How Channel Mis-Match Would Degrade the Bit-Error Rate of the Song-Fung-Wong-Meng-Tseng Transceiver Architecture for Block-Based Single-Carrier Transmission

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Abstract—Song, Fung, Wong, Meng & Tseng in [1] has earlier proposed a new transceiver architecture for block-based single-carrier modulation, to shorten the guard interval or even to remove cyclic prefixing altogether. That architecture involves zero-inserting precoder and a two-stage linear equalizer. However, the second-stage equalizer would require a perfect knowledge of the propagation channel’s impulse response. That prior knowledge is seldom precisely available to real-world systems, due to the receiver’s imperfect channel-estimation, and due to the channel’s own time-variability. This paper investigates how the Song-Fung-Wong-Meng-Tseng scheme of [1] be degraded in its bit-error rate (BER) by imperfect information on the channel impulse response, i.e. when channel-mismatch exists.

Index Terms—equalizers, interchannel interference, interference suppression, intersymbol interference, least square methods.

I. INTRODUCTION

Consider the standard architecture of the single-input single-output block-based single-carrier (SC) transceiver, shown in Figure 1. There in the transmitter, a “cyclic prefix” (CP) is added to each block of information-bearing bits, so as to mitigate inter-symbol interference due to a temporally spreading propagation channel. A total mitigation would require this “cyclic prefix” to exceed the channel’s impulse response in duration, thereby rendering the linear convolution between the block of information-bearing bits with the channel impulse response to become a circular convolution. However, the presence of this “cyclic prefix” incurs overhead costs, by occupying time-slots that could carry information-bearing bits.

In order to shorten the “guard interval”, Song, Fung, Wong, Meng & Tseng [1] has proposed a new architecture, involving a zero-inserting precoder and a two-stage linear equalizer, shown in Figure 2. There, the first-stage equalizer consists of linear single-tap-per-subcarrier frequency-domain equalization (FDE). The second-stage equalizer maximizes the SINR, in the time-domain, based on (i) the data received during the above mentioned zero-padded sub-intervals of the single-carrier modulation, to estimate the interference-plus-noise characteristics; and (ii) the data received during the aforementioned “precoded” zero-energy symbol-intervals, to estimate the combined effects of the signal-of-interest’s self-interference, of any multiple-access-user interference, of any overlaid interference, and of the additive noises. From (i) and (ii) together, the signal-of-interest’s (SOI) energy is reduced in the zero-energy symbol-intervals by a post-FFT linear single-tap-per-subcarrier frequency-domain equalizer (FDE) $W$, which would constitute a linear minimum-mean-square-error (LMMSE) equalizer, if no interference existed, if the precoding length exceeds the channel’s impulse response, and if no channel mismatch exists. Finally, the abovementioned denigrating effects (of the signal-of-interest’s self-interference, of any multiple-access-user interference, of any overlaid interference, and of the additive noises) are “subtracted” from the information-bearing parts of the symbol-block, via a time-domain signal-to-interference-and-noise (SINR) maximizer in the receiver’s second stage.

The architecture in [1] can function with any non-zero number of zero-energy symbols, even if without a cyclic prefix. However, the second-stage equalizer presumes perfect prior knowledge of the propagation channel’s impulse response. This presumption is seldom valid in practical deployment, because of the receiver’s imperfect channel-estimation, and because of the channel’s own time-variability. As to how the Song-Fung-Wong-Meng-Tseng architecture’s bit-error rate (BER) would be degraded by imperfect information on the channel impulse response – that degradation has been partly investigated in [2], via only Monte Carlo simulations, but with comparison to the counterpart degradation for a “purely” minimum mean-square-error (MMSE) receiver under the same channel-mismatch conditions. This paper will show new Monte Carlo simulations on how the Song-Fung-Wong-Meng-Tseng architecture would be degraded, in comparison to how a “purely” zero-forcing (ZF) receiver would be degraded under identical channel-mismatch conditions.

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II. REVIEW OF BLOCK-BASED CYCLICALLY-PREFIXED SINGLE-CARRIER TRANSMISSION THROUGH A TIME-DISPERSED CHANNEL

Referring to Figure 1: The information-bearing symbols \( \{u(j), \forall j\} \) are transmitted at the symbol rate, into disjoint blocks of \( N \) symbols. Represent the \( k \)-th block as an \( N \)-element vector, 
\[
\mathbf{u}(k) = [u_{N-1}(k), \ldots, u_0(k), \ldots, u_{N-1}(k)]^T,
\]
with \( u_n(k) = u(k N + \frac{n}{N} - 1 + n) \), for \( n = 0, 1, \ldots, \frac{N-1}{N} \). \( u(k) \) is modeled as an independent, zero-mean random process, i.e., \( E[u(i)u^*(j)] = \sigma_u^2 \delta(i-j) \), where \( \delta(i) \) symbolizes the Kronecker-delta function, and \( \sigma_u^2 \) denotes the signal power.

Prepend \( \mathbf{u}(k) \) with a length-\( G \) guard interval, which could be a cyclic prefix (CP), which replicates the last \( v \) entries of \( \mathbf{u}(k) \). Mathematically, this cyclic-prefixing may be realized by a multiplication of \( \mathbf{u}(k) \) into an \( (N+G) \times N \) cyclic-prefix-insertion matrix, 
\[
\mathbf{T}_{cp} = \begin{bmatrix} \mathbf{0}_{G \times (N-G)} & \mathbf{I}_N \\ \mathbf{I}_N & \mathbf{0}_{(N-G) \times N} \end{bmatrix},
\]
to produce the \((N+G)\)-element vector, \( \tilde{\mathbf{u}}(k) = \mathbf{T}_{cp}\mathbf{u}(k) \). This CP could reduce or eliminate up to \( G \) taps of inter-block interference (IBI) that might arise due to a frequency-selective-channel fading. The guard interval needs not be a cyclic prefix as above, but could be entirely zero-energy symbols, or some mix of the two.

Consider a frequency-selective time-invariant channel of order \( Q \), with the discrete-time impulse-response taps of \( h_0, h_1, \ldots, h_Q \). This channel's output is modeled as corrupted by additive noise, symbolized by the \((N+G)\)-element noise-vector \( \mathbf{\eta}(k) \), which is zero-mean, temporally white, and statistically independent from \( \mathbf{u}(k) \).

Hence, the received data's \( k \)-th symbol-block would equal \( \tilde{\mathbf{r}}(k) = \mathbf{H}_0 \tilde{\mathbf{u}}(k) + \mathbf{H}_1 \mathbf{T}_{cp}\mathbf{u}(k-1) + \mathbf{\eta}(k) \), where \( \mathbf{H}_0 \in \mathbb{C}^{(N+G)\times(N+G)} \) represents a lower triangular Toeplitz matrix, with its first column being \( [h_0, h_1, \ldots, h_Q, 0, \ldots, 0]^T \); and \( \mathbf{H}_1 \in \mathbb{C}^{(N+G)\times(N+G)} \) denotes an upper triangular Toeplitz matrix, with its first row as \( [0, \ldots, 0, h_Q, \ldots, h_1] \).

The receiver removes the cyclic prefix, via \( \mathbf{R}_{cp} = [0_{N\times G}, \mathbf{I}_{N \times N}] \), from the received signal to yield the \( N \)-element vector 
\[
x(k) = \mathbf{R}_{cp} \bigg[ \mathbf{H}_0 \tilde{\mathbf{u}}(k) + \mathbf{H}_1 \tilde{\mathbf{u}}(k-1) + \mathbf{\eta}(k) \bigg],
\]
with \( \mathbf{H}_0 \in \mathbb{C}^{N\times N} \) being the inter-block interference (IBI) matrix, \( \mathbf{H}_1 \in \mathbb{C}^{N\times N} \), the \( N \times N \) inter-symbol interference (ISI) matrix, and the permutation matrix \( \mathbf{P} = \begin{bmatrix} \mathbf{0}_{G \times (N-G)} & \mathbf{I}_N \\ \mathbf{I}_N & \mathbf{0}_{(N-G) \times G} \end{bmatrix} \).

III. THE PRECODER/EQUALIZER SCHEME OF [1]

A. The Zero-Inserting Precoder in [1]

To suppress the ISI and IBI, but with only a length-\( G \) insufficient cyclic prefix; [1] deploys a guard interval (comprising of zero-energy symbols, plus an optional cyclic prefix) that may be shorter than the channel impulse response.

This zeros-inserting precoding may be realized by an \( N \times (N-P) \) precoding matrix \( \mathbf{T}_{zero} \), formed by inserting \( P \) number of all-zero rows into an \( (N-P) \times (N-P) \) identity matrix. For example, appending all these zeros would require a precoding matrix of 
\[
\mathbf{T}_{zero} = \begin{bmatrix} \mathbf{I}_{(N-P) \times (N-P)} \\ \mathbf{0}_{P \times (N-P)} \end{bmatrix}.
\]

This precoder would not be affected by any channel mismatch.

B. The Two-Stage Equalizer in [1]

At the receiver, (1) remains valid despite the zero-inserting precoder, but now has \( \mathbf{u}(k) = \mathbf{T}_{zero}\mathbf{s}(k) \).

The first stage is a single-tap-per-subcarrier frequency-domain linear equalizer (FDE),
\[
\mathbf{W} = \mathbf{D}^H \left( \mathbf{D} \mathbf{D}^H + \frac{1}{\text{SNR}} \mathbf{I}_N \right)^{-1},
\]
where superscript \( H \) denotes complex-conjugate transposition, \( \text{SNR} \equiv \sigma_\text{I}^2 / \sigma_\text{N}^2 \) refers to the signal power, and \( \sigma_\text{I}^2 \) symbolizes the noise power. The scheme in [1] has assumed the receiver of a prior knowledge or a perfect estimation of \( \mathbf{D} \) and the SNR for use in (2) above. The output of \( \mathbf{W} \) equals
\[
y(k) = \mathbf{W}^H \mathbf{N} \mathbf{W} \mathbf{N} \mathbf{x}(k).
\]

For the second stage:

(a) Form a \( P \times N \) “zero-selection” matrix, to block all information-bearing symbol-intervals (which have non-zero energy at transmission). E.g., \( \mathbf{J}_{zero} = [\mathbf{0}_{P \times (N-P)} \mid \mathbf{1}_{P \times P}] \) would be compatible with the earlier defined \( \mathbf{T}_{zero} \).

(b) Also form a \( (N-P) \times N \) “zero-removal” matrix, to remove the precoder-inserted zeros. E.g., \( \mathbf{R}_{zero} = [\mathbf{I}_{(N-P)\times(N-P)} \mid \mathbf{0}_{(N-P)\times P}] \) would be compatible with the earlier defined \( \mathbf{T}_{zero} \) and \( \mathbf{J}_{zero} \).

Next, form the \((N-P)\times P\) matrix \( \mathbf{U} \), to minimize the mean-squared error \( \xi \) between the signal-output from \( \mathbf{R}_{zero} \) and \( \mathbf{J}_{zero} \), i.e.,
\[
\xi_{\min} = \min \mathbb{E} \left[ \| \mathbf{u}(k) - \mathbf{J}_{zero} \mathbf{y}(k) \|_2^2 \right].
\]

where \( \mathbf{u}(k) \triangleq \mathbf{R}_{zero} \mathbf{W}^H \mathbf{N} \mathbf{x}(k) - \mathbf{H}_{IBI} \mathbf{u}(k-1) + \mathbf{\eta}(k) \) represents the interference and noises in the information-bearing symbols’ durations. The optimization in (4) can be solved via the principle of orthogonality, i.e., \( \mathbf{E}[\mathbf{U} \mathbf{J}_{zero} \mathbf{y}(k)] = (\mathbf{I}(k) - \mathbf{U} \mathbf{J}_{zero} \mathbf{y}(k))^H = 0 \), to yield 
\[
\mathbf{U} = \mathbf{R}_{zero} \mathbf{R}_{U}(k) \mathbf{J}_{zero} \mathbf{R}_{U}(k) \mathbf{y}(k) \mathbf{J}_{zero}^{-1},
\]
where \( \mathbf{R}_{U}(k) \) is a \( (N-P) \times P \) matrix, yielding the optimal solution. 

\[
\mathbf{R}_{U}(k) \mathbf{y}(k) = \mathbf{W}^H \mathbf{N} \mathbf{x}(k). 
\]

where
\[
\mathbf{R}_{U}(k) \mathbf{y}(k) \triangleq \mathbf{R}_{U}(k-1) \mathbf{u}(k-1) = \sigma_\text{I}^2 \mathbf{I}_{N \times N}
\]
\[
\mathbf{R}_{U}(k) \mathbf{u}(k) \triangleq \mathbf{R}_{U}(k-1) \mathbf{u}(k-1) = \sigma_\text{I}^2 \mathbf{I}_{N \times N}
\]
Lastly, the \((N - P) \times 1\) transmitted symbol-vector \(\mathbf{s}(k)\) is estimated by the receiver as \(\hat{\mathbf{s}}(k) = (\mathbf{R}_{\text{zero}} - \mathbf{UJ}_{\text{zero}}) \mathbf{y}(k)\).

IV. CHANNEL-MISMATCH MODELING & CONSEQUENCES

The mismatched impulse response, to be used in the scheme in [1], has its \(q\)th tap represented as
\[
\hat{h}_q = h_q[1 + \delta_q]e^{j\varphi_q}, \quad \forall q = 0, 1, \ldots, Q, \tag{7}
\]
where \(\delta_q\) denotes the \(q\)th tap’s magnitude-error, modeled statistically as zero-mean, with variance \(\sigma^2_{\delta_q}\), and statistically independent over \(q = 0, 1, \ldots, Q\). Similarly, \(\varphi_q\) symbolizes the \(q\)th tap’s phase-error, modeled statistically as zero-mean, with variance \(\sigma^2_{\varphi_q}\), statistically independent over \(q = 0, 1, \ldots, Q\), and statistically independent from \(\{\delta_q, q = 0, 1, \ldots, Q\}\).

With the above channel mismatch, (2) would become
\[
\hat{\mathbf{W}} = \hat{\mathbf{D}}^H \left( \hat{\mathbf{D}}\hat{\mathbf{D}}^H + \frac{1}{\text{SNR}} \mathbf{I}_N \right)^{-1}, \tag{8}
\]
where \(\hat{\mathbf{W}}\) equals an imperfect estimate of \(\mathbf{D}\), \(\text{SNR}\) represents an imperfect estimate of the SNR.

Instead of (3), the output of \(\hat{\mathbf{W}}\) equals
\[
\hat{\mathbf{y}}(k) = \mathbf{W}_{\text{N}}^H \hat{\mathbf{W}} \mathbf{W}_N \mathbf{x}(k), \tag{9}
\]
Due to the channel mismatch, the \((N - P) \times P\) matrix \(\mathbf{U}\) in the 2nd stage of the equalizer in [1] would be changed as
\[
\hat{\mathbf{U}} = \mathbf{R}_{\text{zero}} \mathbf{R}_{\mathbf{u}(k), \mathbf{u}(k)} J_{\text{zero}}^H \left[ \mathbf{J}_{\text{zero}} \mathbf{R}_{\mathbf{y}(k), \mathbf{y}(k)} J_{\text{zero}}^H \right]^{-1}, \tag{10}
\]
in which the cross-correlation matrix in (5) and (6) need to be modified as
\[
\mathbf{R}_{\mathbf{u}(k), \mathbf{u}(k)} = \mathbf{W}_N^H \hat{\mathbf{W}} \mathbf{W}_N \left\{ \mathbf{H}_{\text{BI}} \mathbf{R}_{\mathbf{u}(k), \mathbf{u}(k)} (\mathbf{W}_N^H \hat{\mathbf{W}} \mathbf{W}_N \mathbf{H}_{\text{BI}})^H \right. \\
+ \mathbf{H}_{\text{BI}} \mathbf{R}_{\mathbf{u}(k-1), \mathbf{u}(k-1)} (\mathbf{W}_N^H \hat{\mathbf{W}} \mathbf{W}_N \mathbf{H}_{\text{BI}})^H \\
+ \mathbf{R}_{\mathbf{u}(k), \mathbf{u}(k)} (\mathbf{W}_N^H \hat{\mathbf{W}} \mathbf{W}_N)^H \left. \right\}, \tag{11}
\]
\[
\mathbf{R}_{\mathbf{y}(k), \mathbf{y}(k)} = \mathbf{W}_N^H \hat{\mathbf{W}} \mathbf{W}_N \mathbf{C} \mathbf{R}_{\mathbf{u}(k), \mathbf{u}(k)} (\mathbf{W}_N^H \hat{\mathbf{W}} \mathbf{W}_N \mathbf{C})^H \\
+ \mathbf{R}_{\mathbf{u}(k), \mathbf{u}(k)} \tag{12}
\]
where \(\hat{\mathbf{H}}_{\text{BI}}, \mathbf{H}_{\text{BI}}\) and \(\mathbf{C}\) are formed by substituting \(\hat{h}_q\) for \(h_q\) in \(\mathbf{H}_{\text{BI}}, \mathbf{H}_{\text{BI}}\) and \(\mathbf{H}_0\).

Taking account of channel mismatch, the \((N - P) \times 1\) transmitted symbol-vector \(\hat{\mathbf{s}}(k)\) would be estimated as \(\hat{\mathbf{s}}(k) = (\mathbf{R}_{\text{zero}} - \mathbf{UJ}_{\text{zero}}) \hat{\mathbf{y}}(k)\).

V. MONTE CARLO SIMULATIONS

The information-bearing symbols \{\(s(k)\}\} are modulated with equiprobable QPSK-symbols. The transmitted signal has \(N = 64\). \(P\) number of zero-energy symbols are inserted at the end of symbol bloc. The channel impulse response \{\(h_q, q = 0, 1, \ldots, Q\}\) has \(Q + 1 = 11\) complex-valued taps, each randomly generated and not cross-correlated among themselves. Each \(h(q)\) tap’s real-value part and imaginary-value part are not cross-correlated. Each \(h(q)\) tap is Gaussian, zero-mean. The \(q\)th tap has an exponentially decaying variance of \(\sigma^2_q = \left(1 - e^{-\frac{T_{\text{rms}}}{Q}}\right) e^{-\frac{k}{T_{\text{rms}}}}, \forall q = 0, \ldots, Q\), where \(T_s\) denotes the sampling period, \(T_{\text{rms}}\) symbolizes the root-mean-square delay-spread of the channel, and \(\frac{T_{\text{rms}}}{Q} = \frac{T_s}{4}\).

Figures 3 to 5 show the channel-mismatch degradation of the bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture. Each data point therein represents 50 symbol-blocks. These figures verify that the Song-Fung-Wong-Meng-Tseng architecture of [1], in shifting some of the overhead from cyclic prefixing to zero insertion, lowers the BER, despite channel mismatch. These figures also verify that the BER increases with decreasing additive noise SNR, with increasing \(\sigma_\delta\), or with increasing \(\sigma_\varphi\) approaches zero, as expected.

Figures 6 to 8 show that the Song-Fung-Wong-Meng-Tseng architecture of [1] offers a lower BER than the well known zero-forcing (ZF) FDE does, under identical conditions of channel mismatch. These figures also verify that the BER increases with decreasing additive noise SNR, with increasing \(\sigma_\delta\), or with increasing \(\sigma_\varphi\) approaches zero, as expected. All simulations in these figures are conducted at \(G = 6\) and \(P = 0\).

VI. CONCLUSION

For cyclic-prefixed block-based single-carrier-based communication systems, proposed in [1] is a zero-inserting time-domain precoder and an accompanying two-stage equalizer, to allow an insufficient guard interval, in order to reduce the transmission overhead. Monte Carlo simulations show that the aforementioned scheme offers lower bit-error rates versus zero-forcing frequency-domain equalization, under comparable conditions of mismatch between the channel’s actual impulse response and receiver’s incorrect prior knowledge of this impulse response. This degradation worsens with decreasing additive noise SNR or with increasing amount of mismatch, as expected. For a theoretical analysis, please refer to [3]

REFERENCES

Fig. 3. The bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture [1] at various \( \sigma_\delta \), but no phase uncertainty in the channel impulse response taps.

Fig. 4. The bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture [1] at various \( \sigma_\delta \), but no magnitude uncertainty in the channel impulse response taps.

Fig. 5. The bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture [1] at various \( \sigma_\delta = \sigma_\varphi \).

Fig. 6. The bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture [1] and of zero-forcing DFE at various \( \sigma_\delta \), but no phase uncertainty in the channel impulse response taps.

Fig. 7. The bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture [1] and of zero-forcing DFE at various \( \sigma_\delta \), but no magnitude uncertainty in the channel impulse response taps.

Fig. 8. The bit-error rate (BER) of the Song-Fung-Wong-Meng-Tseng architecture [1] and of zero-forcing DFE at various \( \sigma_\delta = \sigma_\varphi \).