Low-bit Conversion & Noise Shaping

Preface

- Expensive A/D and D/A converters, labor-intensive calibration procedures during manufacture, and sophisticated circuit design required to achieve the performance and maintain it over the life of the converter.

Noise Shaping

- Noise shaping can be achieved with \textit{sigma-delta modulation}.
- As its name implies, \textit{noise shaping} shifts noise away from the audio band (0 to 20 kHz) thus lowering audio band noise.
Sigma-delta modulation (SDM) encoder.

Sigma-delta modulation (SDM) decoder.

Mathematical basis of 1st-order noise shaping

$f_s$ is the sampling frequency. ($=1/T_0$)
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Example: principle of operation of a 1\textsuperscript{st} order noise shaper

The local average of the output equals to the input.

A sequence of fixed output pattern is generated for a constant input.
The higher the order of the noise shaper, the more the quantization noise can be removed from the audio band.

A successful noise shaping circuit thus seeks to balance a high oversampling rate with noise shaping order to reduce in-band noise and shift it away from audible range.

N-th-order noise shaping could employ cascaded sections as in this 3rd-order noise shaper.

Tricks for improving the performance

- The low-level linearity of low-order noise shaping circuits can be degraded by 2 problems known as idle patterns and thresholding.
- A zero or very low level input may result in a regular 1010 pattern. If the period of the repetition of such patterns is long enough, they may be audible as a deterministic or oscillatory tone, rather than as noise.
To remove signal distortion, a noise shaping circuit must employ **dithering**.

Dither can be added to the input data so the circuit always operates with a changing signal even when the audio signal is zero or DC.

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The total noise power is increased as additional noise is added.

But it changes the nature of the quantization error into a white noise, which makes it more inaudible to human.

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**Use of noise shaping**

- Noise shaping is advantageous because a simple shaper can remove quantization noise from the audio band.
- These algorithms are more effective at high sampling rates so there is more spectral space between the highest audio frequency and the Nyquist frequency.
Another possible objective of noise shaping is reduction in the number of bits required to represent the signal. This implies quantization.

Noise shaping is good for quantization and hence can be used for such a purpose.

The so-called 1-bit stream technology used in current CD players is based on this technique.

With any 1-bit system, because of the noise shaping employed, it is difficult to quote a meaningful figure for signal-to-noise ratio because the noise level varies with respect to frequency.

However, in general, a 1-bit system can provide an audio-band noise floor lower than that encountered in 16- or 18-bit conversion.

Revisit conventional system

Summary of spectral characteristics of a conventional D/A conversion system.
Oversampling D/A system

Several manufacturers have developed D/A conversion methods which employ third-order noise shaping.
Their design details differ somewhat, particularly in the noise shaping algorithms and the low-bit output signal.
The MASH system co-developed by NTT and Matsushita is a multistage third-order noise shaping method.

One implementation of this design accepts 16-bit words at a nominal sampling frequency, and a digital filter stage performs 8-times oversampling and outputs 24-bit words.
Noise shaping circuits output data as an 11-value signal, at a 32-times oversampling rate.

Digital-to-analog conversion is accomplished via PWM (pulse width modulation), outputting 1-bit data at a 768-times oversampling rate.
Conversion is then performed by simply passing the 1-bit stream through a low-pass filter.
The outputs of each stage:

\[ Y_1 = X + (1 - z^{-1})N_1 \]
\[ Y_2 = -N_1 + (1 - z^{-1})^2 N_2 \]

Resultant transfer function:

\[ Y = Y_1 + (1 - z^{-1})Y_2 = X + (1 - z^{-1})^3 N_2 \]

Generally, if cascaded noise shaping circuits exceed second order they can be prone to oscillation.

MASH system avoids this through its multistage configuration.

The final element in the system is D/A conversion.

The eleven-value signal is converted into pulses, each with a width corresponding to one value.

This can be accomplished by applying the 4-bit output of the DSP to a ROM to map 11 amplitude values into 22 time values with constant amplitude.
Because the signal is represented by a pulse width modulation waveform, conversion is performed by simply passing the signal through a low-pass filter.

Because great timing accuracy can be achieved through crystal oscillators, the widths are very accurate, and hence the error of the signal is low.

Such a method achieves 20-bit resolution, but greater resolution is possible.
Application of Noise Shaping in Analog-to-digital Conversion

Revisit conventional system

Oversampling A/D system

- An oversampling A/D converter is simple:

- The input signal is first passed through a simple analog anti-aliasing filter, and the input signal is sampled at a very fast rate of \( R \) to extend the Nyquist frequency.

- The analog low-pass filter at the input is to remove the frequency components which cannot be removed by the digital filter, but, because the preliminary sampling rate is high, the analog low-pass filter is low order.
In any case, noise performance hinges on the oversampling rate and order of noise shaping employed.

The higher the order of the sigma-delta modulator we use, the lower the oversampling rate is required to achieve a given S/N performance.

The decimation process low-pass filters the signal and noise in the 1-bit code, bandlimiting the 1-bit code prior to sample rate reduction to remove alias components.

The decimation process also replaces the 1-bit coding with 16-bit coding, for example, provides a lower sampling rate, and generates a PCM output.

The decimation filter can be designed so that its frequencies of maximum attenuation will coincide with the potentially aliasing frequencies.

A comb filter is an expedient choice because its design does not require a multiplier.
In recursive form, the transfer function of a comb filter can be written as:

\[ H(z) = \frac{1 - z^{-R}}{1 - z^{-1}} \]

No multiplication is involved (unity coefficients).

Filtering is generally performed with decimation simultaneously.

Advantages of oversampling noise shaping A/D converter over conventional A/D converters:
- It eliminates brick-wall analog filters.
- It achieves increased resolution compared to SAR methods by extending the spectrum of the quantization error between analog input and digital output far outside the audio band.

A good balance is required.