Tutorial (15/3/2006) Solutions

1. See figure 1. From this constellation diagram we can see that the scheme is not very efficient (the constellation points are not very well spread out), and we could therefore conclude that this scheme would not perform well in the presence of noise.

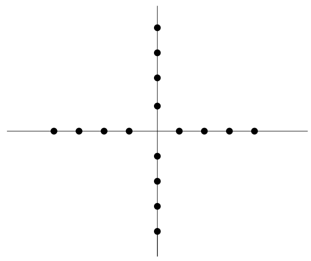

Figure 1: Constellation diagram for question 1.

2. See figure 2. The three schemes are PSK (or 4-QAM),ASK and QAM. They have 2 bits/signal element, 1 bit/signal element and 2 bits/signal element, respectively.

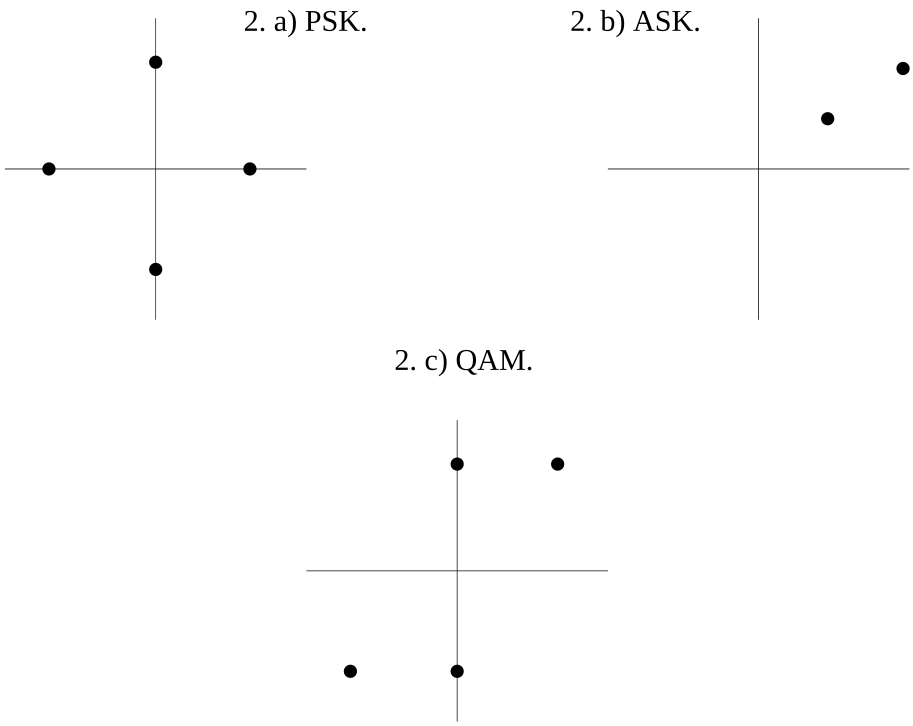

Figure 2: Constellation diagrams for question 2.

3. (a) Differential Manchester has 0.5 bits/signal element and 16-QAM is 4 bits/signal element. Therefore after encoding with Differential Manchester and then encoding the output of that with 16-QAM one achieves only 2 bits/signal element.

(b) Note that $\frac{E_b}{N_0} = \frac{S/R}{N/B} = \frac{SB}{NR}$ where S and N are the signal and noise power and B and R are the bandwidth and the data rate. Suppose that $B = D$ (i.e. each signal element is one entire cycle of the sinusoid), then $\frac{E_b}{N_0} = \frac{8\times10^{-3}}{6\times10^{-8}}\frac{D}{R} = \frac{4}{3} \times 10^5 \times \frac{1}{2} = 6.67 \times 10^5$ or 48 dB.

4. (a) Bandwidth efficiency $\mathcal{E}$ is given by $\mathcal{E} = \frac{R}{B}$ and $\frac{E_b}{N_0} = \frac{S/R}{N/B} = \frac{S}{N}\frac{B}{R} = \frac{\frac{S}{N}}{\mathcal{E}}$ I.e.

$$\mathcal{E} = \frac{\frac{S}{N}}{\frac{E_b}{N_0}}$$

(b) In both cases, a bandwidth efficiency of 0.5 is achieved if $\frac{E_b}{N_0} = 2\frac{S}{N}$ i.e. the SNR should be 6 dB.

(c) Now, if $B = 4$ MHz and $\mathcal{E} = 0.5$, then $R = 2$ Mbps. FSK has one bit per signal element and therefore $D = 2$ Mbaud, 16-QAM has 4 bits per signal element so $D = 500$ kbaud.

5. (a) We have that $C = 2B\log_2 M$ and $C = B\log_2(\text{SNR} + 1)$, therefore $M^2 = \text{SNR} + 1$, we suppose that $\text{SNR} \gg 1$ so $M^2 \approx \text{SNR}$, converting to dB we obtain $(\text{SNR})_{\text{dB}} \approx 20\log_{10} M = 20\log_{10} 2^n = 20n\log_{10} 2 \approx 6.02n$.

(b) Since $98 \approx 6.02n + 1.76$ we have that $n \approx 15.98$. I.e. to achieve sufficient SNR we need *not less than* 16 bits.

(c) Now $40 \approx 6.02n + 1.76$ implies that $n \approx 6.352$. If $n \leq 6$ then the quantisation error will be larger than the signal noise, converselt if $n \geq 7$ then the signal noise will be larger than the quantisation error. Therefore, exactly 7 bits should be used.