Acoustic Near-Field Source-Localization by Two Passive Anchor-Nodes

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A new scheme is herein proposed to localize an acoustic source. This new method blends the “received signal strength indication” (RSSI) approach of geolocation, and the acoustic vector-sensor (AVS) (a.k.a., vector-hydrophone) based direction-finding (DF). Unlike customary RSSI-based source-localization, this proposed approach needs only two (not three or more) passive anchor-nodes: 1) one pressure-sensor, and 2) one physically compact triad of three (collocating, but orthogonally oriented) acoustic velocity-sensors. The latter can estimate the direction-of-arrival (DOA) of an emitter, regardless of that emitter’s arbitrary/unknown center-frequency, bandwidth, spectrum, and near-field/far-field location. This triad’s DOA estimates can be “distributed processed,” locally, apart from the pressure-sensor’s measured power, to estimate the emitter’s radial distance. This proposed algorithm is noniterative, requires no initial estimate, is closed form, and can accommodate any prior known propagation-loss exponent.

I. INTRODUCTION

A. The “Spatially Extended” Acoustic Vector-Sensor

An acoustic vector-sensor (AVS) consists of 1) an acoustic pressure-sensor, and 2) a velocity-sensor triad of three identical but orthogonally oriented acoustic velocity-sensors, all spatially colocated in a point-like geometry. Each acoustic velocity-sensor has an intrinsically directional response to the incident acoustic particle-velocity wavefield, measuring the one Cartesian component (of the incident wavefield’s three-dimensional particle-velocity-field vector) in parallel to that velocity-sensor.

For a velocity-sensor triad located at the coordinates’ origin, its $3 \times 1$ array manifold equals $[15, 39, 82, 90]$: \[
\mathbf{a}_v(\phi, \psi) \triangleq \begin{bmatrix} \sin \psi \cos \phi \\ \sin \psi \sin \phi \\ \cos \psi \end{bmatrix} \tag{1}
\]

where $\psi \in [-\pi/2, \pi/2]$ symbolizes the elevation angle measured from the $x$-$y$ plane, and $\phi \in [0,2\pi)$ denotes the azimuth angle measured from the positive $x$-axis.

The array manifold in (1) allows azimuth-elevation two-dimensional direction-finding (DF), with no prior knowledge of the emitter’s center-frequency, bandwidth, or spectrum. The emitter could be in the velocity-sensor triad’s near field or far field. All these advantages stem from the spatial colocation of the three velocity-sensors. In contrast, an array of spatially displaced pressure-sensors (a spatial finite-impulse-response (FIR filter)) has a directivity affected by the frequency-dependent inter-sensor spatial phase-factor.

Though directional in azimuth-elevation sensitivity, this velocity-sensor triad can isotropically measure the incident signal power, because $\|\mathbf{a}_v(\phi, \psi)\| = 1$, $\forall \phi, \psi$. This property is important to the subsequent synergy with received signal strength indication (RSSI)-based source-localization.

This velocity-sensor technology is well established, dating back to N. A. Umov’s seminal work in 1873 [51, p. 9]. Since then, acoustic velocity-sensor technology has been used in underwater acoustics and air acoustics [1] for over a century, and has recently attracted renewed interest [18, 51]. Many different implementations of acoustic velocity-sensors are available [3], with designs ranging from mechanically-based [4], to thermally-based [50], to optically-based [5], to derivative-based [20, 52, 57].

This velocity-sensor triad concept is practical. It has been implemented in hardware in various forms for underwater or sea-surface applications [6, 7, 10, 13, 42, 59, 60], or for air-acoustic applications [27, 29]. Acoustic velocity-sensor triads are commercially available as the “Uniaxial P-U Probe” from Acoustech.\(^1\) The velocity-sensor triad have

\(^1\)http://www.acoustechcorporation.com.
undergone sea trials [6–9, 11, 13, 14, 16, 17, 25, 32, 56, 58, 59, 64], and in-building room trials or atmospheric trials [52]. Acoustic velocity-sensor triad has been proposed for use for underwater port and waterway security [47] and for underwater wireless communications [19, 65, 77, 88].

This velocity-sensor triad concept is versatile for DF. Algorithms that exploit the vector-sensor’s unique array-manifold have been developed for maximum-likelihood-based direction-of-arrival (DOA) estimation [21, 31, 38, 66], for Capon-based spectrum estimation [24], for ESPRIT-based DOA estimation [22, 23, 28, 30, 35, 53, 57, 62, 68, 71, 73, 80, 86, 88], for MUSIC-based DOA estimation [12, 28, 30, 49, 61, 67, 68], for root-MUSIC-based DOA estimation [26, 84], for quaternion-MUSIC-based DOA estimation [54], for least-squares-based DOA estimation [40, 41], for beamspace-based DOA estimation [24, 44–46, 59, 67, 76, 78, 86], for other approaches of DOA estimation [43, 69, 70, 72, 74–76, 81, 83, 85, 89], and for DOA tracking [33, 55]. The present work will apply the velocity-sensor triad to three-dimensional source localization, not mere azimuth-elevation two-dimensional DF as in these references.

B. Synergy Between RSSI-Base Localization & AVS DF

The present scheme combines these two approaches of source-localization:

1) the uni-AVS azimuth-elevation DOA estimation, and
2) the RSSI algorithm.

The proposed technique offers the following advantages over purely RSSI-based geolocation.

a) A simple closed-form solution now exists.
b) Only 2 anchor-nodes (instead of a minimum of 4 anchor-nodes [34, 37]) are now needed for unambiguous geolocation to within a hemispheric space in the polar coordinates.

Moreover, the proposed technique retains the following advantages of RSSI-based geolocation.

c) Needs no prior knowledge of the emitted signal’s time-frequency structure.
d) Allows the emitted signal to be wideband, even of arbitrary bandwidth or arbitrary center-frequency.
e) Needs no prior information of the emitter’s transmission power.
f) Uses passive sensors.
g) Time-samples the acoustic energy at a sampling rate much slower than the Nyquist frequency of the transmitted waveform.
i) Needs no time synchronization across different nodes.
j) Allows “distributed processing” locally at each anchor-node (where the raw measurements are taken), resulting in low communication rate between any center node (where the three-dimensional estimation is performed) and the two anchor-nodes.

This proposed technique would not be viable if the velocity-sensor triad were substituted by some other directional sensors, which generally cannot isotropically measure the incident signal strength. The velocity-sensor triad can, despite its directionality, because \(|a_i(\phi, \psi)| = 1, \forall \phi, \psi|.

This proposed method has not assumed a planar wavefront of the incident source. Such an assumption would implicitly place the emitter in the receive sensors’ far field, contradicting the near-field data-model inherent in the present source-localization problem.

This proposed three-dimensional source-localization (via a synergy between the AVS and the RSSI method) offers advantages over the alternative “cross-bearing estimation” method, wherein the emitter’s azimuth-elevation DOA is estimated with reference to each of two widely separated anchor-nodes. Each DOA estimate corresponds to a radial line emerging from the corresponding anchor-node. Where these two radial lines intersect, there must the emitter lie. Nonetheless, the present synergy offers these advantages over such cross-bearing methods. 1) One anchor-node here needs only one simple pressure-sensor. Hence, the hardware here is simpler. 2) The other anchor-node consists of the velocity-sensor triad, which is physically compact due to the geometric collocation of the three velocity-sensors. This contrasts with an array of displaced pressure-sensors, taking up extensive space. 3) The velocity-sensor triad can estimate the DOA, with no restriction on, no prior knowledge of, and no real-time estimation of the incident signal’s center-frequency, bandwidth, or spectrum.

This proposed algorithm accommodates any prior known path-loss attenuation exponent value. The typical value 2 is mostly used in the modeling and it’s widely known as the inverse-square law in physics. Reference [63] proposes several techniques for online calibration of the path-loss exponent in wireless sensor networks without relying on distance measurements. Reference [79] proposes a localization algorithm using different path-loss exponents for each link (target to each receive node). Reference [87] proposes a localization algorithm in sensor network which can be used without knowing the exact path-loss model. However, the algorithm is highly iterative and very complicated.

II. THE MEASUREMENT DATA MODEL OF A NEAR-FIELD SOURCE IMPINGING UPON A “SPATIALLY EXTENDED” ACOUSTIC VECTOR-SENSOR

The measurement system may be construed as a “spatially distributed” four-component AVS, which consists of 1) a velocity-sensor triad centered at the Cartesian coordinates (0,0,0), plus 2) an isotropic
emitter-sensor separation of well-known inverse-square law holds at on the specific propagation environment. (The symbolizes a positive exponent, which also depends that depends on the particular field environment and signal speed; velocity-sensor triad; R>D 2 the emitter would be in the sensor’s "far field" if R > 2D 2 /λ [36] or R > D 2 / (2λ) [2].

For an emitter at wavelength λ, and a distance of R from a sensor, the emitter would be in the sensor’s “far field” if R > 2D 2 /λ [36] or R > D 2 / (2λ) [2].

3Or, in the spherical coordinates (r,φ,ψ), where r ≥ 0, 0 ≤ φ < 360° and 0 ≤ ψ ≤ 90°.

For an emitter at wavelength λ, and a distance of R from a sensor, the emitter would be in the sensor’s “far field” if R > 2D 2 /λ [36] or R > D 2 / (2λ) [2].

The velocity-sensor triad produces a 3 × 1 data-measurement at time t:

\[ \mathbf{z}_v(t) = \mathbf{a}_v(\phi_v, \psi_v) \sqrt{P(r_v)} s(t - \tau(r_v)) + \mathbf{n}_v(t) \]  

where \( r_v = \sqrt{x_v^2 + y_v^2 + z_v^2} \) symbolizes the unknown separation between the emitting source and the velocity-sensor triad; \( \tau(r_v) = \frac{r_v}{c} \) refers to the signal’s propagation time from the emitter to the velocity-sensor triad; c represents the propagation speed; \( \mathbf{n}_v(t) \) signifies a zero-mean stochastic sequence of additive noise uncorrelated across its three particle-velocity components; \( P(r_v) = P_s/\left(r_v^nK\right) \) denotes the power received by the velocity-sensor triad; \( K \) equals the power-loss associated with \( P(r) \) for an emitter-sensor separation of \( r \); \( K \) denotes a constant that depends on the particular field environment and could remain unknown to the proposed algorithm, \( n \) symbolizes a positive exponent, which also depends on the specific propagation environment. (The well-known inverse-square law holds at \( n = 2 \).)

Similarly, the pressure-sensor produces a scalar measurement at time t:

\[ z_p(t) = \sqrt{P(r_p)} s(t - \tau(r_p)) + n_p(t) \]  

where \( r_p = \sqrt{(x_p - D)^2 + y_p^2 + z_p^2} \) symbolizes the unknown separation between the emitter and the pressure-sensor, \( P(r_p) = P_s/(r_p^nK) \) denotes the signal power measured by the pressure-sensor, \( \tau(r_p) = r_p/c \) refers to the signal’s propagation time from the emitter to the pressure-sensor, and \( n_p(t) \) refers to a zero-mean noise sequence not cross-correlated with \( \mathbf{n}_v(t) \).

Over \( N \) time-instants, the overall \( 4 \times N \) observed dataset equals:

\[ \mathbf{Z} = \begin{bmatrix} \mathbf{z}_v(t_1) & \cdots & \mathbf{z}_v(t_N) \\ \mathbf{z}_p(t_1) & \cdots & \mathbf{z}_p(t_N) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_v \\ \mathbf{Z}_p \end{bmatrix}. \]  

The present problem is to estimate \( \{\hat{\phi}_v, \hat{\psi}_v, \hat{r}_v\} \) based on \( \mathbf{Z} \).

III. THE PROPOSED ALGORITHM TO LOCALIZE THE SOURCE’S RANGE AND AZIMUTH-ELEVATION DOA

The proposed source-localization algorithm has two stages. The first stage, presented in Section IIIA, estimates the emitter’s DOA from the velocity-sensor triad’s data. The second stage, presented in Section IIIB, locates the emitter by comparing the power received by the velocity-sensor triad versus the power received by the pressure-sensor.

A. Azimuth-Elevation Angle-of-Arrival Estimation Using a Velocity-Sensor Triad

From the velocity-sensor triad’s data, form a \( 3 \times 3 \) correlation matrix

\[ \mathbf{R} = [\mathbf{z}_v(t_1) \cdots \mathbf{z}_v(t_N)][\mathbf{z}_v(t_1) \cdots \mathbf{z}_v(t_N)]^H \]  

where the superscript \(^H\) denotes the Hermitian operation. Eigen-decompose \( \mathbf{R} \) to obtain the eigenvector corresponding to the “largest-magnitude” eigenvalue of \( \mathbf{R} \). Denote this principal eigenvector as \( \hat{\mathbf{a}}_v \). By the definition of a eigenvector, the Frobenius norm \( ||\hat{\mathbf{a}}_v|| = 1 \). Moreover, \( \hat{\mathbf{a}}_v \approx e^{j\eta} \mathbf{a}_v(\phi_v, \psi_v) \), where \( \eta \) symbolizes an unknown phase, with the approximation converging to equality, under noiseless or asymptotic conditions. That is, \( \mathbf{a}_v(\phi_v, \psi_v) \) is here estimated to within a complex-phase factor. (Recall that \( ||\mathbf{a}_v(\phi_v, \psi_v)|| = 1, \forall \phi_v, \psi_v \).

Hence, (1) gives the azimuth-angle and elevation-angle estimates:

\[ \hat{\phi}_v = \arctan \left( \frac{[\hat{\mathbf{a}}_v]_2}{[\hat{\mathbf{a}}_v]_1} \right) \]  
\[ \hat{\psi}_v = \arccos \left( |[\hat{\mathbf{a}}_v]_3| \right) \]  

where \( [\hat{\mathbf{a}}_v]_k \) symbolizes the \( k \)th entry in \( \hat{\mathbf{a}}_v \). The incidence source must therefore lie on the straight half-line,

\[ \ell_1 = \left\{ x, y = x \tan \hat{\phi}_v, z = \frac{x}{\cos \hat{\phi}_v \tan \hat{\psi}_v} > 0 \right\} \]  

expressed in the Cartesian coordinates.
Two-dimensional DF has thus been accomplished using a single velocity-sensor triad, with no prior information of the signal’s spectrum. This is viable because the array manifold in (1) is independent of the impinging signal’s frequency, due to the spatial colocation of the triad’s constituent sensors. The complicating effects of a near-field wave-front’s curvature is avoided, because of the spatial colocation of the three velocity-sensors, to decouple the DOA estimation here in Section IIIA from the range estimation in Section IIIB.

B. Received Signal Strength Indication (RSSI)

\[ P(r_v) \text{ and } P(r_p) \text{ may be estimated from } (4), \text{ as} \]

\[ \hat{P}_v = \frac{1}{N}[z_x z_y z_z]^H \]

\[ \hat{P}_p = \frac{1}{N}[z_x z_y z_z]^H \]

where \( z_x, z_y, \text{ and } z_z \) respectively denote the first, second, and third rows of \( Z_v \).

From the propagation-path power-loss models, \( \hat{P}_v = P_v/(r_v^2 K) \) and \( \hat{P}_p = P_p/(r_p^2 K) \).

The incident source must then lie simultaneously on two spherical surfaces:

S1: a sphere of radius \( \hat{r}_1 \), centered at the velocity-sensor triad at \((0,0,0)\), and defined by

\[ x^2 + y^2 + z^2 = \hat{r}_1^2. \] (11)

S2: another sphere of radius \( \hat{r}_2 \), centered at the pressure-sensor at \((D,0,0)\), and defined as

\[ (x-D)^2 + y^2 + z^2 = \hat{r}_2^2. \] (12)

These two spheres intersect at a circle \( \ell_2 \), perpendicular to the \( x \)-axis, and defined via these two equations:

\[ \ell_2 : \left\{ \begin{array}{l}
 x = \frac{1}{2D} \left[ \left( \frac{P_v}{P_v K} \right)^{2/n} - \left( \frac{P_p}{P_p K} \right)^{2/n} \right] + \frac{D}{2} \\
 y^2 + z^2 = \left( \frac{P_p}{P_v K} \right)^{2/n} - x^2.
\end{array} \right. \]

By varying the emitter’s transmitted power \( P_v \), the circle \( \ell_2 \) would span a curved manifold \( m_3 \). To mathematically define this manifold, combine (11) and (12), then eliminate \( P_v \), giving

\[ m_3 : \frac{x^2 + y^2 + z^2}{(x-D)^2 + y^2 + z^2} = \frac{\beta}{\alpha} \] (13)

where \( \alpha = (\hat{P}_p)^{2/n} \) and \( \beta = (\hat{P}_v)^{2/n} \).

At the intersection between the straight half-line \( \ell_1 \) and the surface \( m_3 \), there the emitter must be located.

For the unlikely case of \( \hat{P}_v = \hat{P}_p \), (13) would degenerate to a flat plane perpendicular to the \( x \)-axis. Combine (8) and (13), to give

\[ (\hat{x}, \hat{y}, \hat{z}) = \frac{D}{2}(1,a,b) \] (14)

where \( a = \tan \hat{\phi}_s \) and \( b = [\cos \hat{\phi}_s \tan \hat{\psi}_s]^{-1}. \)

For the likely case that \( \hat{P}_v \neq \hat{P}_p \), rewrite (13) as

\[ m_3 : \left( x - \frac{D \beta}{\alpha - \beta} \right)^2 + y^2 + z^2 = \left( \frac{D \sqrt{\alpha \beta}}{\alpha - \beta} \right)^2 \] (15)

which represents a sphere. Please see Fig. 1. Combine (8) and (15) to give the estimates:

\[ \hat{x} = \frac{-D \beta \pm D \sqrt{\beta^2 + \beta(\alpha - \beta)(1 + a^2 + b^2)}}{\alpha - \beta(1 + a^2 + b^2)} \] (16)

\[ \hat{y} = a \hat{x} \]

\[ \hat{z} = b \hat{x}. \] (18)

Suppose the emitter is known a priori to impinge from the left hemisphere.\(^4\) The emitter would be nearer the velocity-sensor triad than the pressure-sensor. Hence, the velocity-sensor triad’s received power will exceed the pressure-sensor’s received power, i.e., \( \hat{P}_v > \hat{P}_p \), thus \( \alpha > \beta \). Consequently,

\[ \beta^2 + \beta(\alpha - \beta)(1 + a^2 + b^2) \geq \beta^2 + \beta(\alpha - \beta) = \alpha \beta > \beta^2 > 0. \]

Hence, the square root in (16) would be real value; and there would exist two solutions to (16)–(18):

1) \( (\hat{x}, \hat{y}, \hat{z}) = (x_+, y_+, z_+) = (x_+, ax_+, bx_+) \), where \( x_+ > 0, y_+ > 0, z_+ > 0 \).

2) \( (\hat{x}, \hat{y}, \hat{z}) = (x_-, y_-, z_-) \), where \( x_- < -2D \beta/(\alpha - \beta)(1 + a^2 + b^2) < 0, y_- = ax_- < 0, z_- = bx_- < 0. \)

If the emitter is furthermore known a priori to impinge from either the upper or the lower hemisphere (in polar coordinates), the emitter can be unambiguously located.

C. Summary of the Proposed Algorithm

The proposed algorithm follows these steps:

1) From the \( 4 \times N \) observed data \( Z \) in (4). Form the covariance-matrix \( R \) as in (5).

2) Eigen-decompose \( R \), to estimate the source’s \( 3 \times 1 \) velocity-sensor-triad steering vector \( a_\ast \). Compute the azimuth-elevation DOA estimates \( \hat{\phi}_s \) and \( \hat{\psi}_s \) using (6) and (7).

3) Estimate \( \hat{P}_v \) and \( \hat{P}_p \) at the velocity-sensor triad and the pressure-sensor using (9) and (10), respectively.

\(^4\)If the emitter is known to impinge from the right-hemisphere, interchange the location of the velocity-sensor triad and the pressure-sensor.
4) If $\hat{P}_v = \hat{P}_p$, use (14) to estimate the emitter’s three-dimensional position.

5) If $\hat{P}_v > \hat{P}_p$, use (16)–(18) to estimate the emitter’s three-dimensional position.

IV. MONTE CARLO SIMULATIONS

The above-proposed scheme is applicable to any constant-power isotropic emitter of signals of any time-frequency structure. To avoid unnecessary distraction from the proposed scheme’s central features, the Monte Carlo simulations use a very simple signal model:

A1: The source signal is sinusoid with power $P_s$, radial frequency $\omega_s$, and initial phase $\epsilon_s$, all unknown deterministic constants.

A2: $n_v(t)$ and $n_p(t)$ are zero-mean white Gaussian processes, not cross-correlated with each other. The spatial covariance matrix for $[n_v^T(t), n_p(t)]^T$ is $\Gamma = \sigma^2 I$, where $I_p$ symbolizes a $p \times p$ identity matrix, and $\sigma^2$ represents an unknown deterministic constant.

In all subsequent Monte Carlo simulations, $P_s = 10^4$, with the emitter located in the near field at $(\phi_s, \psi_s, r_v) = (10^2, 23^3, 12)$ in the spherical coordinates or, equivalently, $(x_s, y_s, z_s) = (-0.9749, 4.5863, 11.0461)$ in the Cartesian coordinates. The separation between the velocity-sensor triad and the pressure-sensor equals $D = 20$. The free-space propagation model gives a path-loss exponent of $n = 2$ and $K = 1$. The signal-to-noise (SNR) power ratio is defined as $(P(r_v) + P(r_p))/2\sigma^2$. Altogether, $N = 500$ time-samples are available in each of the 100 independent Monte Carlo trials.

The proposed scheme’s estimation error standard deviation is plotted in Figs. 2(a)–(c) for the azimuth-angle $\phi_s$, the elevation-angle $\psi_s$, and the radial distance $r_v$, respectively—along with the Cramér-Rao bounds as computed in the Appendix. The corresponding estimation bias is plotted in absolute value in Figs. 3(a)–(c). From these figures, 20 dB SNR would result in a standard deviation of
0.57° for $\hat{\phi}_s$, 0.21° for $\hat{\psi}_s$, and 0.14 m for $\hat{r}_v$, with corresponding biases of 0.09°, 0.02°, and 0.04 m. The proposed estimates are within 1 dB of the Cramér-Rao bounds, for any received SNR exceeding 20 dB. These figures also show that the proposed scheme would break down below 0 dB. The SNR threshold could be lowered, by taking more frequent time-samples (i.e. by increasing $N$) for more noise averaging.

V. CONCLUSION

A new synergy is herein proposed between the RSSI method of source-localization and the AVS to locate a noncooperative acoustic emitter in three-dimensional space. This schemes requires only two passive anchor-nodes; each is physically compact and may be arbitrarily placed at a known location. One anchor-node contains only one pressure-sensor, to measure the incident acoustic pressure-field. The other anchor-node consists of a velocity-sensor triad, to measure the acoustic particle velocity-field. The former anchor-node reports only its RSSI to the center-node, whereas the latter anchor-node reports also its azimuth-elevation DOA estimate. The three-dimensional source-localization estimation, performed at the center-node, is closed form, requiring no iteration and no initial estimates. This proposed scheme can accommodate non-free-space propagation models at any arbitrary (but known) path-loss exponent.

APPENDIX. CRAMÉR-RAO BOUND DERIVATION

Under assumptions A1 and A2 of Section IV, the data-vector, collected at time $t$ in (2) and (3), may be represented as this $4 \times 1$ vector:

$$\mathbf{z}(t) = \begin{bmatrix} n_{\phi}(t) \\ n_{\psi}(t) \\ n_p(t) \end{bmatrix} = \begin{bmatrix} a_{\phi}(\phi_s, \psi_s) A_s \sin \left[ \omega_s \left( t - \frac{r_v}{c} \right) + \epsilon_s \right] \\ A_v \sin \left[ \omega_v \left( t - \frac{r_v}{c} \right) + \epsilon_v \right] \end{bmatrix}$$
where $A_s = \sqrt{2P/(r_0K)}$, $A_p = \sqrt{2P/(r_p^2K)}$, and $r_p = \sqrt{r_c^2 - 2Dr_c \cos \phi_s \sin \psi_s} + D^2$. With $N$ number of time-samples, the $4 \times N$ data matrix $Z$ of (4) may be reformed into a $4N \times 1$ “long” vector $\tilde{z}$.

Collect the data-model’s seven deterministic unknown scalars into a vector $\theta = [\phi_s, \psi_s, r_s, P_s, \omega_s, \epsilon_s, \sigma^2]^T$, with $P_s$, $\omega_s$, $\epsilon_s$, and $\sigma^2$ being nuisance parameters.

Under assumptions A1 and A2, the collected data $\tilde{z}$ is Gaussian distributed, with covariance $\Gamma(\theta) = \sigma^2I_{4N}$, and mean

$$\mu(\theta) = \begin{bmatrix} A_A \cdot s_1 \\ A_p \cdot s_2 \end{bmatrix}$$

where $\otimes$ denotes the Kronecker product, and

$$s_1 = \begin{bmatrix} \sin(\omega_s(t_1 - r_s/c) + \epsilon_s) \\ \sin(\omega_s(t_2 - r_s/c) + \epsilon_s) \\ \vdots \\ \sin(\omega_s(t_N - r_s/c) + \epsilon_s) \end{bmatrix},$$

$$s_2 = \begin{bmatrix} \sin(\omega_s(t_1 - r_p/c) + \epsilon_s) \\ \sin(\omega_s(t_2 - r_p/c) + \epsilon_s) \\ \vdots \\ \sin(\omega_s(t_N - r_p/c) + \epsilon_s) \end{bmatrix}.$$}

Hence, the Fisher information matrix has an $(i,j)$th element [12],

$$[J(\theta)]_{i,j} = \left( \frac{\partial \mu(\theta)}{\partial \theta_i} \right)^T \Gamma^{-1}(\theta) \left( \frac{\partial \mu(\theta)}{\partial \theta_j} \right) + \frac{1}{2} \text{tr} \left[ \Gamma^{-1}(\theta) \frac{\partial \Gamma(\theta)}{\partial \theta_i} \Gamma^{-1}(\theta) \frac{\partial \Gamma(\theta)}{\partial \theta_j} \right]$$

with

$$\frac{\partial \mu(\theta)}{\partial \phi_s} = \begin{bmatrix} A_A \cdot \partial \phi_s \otimes s_1 \\ A_p \cdot \partial \phi_s \cdot s_2 + A_p \cdot \partial \phi_s \cdot s_2 \end{bmatrix},$$

$$\frac{\partial \mu(\theta)}{\partial \psi_s} = \begin{bmatrix} A_A \cdot \partial \psi_s \otimes s_1 \\ A_p \cdot \partial \psi_s \cdot s_2 + A_p \cdot \partial \psi_s \cdot s_2 \end{bmatrix},$$

$$\frac{\partial \mu(\theta)}{\partial r_s} = \begin{bmatrix} A_A \cdot \partial r_s \otimes s_1 + A_A \cdot a_s \otimes \partial s_1 \\ A_p \cdot a_s \cdot \partial s_2 + A_p \cdot \partial r_s \cdot s_2 \end{bmatrix}.$$

The Cramér-Rao bounds thus equal

$$\text{CRB}(\phi_s) = [J^{-1}(\theta)]_{1,1},$$

$$\text{CRB}(\psi_s) = [J^{-1}(\theta)]_{2,2},$$

$$\text{CRB}(r_s) = [J^{-1}(\theta)]_{3,3}.$$
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