

FAST ALGORITHM FOR BINARY FIELD WAVELET TRANSFORM FOR IMAGE PROCESSING

N.F. Law¹, A.W.C. Liew² and Wan-Chi Siu¹

¹Centre for Multimedia Signal Processing,
Department of Electronic and Information Engineering,
The Hong Kong Polytechnic University, Hung Hom, Hong Kong.

²Department of Electronic Engineering,
City University of Hong Kong, Kowloon Tong, Hong Kong.

ABSTRACT

Lifting scheme for the real field wavelet transform has provided a new insight into its practical implementation. This paper shows that a similar scheme can be developed for the binary field wavelet transform. In particular, by using the Euclidean algorithm, the binary filters can be decomposed into a finite sequence of simple lifting steps over the binary field. This provides an alternative method for the implementations of the binary field wavelet transform for image processing applications. It is found that the new implementation can reduce the number of arithmetic operations involved in the transform and allow an efficient in-place implementation structure.

1. INTRODUCTION

Wavelet transform (WT) has recently been generalized to finite fields [1-4] to take into account the fact that images in most applications are represented by a finite number of bits. In particular, [3] proposed a binary wavelet transform (BWT) for binary images in GF(2). GF(2) denotes the Galois field of order 2, i.e., there are only two elements, 0 and 1, in the binary fields. They satisfy all of the field axioms for addition and multiplication with the modulo-2 operation.

The binary transform has several advantages. First, the intermediate and the transformed results produced by the binary transform are binary. No quantization error is introduced. Second, the arithmetic operation in the binary field is restricted to be modulo-2. Thus the transform can be performed efficiently by using simple Boolean operations. The potential use of the binary transform in lossless image coding has been demonstrated in [3,4]. The construction of a BWT is equivalent to the design of a two-band perfect reconstruction filter bank, except that the

original and the transformed signals are restricted to be binary.

In order to maintain an invertible BWT with desirable multiresolution properties, [3] proposed three constraints for the binary filters. They are the bandwidth, the perfect reconstruction and the vanishing moment constraints. However, the form of the binary filters designed by using only these three constraints could change with the signal length as pointed out in [4]. To overcome this difficulty, the perpendicular constraint is added to make the form of the binary filters independent of the signal length. Examples of binary filters designed using these four constraints are given in [4].

The design of the binary filters in [3,4] is in the sequency domain. Similar to the lifting scheme in the real field WT, a pure spatial interpretation of the binary filters should provide new insights into examinations of the BWT. The objective of this paper is to develop such a pure spatial interpretation. By using the Euclidean algorithm, we show that the binary filters can be decomposed into a finite sequence of simple lifting steps over the binary field. This decomposition produces an alternative implementation for the binary filters, reduces the number of arithmetic operations involved in the transform and allows an efficient in-place implementation structure.

2. BINARY FIELD WAVELET TRANSFORM

The construction of the BWT is equivalent to the design of a two-band perfect reconstruction filter bank. Mathematically, a transform matrix T is constructed as,

$$T = \begin{bmatrix} H \\ G \end{bmatrix} \quad (1)$$

where

$$\begin{aligned} H &= (\bar{h}|_{s=0}, \bar{h}|_{s=2}, \dots, \bar{h}|_{s=N-2}) \\ G &= (\bar{g}|_{s=0}, \bar{g}|_{s=2}, \dots, \bar{g}|_{s=N-2}) \end{aligned} \quad (2)$$

$\bar{a}|_{s=k}$ defines a vector with element formed from a circular shifted sequence of \bar{a} by an integer k , A' is the transpose of A and,

$$\begin{aligned} \bar{h} &= \{h_0, h_1, h_2, h_3, \dots, h_{N-2}, h_{N-1}\}' \\ \bar{g} &= \{g_0, g_1, g_2, g_3, \dots, g_{N-2}, g_{N-1}\}' \end{aligned} \quad (3)$$

h_i and g_i are respectively the scaling and the wavelet coefficients. The BWT is then defined as,

$$y = Tx \quad (4)$$

where x is the original signal and y is the transformed signal. To guarantee a multiresolution decomposition that inherits the characteristics of the WT as in the real field case, four constraints are imposed on the binary filters. They are the bandwidth, the perfect reconstruction, the vanishing moment and the perpendicular constraints [3,4].

Filter design is bounded by incorporating all of these constraints. In [4], it was shown that there are 4 and 32 different pairs of binary filters for filter lengths which are equal to 4 and 8 respectively. The four feasible designs for the filter length which equals 4 are,

$$\left\{ \begin{array}{l} \text{lowpass,} \\ \text{bandpass} \end{array} \right\} = \left\{ \begin{array}{l} 4.1. \{0,0,0,1\} \{0,0,1,1\} \\ 4.2. \{0,1,0,0\} \{1,1,0,0\} \\ 4.3. \{1,0,1,1\} \{0,0,1,1\} \\ 4.4. \{1,1,1,0\} \{1,1,0,0\} \end{array} \right\} \quad (5)$$

The 32 binary filters for the filter length which is equal to 8 are grouped into four classes depending on the number of "ones" in the binary filters. Examples of the binary filters in each group are given below,

$$\left\{ \begin{array}{l} \text{lowpass,} \\ \text{bandpass} \end{array} \right\} = \left\{ \begin{array}{l} 8.1. \{0,1,0,0,0,0,0,0\} \{1,1,0,0,0,0,0,0\} \\ 8.2. \{1,1,1,0,0,0,0,0\} \{1,1,0,0,0,0,0,0\} \\ 8.3. \{1,1,1,1,0,0,0,1\} \{1,1,0,0,0,0,0,0\} \\ 8.4. \{1,1,1,1,1,1,1,0\} \{1,1,0,0,0,0,0,0\} \end{array} \right\} \quad (6)$$

These binary filters can be expressed in the form of the Laurent polynomials as follows,

$$H(z) = \sum_{k_b}^{k_e} h_k z^k \quad (7)$$

where k_b and k_e are respectively the smallest and the largest integer numbers for which h_k is non-zero. h_k belongs to the binary field and the summation is defined over the binary field, i.e., the addition and multiplication operations are performed with the modulo-2 operator. The degree of $H(z)$ is defined as,

$$|H(z)| = k_e - k_b \quad (8)$$

Thus a monomial, z^l , has a zero degree.

3. PROPOSED IMPLEMENTATION SCHEME

The polyphase representation is employed to study the spatial structure embedded in the BWT. A signal is split into two disjoint sets, which are called the polyphase

components of the signal. Mathematically, the polyphase representation is given by,

$$X(z) = X_e(z^2) + z^{-1}X_o(z^2) \quad (9)$$

where $X_e(z)$ and $X_o(z)$ denote respectively the even and the odd components of the signal. Figure 1 shows the filter bank structure of the BWT while Figure 2 shows the corresponding polyphase representation. The two polyphase matrices, $A(z)$ and $B(z)$, can be written in terms of the forward and the inverse filters as follows,

$$\begin{aligned} \begin{pmatrix} L(z) \\ B(z) \end{pmatrix} &= A(z)' \begin{pmatrix} X_e(z) \\ X_o(z) \end{pmatrix} = \begin{pmatrix} h_e(z) & z^{-1}h_o(z) \\ g_e(z) & z^{-1}g_o(z) \end{pmatrix} \begin{pmatrix} X_e(z) \\ X_o(z) \end{pmatrix} \\ \begin{pmatrix} X_e(z) \\ X_o(z) \end{pmatrix} &= B(z) \begin{pmatrix} L(z) \\ B(z) \end{pmatrix} = \begin{pmatrix} r_e(z) & r_o(z) \\ s_e(z) & s_o(z) \end{pmatrix} \begin{pmatrix} L(z) \\ B(z) \end{pmatrix} \end{aligned} \quad (10)$$

For an invertible system, it is required that,

$$A(z)' B(z) = I \quad (11)$$

where I denotes the identity matrix. Eqn.11 is always satisfied due to the perfect reconstruction constraint used in the filters design. The determinant of the polyphase matrices needs to be found so that the spatial structure embedded in these matrices can be analyzed. Since the binary filters in the BWT are FIR in structure, the determinant of $A(z)$ and $B(z)$ can be expressed in the form of the Laurent polynomials. The determinant of $B(z)$ is equal to the reciprocal of that of $A(z)$ as shown in eqn.11. If the determinant of $A(z)$ is not a monomial, i.e., z^l for some integers l , its reciprocal would result in an infinite series. Therefore, the determinant of $A(z)$ can always be expressed in the form of z^l .

Without loss of generality, we can assume that the determinant of $A(z)$ is 1. If this is not the case, the polynomials are always divisible by their determinants. This is equivalent to shifting the corresponding filters in the spatial domain. Let the modified polyphase matrix be written as follows,

$$P(z) = \begin{pmatrix} m_e(z) & m_o(z) \\ n_e(z) & n_o(z) \end{pmatrix} \quad (12)$$

Depending on the relative degrees between $m_e(z)$, $m_o(z)$, $n_e(z)$ and $n_o(z)$, we can define two different factorization structures for $P(z)$ by using the Euclidean algorithm as follows.

Factorization 1: If $|m_e(z)| \geq |n_e(z)| \geq 0$ and $|m_o(z)| \geq |n_o(z)| \geq 0$, then there exists the Laurent polynomials, $s_1(z)$, $m_e^{new}(z)$ and $m_o^{new}(z)$ with,

$$0 \leq |m_e^{new}(z)| \leq |n_e(z)|$$

$$0 \leq |m_o^{new}(z)| \leq |n_o(z)|$$

such that the following factorization holds,

$$\begin{aligned} m_e(z) &= m_e^{new}(z) + s_1(z)n_e(z) \\ m_o(z) &= m_o^{new}(z) + s_1(z)n_o(z) \end{aligned} \quad (13)$$

This effectively reduces the degrees of $m_e(z)$ and $m_o(z)$ such that they are smaller than or equal to $n_e(z)$ and $n_o(z)$ respectively. In matrix notation, eqn.13 can be written as,

$$\begin{pmatrix} m_e(z) & m_o(z) \\ n_e(z) & n_o(z) \end{pmatrix} = \begin{pmatrix} 1 & s_1(z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m_e^{new}(z) & m_o^{new}(z) \\ n_e(z) & n_o(z) \end{pmatrix} \quad (14)$$

Factorization 2: If $|n_e(z)| \geq |m_e(z)| \geq 0$ and $|n_o(z)| \geq |m_o(z)| \geq 0$, then there exists the Laurent polynomials, $t_1(z)$, $n_e^{new}(z)$ and $n_o^{new}(z)$ with

$$\begin{aligned} 0 &\leq |n_e^{new}(z)| \leq |m_e(z)| \\ 0 &\leq |n_o^{new}(z)| \leq |m_o(z)| \end{aligned}$$

such that the following factorization holds,

$$\begin{aligned} n_e(z) &= n_e^{new}(z) + t_1(z)m_e(z) \\ n_o(z) &= n_o^{new}(z) + t_1(z)m_o(z) \end{aligned} \quad (15)$$

This reduces the degrees of $n_e(z)$ and $n_o(z)$ such that they are smaller than or equal to $m_e(z)$ and $m_o(z)$ respectively. In matrix notation,

$$\begin{pmatrix} m_e(z) & m_o(z) \\ n_e(z) & n_o(z) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ t_1(z) & 1 \end{pmatrix} \begin{pmatrix} m_e(z) & m_o(z) \\ n_e^{new}(z) & n_o^{new}(z) \end{pmatrix} \quad (16)$$

By combining eqn.14 and eqn.16, the polyphase matrix can always be factorized in terms of two types of elementary matrices as follows,

$$P(z) = \begin{pmatrix} m_e(z) & m_o(z) \\ n_e(z) & n_o(z) \end{pmatrix} = \prod_{i=1}^n \begin{pmatrix} 1 & s_i(z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ t_i(z) & 1 \end{pmatrix} \quad (17)$$

Note that the factorization in eqn.17 is different from the lifting scheme in the real-field WT. This is due to the fact that there are only two elements in the binary field. As a result, there is only one way to factorize element 1, i.e., 1×1 , in contrast to the real field case where there are infinite ways for factorization, i.e., $1 = K \times K^{-1}$ [5]. Figure 3 shows a new implementation structure resulting from eqn.17. It provides a pure spatial interpretation for the BWT and thus an alternative implementation structure.

4. DESIGN EXAMPLE

Due to the limitation of spaces, only one example is given. In fact, the factorization in eqn.17 is applicable to all the binary filters in the BWT. We consider the design (8.4) in eqn.6. The forward lowpass and the forward bandpass filters can be written as,

$$\begin{aligned} H(z) &= 1 + z + z^2 + z^3 + z^4 + z^5 + z^6 \\ G(z) &= 1 + z \end{aligned}$$

The polyphase matrix then becomes,

$$\begin{pmatrix} 1 + z + z^2 + z^3 & 1 + z + z^2 \\ 1 & 1 \end{pmatrix}$$

The determinant is, however, z^3 . So it is scaled to 1. The modified matrix is then factorized as follows,

$$\begin{pmatrix} 1 & 1 + z + z^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ z^{-3} & 1 \end{pmatrix}$$

The implementation structure is shown in Figure 4. This corresponds to the following implementation,

$$\begin{aligned} L_k^{(0)} &= x_{2k} \\ B_k^{(0)} &= x_{2k+1} \\ L_k^{(1)} &= L_{k+3}^{(0)} \\ B_k &= B_k^{(0)} + L_{k-3}^{(0)} \\ L_k &= L_k^{(1)} + B_k + B_{k+1} + B_{k+2} \end{aligned} \quad (18)$$

Examining closely eqn.18, it can be seen that an in-place structure exists in the implementation, i.e., there is no need to assign extra memory spaces for storing any intermediate or the final results. All newly calculated sequences can be stored in the location of the "old" sequences. This greatly reduces the memory cost. Also, once the forward transform is defined, finding the inverse is straightforward. To determine the original signal, the whole implementation is simply reversed.

5. EXPERIMENTAL RESULTS

Let us discuss various aspects of the BWT in this section. First, the BWT is applied to images. The 2D BWT can be computed as the tensor products of 1D binary wavelets. Figure 5 shows two transformed images, one is a binary image and the other is the Lena image. Note that the transformed coefficients resulting from the BWT are binary. There is no expansion in their range. Most of the large value coefficients using the BWT correspond to the high frequency transition. We can clearly see that the high frequency edge transitions were mapped onto the high frequency regions in the transformed data. In particular, all three bandpass subbands show the horizontal, the vertical and the diagonal edges of the original image.

The computational complexity of the BWT computed by using the filter bank approach is compared with that using the proposed scheme. A modulo-2 addition is equivalent to an exclusive-or (XOR) operation, so the number of XOR operation is counted. Table 1 summarizes the number of XOR operations in both approaches. It can be seen that by using the proposed scheme, the computational complexity is greatly reduced. Table 2 shows the computational time spent on

transforming the eight-bit 256×256 Lena image using the proposed scheme with a PC of 300 MHz. One eight-bit gray level image consists of eight bit planes in which the BWT can be applied independently to each bit plane. It can be seen that for all designs, the overall computational time for both the forward and the inverse transforms is less than 20ms.

6. CONCLUSIONS

The objective of this paper is to develop a pure spatial interpretation of the binary filters for the binary wavelet transform for image processing applications. By using the Euclidean algorithm, we show that the binary filters can be decomposed into a finite sequence of simple lifting steps over the binary field. This decomposition provides a new spatial implementation structure for the binary wavelet transform. It is found that the proposed scheme can reduce the number of arithmetic operations and allow an efficient in-place implementation structure for the binary field wavelet transform.

7. ACKNOWLEDGMENTS

N.F. Law thanks the Centre for Multimedia Signal Processing, Department of Electronic and Information Engineering, the Hong Kong Polytechnic University for the support she receives under its research fellowship scheme.

8. REFERENCES

- [1] See-May Phoong and P.P. Vaidyanathan, "Paraunitary Filter Banks over Finite Fields", *IEEE Trans. Signal Proc.*, Vol. 45, 1443-1457, 1997.
- [2] T. Cooklev, A. Nishihara and H. Sablatash, "Theory of Filter Banks over Finite Fields" *Proc. Asia Pacific Conf. Circuits Syst.*, Taiwan, 260-265, 1994.
- [3] M.D. Swanson and A.H. Twefik, "A Binary Wavelet Decomposition of Binary Images", *IEEE Trans. Image Proc.*, Vol. 5, 1637-1650, 1996.
- [4] N.F. Law and W.C. Siu, "A Filter Design Strategy for Binary Field Wavelet Transform Using the Perpendicular Constraint", *submitted to IEEE Trans. Image Proc.*, 2000.
- [5] W. Swelden, "The Lifting Scheme: A Construction of Second Generation Wavelets", *SIAM J. Math. Anal.*, Vol. 29, 511-546, 1997.

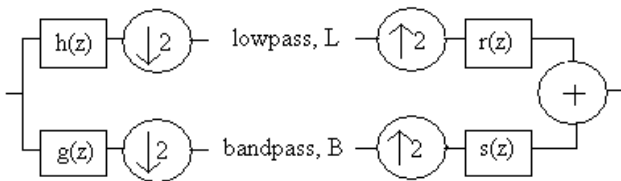


Figure 1: The filter bank implementation of the BWT.

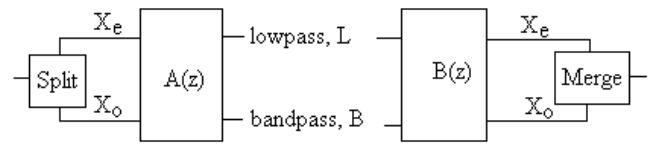


Figure 2: Polyphase representation of the BWT.

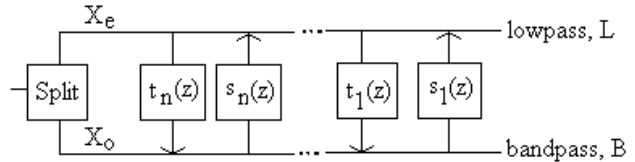


Figure 3: The proposed spatial implementation.

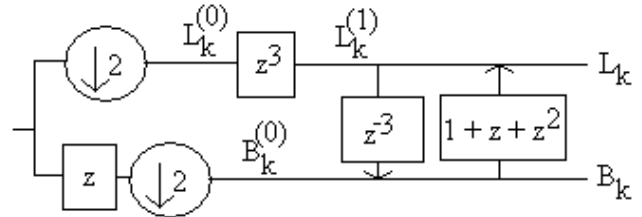


Figure 4: The spatial domain implementation of design 8.4

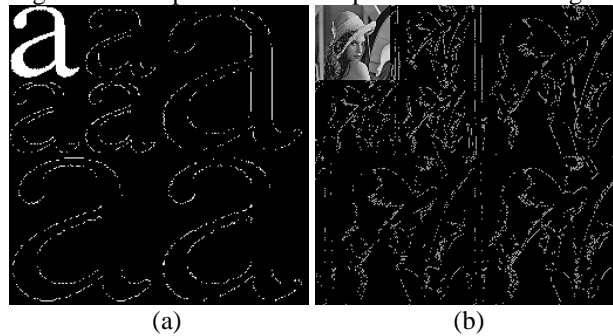


Figure 5: Two sets of binary wavelet transformed images, (a) the binary image "a" and (b) the Lena image.

Filters	No of XOR in filter bank approach	No of XOR in proposed scheme	Speed-up
Design 8.1	1	1	0 %
Design 8.2	3	2	33.3 %
Design 8.3	5	3	40 %
Design 8.4	7	4	42.9 %

Table 1: Number of XOR operations using the filter bank approach and the proposed approach.

Filters	Forward BWT (ms)	Inverse BWT (ms)
Design 8.1	15.6	16.1
Design 8.2	16.4	17.7
Design 8.3	19.1	19.0
Design 8.4	19.9	19.9

Table 2: The computational complexity of the BWT in transforming the eight-bit 256×256 Lena image using the proposed scheme.