Acoustic Direction Finding Using a Pressure Sensor and a Uniaxial Particle Velocity Sensor

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One isotropic pressure sensor and one uniaxial particle velocity sensor is used in order to estimate the azimuth-elevation angle of arrival of a source impinging from the far field. This work introduces new estimation formulas in closed forms applicable to estimators based on an eigen-decomposition of the data covariance matrix. The validity region for unambiguous estimation is also identified, and the corresponding Cramér-Rao bound is derived as well.

I. INTRODUCTION
A. The Acoustic Pressure Scalar Versus the Acoustic Particle Velocity Vector

An acoustic wavefield is measured by a hydrophone or a microphone as the scalar entity of pressure. Thereby, what is overlooked is the vector entity of the underlying particle velocity, which represents the acoustic pressure’s gradient in three-dimensional space. Each Cartesian component of this particle velocity vector may be measured by a uniaxial particle velocity sensor (abbreviated hereafter as velocity sensor) oriented along that Cartesian coordinate. Velocity sensor technology has been used in acoustics for over a century [2] and continues to attract interest [3, 4]. It is commercially available from Acoustech for underwater applications and from Microflown for air acoustic applications. Regarding the velocity sensor’s hardware implementation, for information on field/sea testing as well as signal-processing algorithms for source localization, tracking, and beamforming, refer to the comprehensive surveys in [5–7].

More mathematically, consider a point-like source incident from the far field arriving upon a sensor with unit power at an elevation angle of \(\theta \in [0, \pi]\) with respect to the positive \(z\)-axis and at an azimuth angle of \(\phi \in [0, 2\pi]\) with regard to the positive \(x\)-axis. The resulting particle velocity vector equals [6, 8–11]

\[
a^{(APVF)} = \begin{bmatrix} u(\theta, \phi) \\ v(\theta, \phi) \\ w(\theta) \end{bmatrix} = \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix}. \tag{1} \]

Each element in the above \(3 \times 1\) vector represents a first-order partial derivative of the pressure field along its respective Cartesian coordinate. The above particle velocity sensor contrasts with an isotropic pressure sensor, which will always give a response of 1 for any unit power incident source \(\forall \theta, \phi\).

B. The p-u Probe

A p-u probe is a popular acoustic sensing system consisting of one pressure sensor and one uniaxial velocity sensor, perhaps in spatial collocation or otherwise. For the p-u probe’s various hardware implementations, refer to [12–17], and for the p-u probe’s various applications, see [18–69].

If 1) the velocity sensor’s orientation is limited to one of the three Cartesian coordinates and 2) the axis linking the two sensors is limited also to a Cartesian coordinate, there would still be nine possible spatial/directional configurations for the p-u probe. For these distinct configurations, no comprehensive study is yet available in the open literature on direction finding formulas in closed
form for use with an estimator based on an eigen-decomposition of the empirical data’s covariance matrix. This present work will bridge this literature gap. This paper will also give the corresponding Cramér-Rao lower bounds, also in closed forms.

II. THE p-u PROBE’S ARRAY MANIFOLD

Any of the p-u probe’s nine aforementioned configurations may have its array manifold represented mathematically as a $2 \times 1$ vector,

$$a^{(\text{collocate})}_{P,V}(\theta, \phi) = D S a^{(\text{collocate})}_{P,V} (\theta, \phi)$$

where

$$a^{(\text{collocate})}_{P,V} (\theta, \phi) = \begin{bmatrix} \sin (\theta) \cos (\phi) \\ \sin (\theta) \sin (\phi) \cos (\phi) \end{bmatrix},$$

and

$$D = \begin{bmatrix} 1 & 0 \\ 0 & e^{j \frac{2 \pi}{\Delta_1}} \sin (\theta) \phi + \Delta_1 \sin (\theta) \phi + \Delta_1 \sin (\phi) \phi \end{bmatrix},$$

where $(\delta_x, \delta_y, \delta_z) = \{(\Delta_x, 0, 0), (0, \Delta_y, 0), (0, 0, \Delta_z)\}$ refers to the spatial location of the velocity sensor, while the pressure sensor is placed at the Cartesian origins. (The case where these two sensors switch their locations will be discussed subsequently.) The spatial phase factor, $e^{j \frac{2 \pi}{\Delta_1}} (\delta_x \sin (\theta) \phi + \delta_y \sin (\theta) \phi + \delta_z \sin (\phi) \phi)$, arises here due to a possible spatial displacement between the two sensors. If the two sensors are collocated, $D$ would become an identity matrix. Furthermore, $S$ symbolizes a $2 \times 4$ selection matrix whose $(1,1)$th entry must be $1$, and its second row has exactly one $1$ but not in the first column, and zeroes elsewhere. For example, if the pressure sensor and the y-axis oriented velocity sensor are collocated, its $S$ would be $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Lastly, the subscript $V_c$ refers to the velocity sensor being oriented along the $\zeta$-axis, with $\zeta \in \{x, y, z\}$, and the superscript $\epsilon$ refers to the Cartesian axis on which the velocity sensor lies.

For example, if the p-u probe consists of an $x$-axis oriented velocity sensor and a pressure sensor, the three possible array manifolds (corresponding to Fig. 1a–c) are

$$a^{(x)}_{P,V_x}(\theta, \phi) = \begin{bmatrix} 1 \\ \sin (\theta) \cos (\phi) \end{bmatrix},$$

$$a^{(y)}_{P,V_y}(\theta, \phi) = \begin{bmatrix} 1 \\ \sin (\theta) \cos (\phi) \end{bmatrix},$$

$$a^{(z)}_{P,V_z}(\theta, \phi) = \begin{bmatrix} 1 \\ \sin (\theta) \cos (\phi) \end{bmatrix}.$$

A number of relationships exist among the array manifolds of the nine configurations:

$$a^{(x)}_{P,V_x}(\theta, \phi) = a^{(y)}_{P,V_y}(\theta, \frac{\pi}{2} - \phi),$$

$$a^{(y)}_{P,V_y}(\theta, \phi) = a^{(z)}_{P,V_z}(\theta, \frac{\pi}{2} - \phi),$$

$$a^{(z)}_{P,V_z}(\theta, \phi) = a^{(x)}_{P,V_x}(\theta, \frac{\pi}{2} - \phi).$$

III. DIRECTION OF ARRIVAL ESTIMATION FORMULAS FOR VARIOUS ORIENTATION/LOCATION CONFIGURATIONS OF SECTION II

In eigen-based parameter estimation, the algorithm would involve an intermediate step that estimates each incident source’s steering vector, but correct to only an unknown complex value scalar, to be denoted here as $c$, i.e., available (for each incident source) as an estimate $\hat{a}^{(\epsilon)}_{P,V_{\zeta}} = c a^{(\epsilon)}_{P,V_{\zeta}}(\theta, \phi)$ from which $\theta$ and $\phi$ are to be estimated below. (This approximation would become an

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2 This does not require only one source impinging upon the receiver. There could be multiple sources, which could also be cross-correlated, broadband, and/or time varying.
equality in the hypothetical cases of noiselessness or an infinite number of snapshots.)

This section will show how to estimate \((\theta, \phi)\) from \(\hat{a}_{P,V_i}^{(e)}\). Toward this end, rewrite \(\hat{a}_{P,V_i}^{(e)}\) as

\[
\hat{a}_{P,V_i}^{(e)} = c \left[ \frac{1}{\eta_1 \ e^{j2\pi \hat{\eta}_2}} \right],
\]

(3)

where

\[
\eta_1 = \begin{cases}
\sin(\theta) \cos(\phi), & \text{if } \zeta = x; \\
\sin(\theta) \sin(\phi), & \text{if } \zeta = y; \\
\cos(\theta), & \text{if } \zeta = z.
\end{cases}
\]

\[
\eta_2 = \begin{cases}
\sin(\theta) \cos(\phi), & \text{if } \epsilon = x; \\
\sin(\theta) \sin(\phi), & \text{if } \epsilon = y; \\
\cos(\theta), & \text{if } \epsilon = z.
\end{cases}
\]

The unknown complex value scalar \(c\), aforementioned in (3), may be eliminated as follows:

\[
\left[ \hat{a}_{P,V_i}^{(e)} \right]_2 = \kappa = \eta_1 \ e^{j2\pi \hat{\eta}_2},
\]

(4)

where \([\cdot]\) represents the \(i\)th element of the vector inside the square brackets.

If the two sensors exchange locations, the following holds: Suppose that \(\hat{\theta}\) and \(\hat{\phi}\) applies for \(\hat{a}_{P,V_i}^{(e)}\), then simply change \(\Delta_\epsilon\) to \(-\Delta_\epsilon\) in the estimation formulas for use for \(\hat{a}_{V_i,P}^{(e)}\). The above holds for any \(\epsilon \in \{x,y,z\}\) and for any \(\zeta \in \{x,y,z\}\).

A. When the Two Sensors’ Separation \(\Delta_\epsilon \leq \frac{1}{2}\)

At \(\Delta_\epsilon \leq \frac{1}{2}\), a one-to-one mapping exists between \(\eta_2\) and the spatial phase factor \(e^{j2\pi \frac{\eta_2}{\Delta_\epsilon}}\). This allows for the estimation of \(\eta_1\) and \(\eta_2\), from (4), as

\[
\hat{\eta}_1 = |\kappa| \ \text{sgn}(\eta_1),
\]

(5)

\[
\hat{\eta}_2 = \frac{\lambda}{2\pi \Delta_\epsilon} - \angle \left[ \kappa \ \text{sgn}(\eta_1) \right],
\]

(6)

where \(\text{sgn}()\) represents the sign of the real-value scalar inside the parentheses.

The estimators in (5) and (6) require a prior knowledge of the sign of \(\eta_1\). This is equivalent to a prior knowledge of the origin of the hemisphere (in spherical coordinates) from which the source impinges. The estimators in (5) and (6) also require an a priori knowledge of \(\frac{\lambda}{\Delta_\epsilon}\).

From (5) and (6), the incident source’s azimuth angle and elevation angle can be estimated iff the axis on which the velocity sensor lies differs from the axis along which the velocity sensor is oriented, i.e., \(\epsilon \neq \zeta\), which is equivalent to \(\eta_1 \neq \eta_2\). Otherwise, (5) and (6) together offer only one constraint, which alone would be insufficient to identify \(\theta\) and \(\phi\). Where \(\epsilon \neq \zeta\), the estimators \(\hat{\theta}\) and \(\hat{\phi}\) are given in closed form in Tables I to
Closed-Form Formulas for Eigen-Based Direction Finding of a Far-Field Point Source Using a Pressure Sensor and a Uniaxial Velocity Sensor Oriented Along the $y$-Axis

TABLE II

<table>
<thead>
<tr>
<th>Location of Velocity Sensor</th>
<th>Orientation of Velocity Sensor</th>
<th>Estimation Formulas</th>
<th>Prior Info. Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-axis</td>
<td>y-axis</td>
<td>$\theta = \begin{cases} \sin^{-1} \left( \text{sgn}(\nu) \csc(\phi) \left</td>
<td>\begin{array}{c} \frac{\dot{a}<em>{\nu x P,V,y}}{\dot{a}</em>{\nu y P,V,y}} \ \frac{\dot{a}<em>{\nu y P,V,y}}{\dot{a}</em>{\nu x P,V,y}} \end{array} \right</td>
</tr>
<tr>
<td>y-axis</td>
<td>z-axis</td>
<td>$\phi = \begin{cases} 3 + \text{sgn} \left( \frac{\dot{a}<em>{\nu x P,V,y}}{\dot{a}</em>{\nu y P,V,y}} \right) \right</td>
<td>\right) , &amp; \text{if } \phi \in [0, \pi) \ -\tan^{-1} \left( 2\pi \Delta \hat{a}_{\nu} \left</td>
</tr>
<tr>
<td>z-axis</td>
<td>y-axis</td>
<td>$\hat{\theta}, \hat{\phi}$ unobtainable</td>
<td>$\theta \in [0, \pi]$</td>
</tr>
</tbody>
</table>

B. When the Two Sensors’ Separation $\Delta_{e} > \frac{\lambda}{2}$

If $\Delta_{e} > \frac{\lambda}{2}$, there no longer exists a one-to-one mapping between $\eta_{2}$ and the spatial phase factor $e^{j\frac{2\pi}{\lambda}\eta_{2}}$ in (4). Consequentially, the estimate $\hat{\theta}_{1}$ is insufficient to identify both $(\theta, \phi)$. However, $\Delta_{e} > \frac{\lambda}{2}$ may be accommodated by using the extended-aperture methods in [71–74] if multiple p-u probes are deployed.

IV. CRAMÉR-RAO BOUNDS FOR VARIOUS ORIENTATION/LOCATION CONFIGURATIONS OF SECTION II

For those configurations that $\hat{\theta}$ and $\hat{\phi}$ exist, their corresponding Cramér-Rao bounds (CRBs) will be derived in this section. To avoid unnecessary distraction from the present objective of comparing the various configurations, a simple signal statistical model will be used here: The emitted signal $s(t) = e^{j(\omega t + \varphi)}$ equals a pure tone at angular frequency $\omega$ and an initial phase of $\varphi$. Both $\omega$ and $\varphi$ represent known constants. At the $m$th time moment, $t = mT_{s}$, a $2 \times 1$ data vector $\tilde{z}(mT_{s})$ is observed:

$$\tilde{z}(mT_{s}) = as(mT_{s}) + \tilde{n}(mT_{s}),$$

(7)

where $T_{s}$ symbolizes the time sampling period and $\tilde{n}(t)$ signifies a $2 \times 1$ vector of additive noise, modeled as zero in mean, spatio-temporally uncorrelated, Gaussian distributed, with a known covariance matrix $\Gamma_{0} = \text{diag}(\sigma^{2}, \sigma^{2})$. That is, $\sigma^{2}$ represents the known noise variance at each sensor.

With $M$ number of time samples, the $2M \times 1$ data set equals

$$\mathbf{z} = \left[ \tilde{z}(T_{s})^{T}, \cdots, \tilde{z}(MT_{s})^{T} \right]^{T}$$

$$= \mathbf{s} \otimes \mathbf{a} + \left[ \tilde{n}(T_{s})^{T}, \cdots, \tilde{n}(MT_{s})^{T} \right]^{T},$$

(8)

where $\mathbf{s} = e^{jT_{s}m\omega}, e^{j2T_{s}m\omega}, \cdots, e^{jMT_{s}m\omega})^{T}$, $\otimes$ denotes the Kronecker product, $\mathbf{n}$ stands for a $2M \times 1$ noise vector with a spatio-temporal covariance matrix $\Gamma = \mathbf{I}_{M} \otimes \Gamma_{0}$, and $\mathbf{I}_{M}$ refers to an $M \times M$ identity matrix. Thus, the $2M$ short-time discrete Fourier transform), then processing each frequency bin’s data as in Section III.

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1. Wideband signals may be handled by the well-known approach of first decomposing the measured data into distinct frequency bins (e.g., via a short-time discrete Fourier transform), then processing each frequency bin’s data as in Section III.

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TABLE III
Closed-Form Formulas for Eigen-Based Direction Finding of a Far-Field Point Source Using a Pressure Sensor and a Uniaxial Velocity Sensor Oriented Along the $z$-Axis

<table>
<thead>
<tr>
<th>Location of Velocity Sensor</th>
<th>Orientation of Velocity Sensor</th>
<th>Estimation Formulas</th>
<th>Prior Info. Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-axis</td>
<td>$z$-axis</td>
<td>$\hat{\theta} = \cos^{-1}\left(\text{sgn}(u)\left[\begin{array}{c} a_{x,V_z}^0 \ a_{y,V_z}^0 \end{array}\right]\right)$</td>
<td>$\theta \in \left[0, \frac{\pi}{2}\right)$ or $\left[\frac{\pi}{2}, \pi\right)$</td>
</tr>
</tbody>
</table>
|                             |                                | $\hat{\phi} = \left\{\begin{array}{ll}
\cos^{-1}\left(\frac{1}{\pi} \frac{1}{\lambda} \csc(\hat{\theta}) \left(\text{sgn}(u)\left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right]\right)\right), & \text{if } \phi \in \left[0, \pi\right) \\
2\pi - \cos^{-1}\left(\frac{1}{\pi} \frac{1}{\lambda} \csc(\hat{\theta}) \left(\text{sgn}(u)\left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right]\right)\right), & \text{if } \phi \in \left[\pi, 2\pi\right)
\end{array}\right.$ | $\phi \in \left[0, \pi\right)$ or $\left[\pi, 2\pi\right)$ |
| $y$-axis                    | $z$-axis                       | $\hat{\theta} = \cos^{-1}\left(\text{sgn}(u)\left[\begin{array}{c} a_{y,V_z}^0 \\ a_{z,V_z}^0 \end{array}\right]\right)$ | $\theta \in \left[0, \frac{\pi}{2}\right)$ or $\left[\frac{\pi}{2}, \pi\right)$ |
|                             |                                | $\hat{\phi} = \left\{\begin{array}{ll}
\sin^{-1}\left(\frac{1}{\pi} \frac{1}{\lambda} \csc(\hat{\theta}) \left(\text{sgn}(u)\left[\begin{array}{c} a_{y,V_z}^0 \\ a_{z,V_z}^0 \end{array}\right]\right)\right), & \text{if } \phi \in \left[0, \frac{\pi}{2}\right) \cup \left[\frac{3\pi}{2}, 2\pi\right) \\
2\pi - \sin^{-1}\left(\frac{1}{\pi} \frac{1}{\lambda} \csc(\hat{\theta}) \left(\text{sgn}(u)\left[\begin{array}{c} a_{y,V_z}^0 \\ a_{z,V_z}^0 \end{array}\right]\right)\right), & \text{if } \phi \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right)
\end{array}\right.$ | $\phi \in \left[0, \frac{\pi}{2}\right) \cup \left[\frac{3\pi}{2}, 2\pi\right)$ |
| $z$-axis                    | $z$-axis                       | $\hat{\theta} = \cos^{-1}\left(\left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right]\right)$ | $\theta \in \left[0, \pi\right]$ |
|                             |                                | $\hat{\phi} = \left\{\begin{array}{ll}
\cos^{-1}\left(\left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right]\right), & \text{if } \left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right] \not\in \left[\frac{\pi}{2}, \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \\
\cos^{-1}\left(\left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right]\right) + \frac{\pi}{2}, & \text{if } \left[\begin{array}{c} a_{x,V_z}^0 \\ a_{y,V_z}^0 \end{array}\right] \in \left[\frac{\pi}{2}, \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]
\end{array}\right.$ | $\phi \text{ unobtainable}$ |

TABLE IV
CRBs for Azimuth-Elevation Direction-of-Arrival Estimation for Nine $p-u$ Probe Configurations

<table>
<thead>
<tr>
<th>Types/Locations of Sensors</th>
<th>$\text{CRB}(\theta)$</th>
<th>$\text{CRB}(\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ at $(0,0,0)$, $V_z$ at $(\Delta_z,0,0)$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_y$ at $(0,\Delta_y,0)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_y$ at $(0,0,\Delta_y)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_y$ at $(\Delta_y,0,0)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_y$ at $(0,\Delta_y,0)$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_y$ at $(0,0,\Delta_y)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \cos^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_y}\right)^2}{2M \left(\frac{2\pi}{\Delta_y}\right)^2}$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_z$ at $(\Delta_z,0,0)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2}{2M \left(\frac{2\pi}{\Delta_z}\right)^2}$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2}{2M \left(\frac{2\pi}{\Delta_z}\right)^2}$</td>
</tr>
<tr>
<td>$P$ at $(0,0,0)$, $V_z$ at $(0,\Delta_z,0)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \cos^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2}{2M \left(\frac{2\pi}{\Delta_z}\right)^2}$</td>
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<td>$P$ at $(0,0,0)$, $V_z$ at $(\Delta_z,0,0)$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2}{2M \left(\frac{2\pi}{\Delta_z}\right)^2}$</td>
<td>$\frac{\sigma^4 \cos^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2 \cos^2 \phi + \csc^2 \phi \left(\frac{2\pi}{\Delta_z}\right)^2}{2M \left(\frac{2\pi}{\Delta_z}\right)^2}$</td>
</tr>
</tbody>
</table>
Fig. 2. Estimation RMSE versus √CRB(θ) for p-u probe configuration corresponding to first case of Table III. CRB(θ) is taken from third-to-last case of Table IV.

Each icon in Figs. 2 and 3 consists of 500 statistically independent Monte Carlo trials. Figs. 2 and 3 show that the estimation RMSE approximates the CRB for signal-to-noise ratio (SNR) > 0 dB, but the estimator breaks down for SNR < −5 dB. This SNR threshold may be lowered by allowing more snapshots.

V. CONCLUSION

This work systematically investigates various pairings of a uniaxial particle velocity sensor with an isotropic pressure sensor for eigen-based direction finding of a source emitting in the far field. The orientation of the particle velocity sensor must not be the same as the orientation for the displacement between the two sensors, and the intersensor spacing must not exceed half a wavelength, otherwise, the azimuth-elevation direction finding would not be viable. Also required is a prior knowledge of the intersensor spacing in units of the incident signal’s wavelength and of the hemisphere the source lies in.

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