\lambda(i, j)$ to zero, otherwise, the smoothness constraint is enforced by setting $\lambda(i, j)$ to one. We also adopt this approach except that $\lambda(i, j)$ in our case is determined by the wavelet maxima contours. Where the wavelet maxima contours are found, λ is set to zero to turn off the smoothness constraint, and where the wavelet maxima contours are not found, λ is set to one to ensure surface smoothness. In this way, λ is set based on the inherent multiresolution nature in surface reconstruction. Also, it should be stressed that the features found in using the feature tracking outlined above ensure that features are connected, rather than isolated.

3.4. Smoothing

Coefficient shaping or smoothing is essential in two ways. First, it is needed to ensure that significant features are not created from fine to coarse scales (the causality principle). Second, it is needed to remove some small variations in the wavelet coefficients which are not genuine significant features that correspond to “strong” features such as jump discontinuity.

When we decide to remove a particular contour, each wavelet modulus maximum point along the contour needs to be smoothed out. As a separable wavelet transform scheme is chosen in our implementation, the smoothing can be simplified to one-dimensional implementation, i.e., the smoothing is done along the row for $W_a^1(x, y)$ and across the column for $W_a^2(x, y)$. The simplest way to do this is to first locate the unwanted modulus maximum point and one minimum point on either side of the unwanted point. In case both the left and the right sides have minimum points, we choose the one which has a magnitude close to the unwanted modulus maximum point. This ensures that the smoothing does not bring

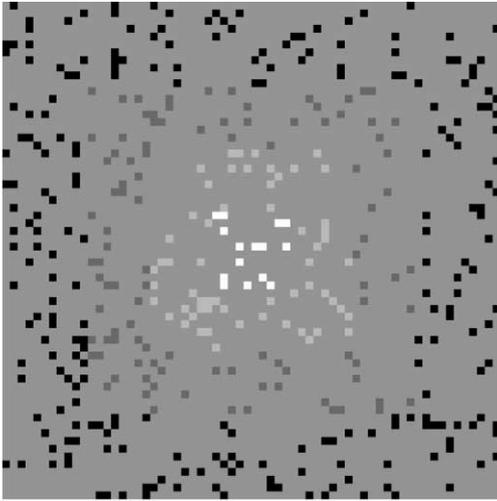
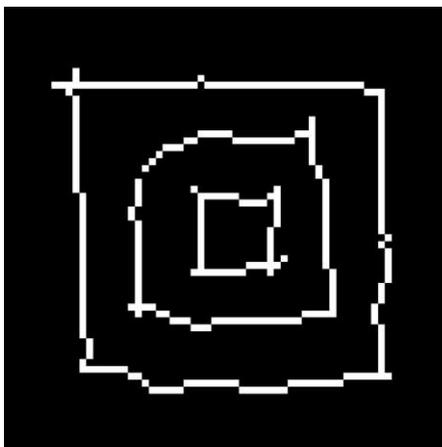
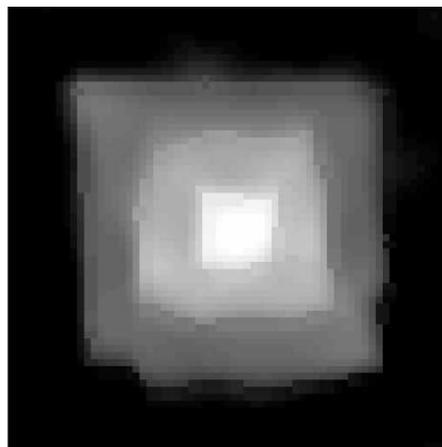


Fig. 3. 10% sampling density of the wedding cake image. Dots indicate the sampled data, and the brightness on them indicates the depth. The brighter the point is, the closer to the viewer it is.



(a)



(b)

Fig. 4. The results obtained with the proposed algorithm for the 10% wedding cake range data: (a) the discontinuities found and (b) the reconstructed surface.

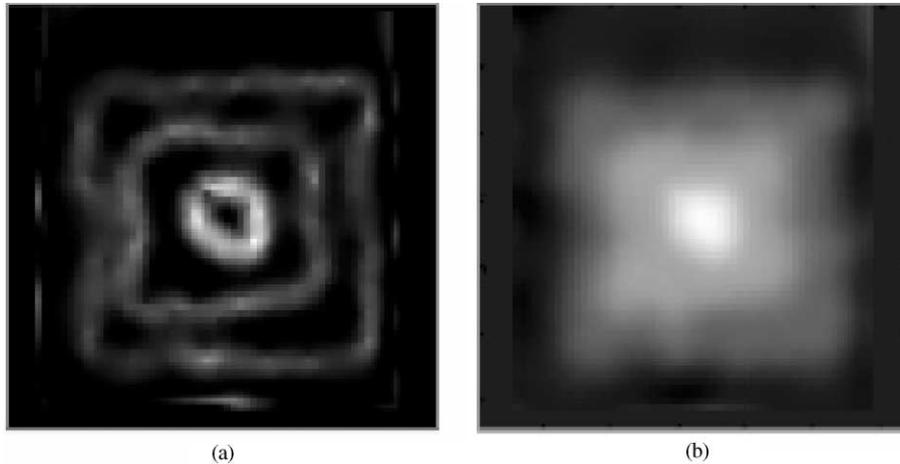


Fig. 5. The results obtained with the classic algorithm by using global smoothness assumption for the 10% wedding cake range data: (a) the discontinuities found and (b) the reconstructed surface.

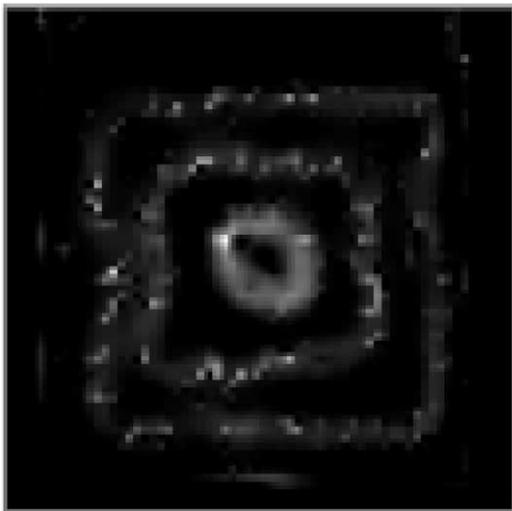


Fig. 6. The resultant discontinuity map obtained by decreasing the bending moment threshold in the classic method.

a great change to the original sequence. After locating the two points, the sequence between these two points is replaced by a constant value sequence. For simplicity, the constant value can be chosen to be the mean value of the maximum and the minimum points. In this way, the unwanted maximum point is smoothed out with a small change in the original sequence. A more sophisticated way to do the smoothing is proposed in Ref. [8]. However, the actual implementation of the smoothing operator is not crucial in our case as the smoothing operator is applied once only in estimating each wavelet subband, unlike the case in Ref. [8].

4. Experimental results

We provide some experimental results to illustrate the edge-detection/edge-preserving performance of our surface reconstruction algorithm. The proposed algorithm is applied to various data sets including both synthetic and real images. Its performance as a surface reconstruction algorithm was demonstrated on four images as shown in Fig. 2. The first two images, the wedding cake and the checkerboard image, are range data while the last two images, the bird and the camera man images, are intensity data. Range data usually consist of boundaries that are long and connected, while the intensity data can sometimes consist of long and connected boundaries as well as short and isolated edges.

Fig. 3 shows synthetic range measurements where zero-value means no sample data and the intensity denotes the range. A sampling density of only 10% of the range image is used in which the sparseness of the sample data makes reconstruction and discontinuity detection difficult. Fig. 4 shows the discontinuities found and the reconstructed surface using our algorithm. Despite the sparse data available over a surface with a lot of discontinuities, our algorithm gives a good reconstruction and the discontinuities detected are mostly connected.

For comparison purposes, a classic method using bending moment for discontinuity detection is applied to the same image [3]. Fig. 5 shows the reconstructed surface by using global smoothness assumption and the discontinuity detected using bending moment. A few more iterations are applied by relaxing the global smoothness assumption through decreasing the threshold for the bending moment as suggested in Ref. [3]. The threshold setting is thus not very crucial as this method relies on successively decreasing the threshold

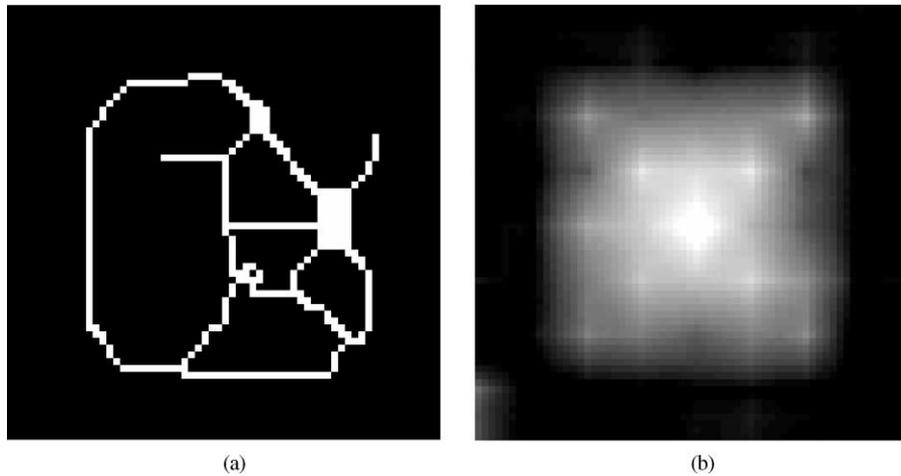


Fig. 7. The results obtained with the bending moment-based algorithm for the 10% wedding cake range data: (a) the discontinuities found and (b) the reconstructed surface. The resolution level is set to be 3 and the edge threshold is 150.

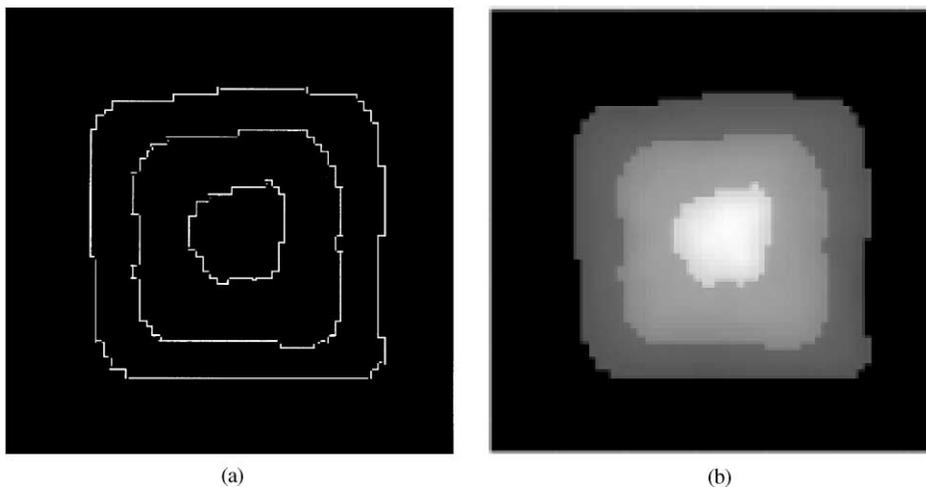


Fig. 8. The results obtained with the proposed algorithm for the 15% wedding cake range data: (a) the discontinuities found and (b) the reconstructed surface.

for surface reconstruction. Fig. 6 shows the resultant discontinuity map. It can be seen that the discontinuities detected are not connected. Unlike our formulation, connectness is not enforced in the formulation. As a result, the discontinuity map would consist of isolated edges.

A comparison is also made with a recently published wavelet-based reconstruction algorithm which uses bending moment for discontinuity detection [5]. Fig. 7 shows the discontinuities found and the reconstructed surface for the 10% sampling data. A number of edge thresholds have been tried and the best discontinuity map is obtained by setting the threshold to be 150. It should be noted that the off-diagonal terms of the pre-

conditioned equation system are all approximated to be zero in this formulation. The effect of this diagonalization depends on the regularization constant, the sampling density and the reconstruction level chosen. Thus, the diagonal assumption may not be valid at all times. As to this case in which the sampling is very sparse, the reconstruction obtained would not be good.

The sampling density is varied to test the performance of our algorithm under different values of sampling density. Fig. 8 shows the discontinuities found and the reconstructed surface when the sampling density is increased from 10 to 15%. As expected, the quality of the reconstructed surface is improved. The three layers of the

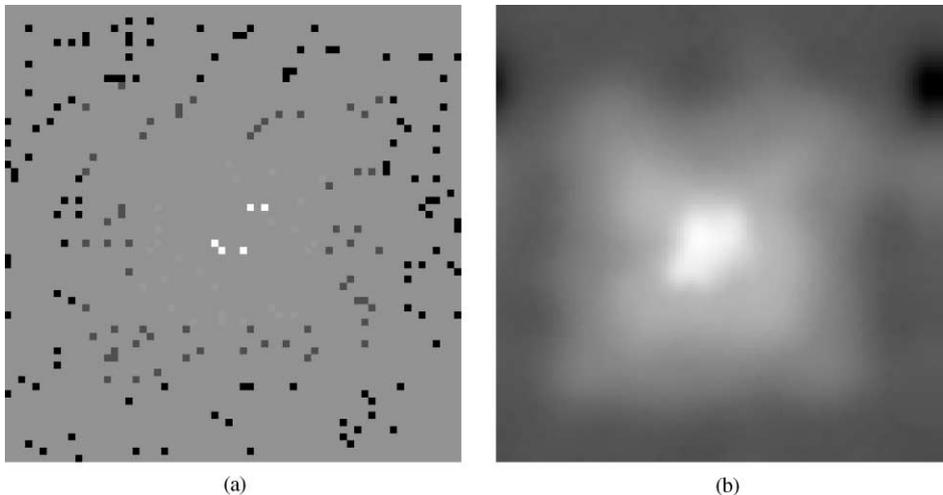


Fig. 9. The results obtained with the proposed algorithm for the 7.5% wedding cake range data: (a) the sampling pattern and (b) the reconstructed surface.

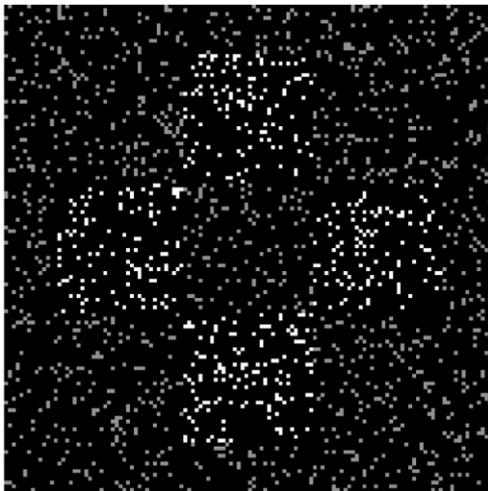


Fig. 10. 10% sampling density of the checkerboard image.

wedding cake can be clearly distinguished. Compared to the reconstructed surface from the 10% sampling case shown in Fig. 4, the surface of each layer is “flatter” and has less ripples. For this wedding cake example, 10–15% sampling density provides enough measurements for a good reconstruction and an accurate discontinuities detection.

The sampling density is then decreased to 7.5%. Fig. 9 shows the sampling pattern and the reconstructed surface. It can be seen that the discontinuities cannot be located precisely and thus the boundaries between layers

are blurred. Due to the sparse sampling pattern, there are no measurements in some regions. The algorithm cannot determine whether there are any discontinuities in these regions. As a result, a lowpass function is simply fitted in these regions which makes the three layers of the reconstructed surface indistinguishable. This shows that under a very low sampling density case, a ‘smoothed’ version of the original surface is reconstructed by our algorithm.

In the second example, a checkerboard surface of size 128×128 is interpolated. The sampling density is 10% and the sampled image is shown in Fig. 10. Fig. 11 shows the discontinuities found and the reconstructed surface using our algorithm. It can be seen that the discontinuities detected are mostly connected and discontinuities are preserved in the reconstructed surface despite the sparseness of the sample data.

The image considered in the third example is the bird image and 30% sampling density is used. As a high sampling density is used, a comparison is made between our proposed algorithm and another wavelet-based algorithm which uses bending moment for discontinuity detection. Fig. 12 shows the reconstructed surfaces for the two algorithms. It can be seen that the reconstruction of our proposed algorithm seems to have less artifacts and has a better reconstruction.

The next image considered is the camera man image and 30% sampling density is used. This image consists of a foreground — the camera man and his camera, and a background — the sky, the glass field and a few buildings. The discontinuities map of the camera man image is complicated as can be seen in Fig. 13(a). There are long and connected discontinuities as well as short and isolated discontinuities. Some of the discontinuities are clustered together. In the feature characterization and

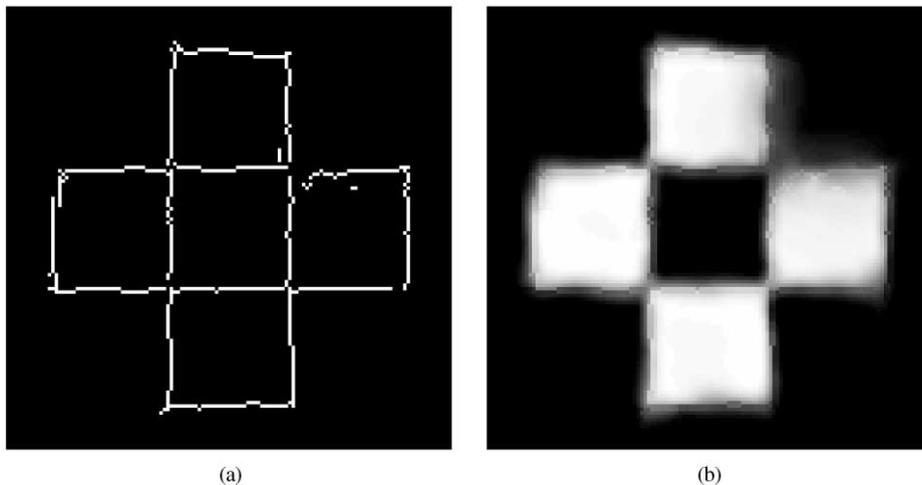


Fig. 11. The results obtained with the proposed algorithm for the 10% sampling density of the checkerboard image: (a) the discontinuities found and (b) the reconstructed surface.

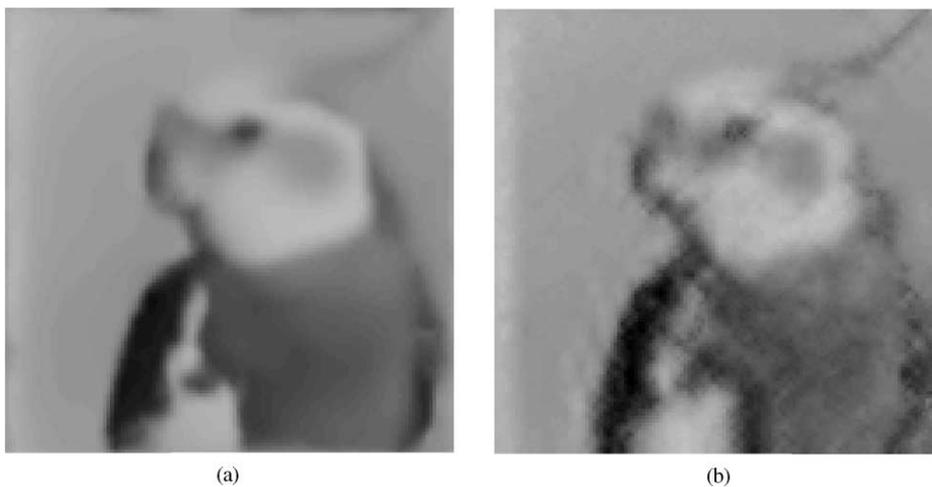


Fig. 12. The reconstructed surfaces obtained by (a) our proposed algorithm and (b) the bending moment-based algorithm for a 30% sampling density of the bird image.

the smoothing parts of our algorithm, weak and short contours are discarded while long and connected contours are kept. As shown in Fig. 13(b), the sharpness around the camera man in the reconstructed image is kept very well, but the short and isolated contours such as those from the buildings and the glass field are mostly discarded by our algorithm. In most of the applications, the long and connected discontinuities correspond to important structure while short and isolated discontinuities correspond to wrong measurements, noises, or the background information. Our algorithm can reconstruct a surface that preserves the sharpness of the important

structure represented by long and connected discontinuities.

5. Conclusion

Global smoothness constraints intrinsic to standard regularization are inadequate near discontinuities. Since discontinuities play a vital role in inverse visual problems, some ways are needed to detect their locations so as to avoid smoothing across the discontinuous points. We proposed to use Mallat–Zhong WTMM for surface

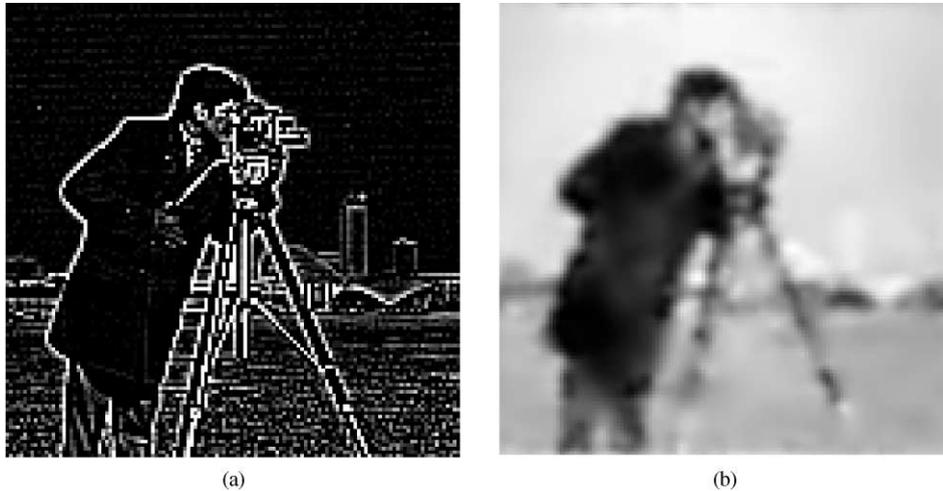


Fig. 13. (a) The original discontinuities map of the camera man image; and (b) the reconstructed image.

representation. The multiresolution feature analysis inherent in the representation helps in characterizing the strength of different significant features which can then be used to control the application of smoothness constraint spatially.

The new algorithm starts with an overly smoothed surface. Discontinuities information which are characterized by the WTMM are added to the reconstructed surface progressively. Thus, the discontinuity map undergoes refinement during surface reconstruction. It should also be noted that WTMM contours, instead of WTMM points, are used for discontinuities characterization. Therefore, the discontinuities detected are mostly connected, which matches the observation that discontinuities are mainly extended, rather than isolated, features.

The new algorithm has been tested on a number of synthetic and real data sets. Simulation results show that the approach can preserve the discontinuities while ensuring smoothness in most cases. Comparison of the proposed algorithm with a classic method [3] and a recently published algorithm [5] have also been presented. It is observed that the sparseness of sample data has made the assumption in the latter algorithm invalid. As the connectedness of discontinuities is not enforced in their detection, the discontinuity map produced may not be connected. In contrast, the discontinuity map produced from our algorithm is encouraged to be connected.

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