Acoustic Direction Finding Using a Spatially Spread Tri-Axial Velocity Sensor

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This paper shows how a triad of orthogonally oriented uniaxial velocity sensors may be spatially separated, yet facilitate direction finding of incident emitters via closed-form subspace-based parameter estimation algorithms, while extending the triad’s spatial aperture in three-dimensional space to enhance the resolution of the azimuth/elevation direction of arrival estimates.
velocity sensors—all collocated in a point-like spatial geometry. The entire tri-axial velocity sensor, thus, distinctly measures all three Cartesian components of the acoustic particle velocity vector, which equals the spatial gradient of the acoustic pressure. This tri-axial velocity sensor, therefore, treats the acoustic wave field as a vector field (namely, the acoustic particle velocity field), not merely as a scalar field (namely, the pressure field) by the customary microphone or hydrophone.

Mathematically, a collocating tri-axial velocity sensor (placed at the origin of the three-dimensional Cartesian coordinates) would have this $3 \times 1$ array manifold [5, 6, 7]

$$
\mathbf{a} = \begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix}
$$

in response to a unit-power incident acoustic wave that has traveled through an homogeneous isotropic medium. In the previously mentioned, $\theta \in [0, \pi]$ symbolizes the elevation angle measured from the positive $z$-axis, $\phi \in [0, 2\pi]$ denotes the azimuth angle measured from the positive $x$-axis, and $u$, $(v, w)$ refers to the direction cosine along the $x$-axis ($y$-axis and $z$-axis). The first, second, and third components in (1) correspond to the uniaxial velocity sensors aligned along the $x$-axis, the $y$-axis, and the $z$-axis, respectively. These three components together give a Frobenius norm of $u^2 + v^2 + w^2 = 1$, $\forall \theta, \phi$. This identity will be shown to remain valid, regardless of any spatial displacement among the three uniaxial velocity sensors. This property will be essential to the subsequently proposed algorithm and will be used in (5), (16), and (17).

III. THE NEW SCHEME’S UNDERLYING PHILOSOPHY—ILLUSTRATED BY A SIMPLE ARRAY CONFIGURATION

To motivate the proposed scheme, this section will focus on one particularly simple array configuration to spatially spread the three uniaxial velocity sensors. The proposed scheme will later be developed in Section IV to any arbitrary array configuration.

A. The New Array Manifold for the Illustrative Example of Fig. 1

In the simple array configuration of Fig. 1, a $z$-axis oriented velocity sensor is placed at the Cartesian origin, whereas an $x$-axis ($y$-axis) oriented velocity sensor lies on the $x$-axis ($y$-axis) at a distance $\Delta_x$ ($\Delta_y$) from the Cartesian origin. The displacements $\Delta_x$ and $\Delta_y$ may each be positive or negative, though the special case of $\Delta_x > 0$ and $\Delta_y > 0$ is illustrated in Fig. 1, which labels the uniaxial velocity sensor oriented along the $x$-axis ($y$-axis and $z$-axis) as $V_x$ ($V_y$, $V_z$).

This spatially distributed tri-axial velocity sensor has an array manifold different from the spatially collocated array manifold of (1), because the two off-origin uniaxial velocity sensors introduce the spatial phase factors of $e^{j \frac{2\pi}{\lambda} \Delta_x u}$ and $e^{j \frac{2\pi}{\lambda} \Delta_y v}$. Hence, Fig. 1 has the array manifold

$$
\mathbf{a}_{\text{triangle}} = \begin{bmatrix}
    2\pi u e^{j \frac{2\pi}{\lambda} \Delta_x u} \\
    2\pi v e^{j \frac{2\pi}{\lambda} \Delta_y v} \\
    w
\end{bmatrix}.
$$

This new array manifold in (2) now depends on the incident signal’s wavelength $\lambda$. However, its Frobenius norm, nonetheless, equals unity $\forall \theta, \phi, \lambda$ (or equivalently $\forall u, v, w, \lambda$), just as for the collocating triad in Section II.

B. The Proposed Self-Normalization Direction-Finding Algorithm for the Illustrative Example of Fig. 1

In eigen-based approaches of direction finding, the uniaxial velocity sensor’s observed data would be formed into a $3 \times 3$ spatial correlation matrix. Then, eigen-decompose this $3 \times 3$ matrix (plus any additional processing to decorrelate any cross correlated incident signals, and/or to decouple simultaneous sources’ steering vectors). The steering vector estimate for each incident source would, thereby, be obtainable

$$
\hat{\mathbf{a}} \approx \begin{bmatrix}
    p_x \\
    p_y \\
    p_z
\end{bmatrix} \equiv c \mathbf{a}_{\text{triangle}},
$$

which is correct with respect to the true value $\mathbf{a}_{\text{triangle}}$, to within an unknown complex-value scalar of $c$. The unknown $c$ arises from the eigen-decomposition but may be estimated as

$$
c = \sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2} e^{j \angle (p_x, p_y)},
$$

if prior knowledge is available on the sign $(s_x)$ of $w$. If there were no noise and/or if there were an infinite number of snapshots, the approximation in (3) would become an equality. The subsequent exposition will write all such approximations as equalities, for simplicity.

Next, normalize the first component in (3) by $c$ to give

$$
\frac{p_x e^{-j \angle (s_x, p_x)} }{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} = u e^{j \frac{2\pi}{\lambda} \Delta_x u}.
$$

From (5), two complementary estimators of $u$ may be obtained, simultaneously, in parallel:

1) A one-to-many relationship links the complex phase factor $e^{j \frac{2\pi}{\lambda} \Delta_x u}$ and $u$ in $[-1, 1]$ for the extended aperture case of $\frac{\lambda}{\Delta_x} > \frac{1}{2}$. Hence, exploiting the complex phase of only the first element of $\hat{\mathbf{a}}$

$$
\hat{u}_{\text{phs}} = \frac{1}{2\pi} \frac{\lambda}{\Delta_x} \left( \frac{p_x e^{-j \angle (s_x, p_x)} }{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right) = m + u
$$
can be obtained as an estimate \( u \), but only ambiguously to within some (to-be-determined) integer multiple (\( m \times \)) of \( \pm \frac{2\pi}{\Delta_x} \), where \( m \) denotes a to-be-determined integer in \( \{ \left\lfloor \frac{2}{n} - \frac{2\pi}{\Delta_x} \right\rfloor, \ldots, \left\lceil \frac{2}{n} + \frac{2\pi}{\Delta_x} \right\rceil \} \).

2 A one-to-two relationship connects the magnitude \( |u| \) and \( u \in [-1, 1] \). Hence, exploiting the relative magnitude of \( p_x \),

\[
\hat{u}_{\text{mag}} = \frac{p_x}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} = \pm u \quad (7)
\]

can be obtained as an estimate \( u \), but only ambiguously to within some (to-be-determined) integer multiple (\( \pm \) sign).

These two estimates \( \hat{u}_{\text{phs}} \) and \( \hat{u}_{\text{mag}} \) can disambiguate each other, as in the following:

(a) If \( \hat{u}_{\text{mag}} = u \), resolve the cyclic ambiguity via

\[
\hat{m}_u \overset{\text{def}}{=} \arg\min_m \left\{ \left( \frac{m}{\Delta_x} + \frac{\lambda}{2\pi \Delta_x} \right) \frac{p_x e^{-j \lambda s(u, p_x)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right\} - \frac{p_x}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} = \frac{\hat{u}_{\text{phs}}}{\hat{m}_u}.
\]

(b) If \( \hat{u}_{\text{mag}} = -u \), resolve the cyclic ambiguity through

\[
\hat{m}_u \overset{\text{def}}{=} \arg\min_m \left\{ \left( \frac{m}{\Delta_x} + \frac{\lambda}{2\pi \Delta_x} \right) \frac{p_x e^{j \lambda s(u, p_x)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right\} - \frac{p_x}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} = \frac{\hat{u}_{\text{phs}}}{\hat{m}_u}.
\]

(c) How to decide between \( \hat{u}_{\text{mag}} = u \) versus \( \hat{u}_{\text{mag}} = -u \)? Decide in favor of \( \hat{u}_{\text{mag}} = u \), if \( \epsilon_u^+ (\hat{m}_u^+) < \epsilon_u^- (\hat{m}_u^-) \). Choose \( \hat{u}_{\text{mag}} = -u \), if \( \epsilon_u^+ (\hat{m}_u^-) \geq \epsilon_u^- (\hat{m}_u^-) \).

(d) Hence, \( u \) is unambiguously estimated from the phase factor as

\[
\hat{u} = \begin{cases} 
\left( \hat{m}_u^+ + \frac{\lambda}{2\pi} \frac{p_x e^{-j \lambda s(u, p_x)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right) \frac{\lambda}{\Delta_x}, & \text{if } \epsilon_u^+ (\hat{m}_u^+) < \epsilon_u^- (\hat{m}_u^-); \\
\left( \hat{m}_u^- + \frac{\lambda}{2\pi} \frac{p_x e^{-j \lambda s(u, p_x)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right) \frac{\lambda}{\Delta_x}, & \text{if } \epsilon_u^+ (\hat{m}_u^+) \geq \epsilon_u^- (\hat{m}_u^-).
\end{cases}
\]

The y-axis Cartesian direction cosine estimate \( \hat{v} \) may be obtained analogously via (5)–(10).

(e) Items \{c\} and \{d\} indicate that both the magnitude and the phase of the first two entries of \( \mathbf{a}_{\text{triangle}} \) can offer information on \( u, v \). Between \( u_{\text{mag}} \) and \( \hat{u} \), which should be used? It makes intuitive sense that the choice should result in a steering vector closest to orthogonality to the data’s noise subspace. Using this philosophy, proceed as follows: let \( \mathbf{E}_n \) denote \( 3 \times (3 - K) \) noise subspace eigenvector matrix that contains the \( 3 - K \) number of eigenvectors spanning the noise subspace of the data correlation matrix, where \( K \) is the number of incident sources. Then, \( \{u, v\} \) may be estimated as

\[
(\hat{u}, \hat{v}) = \begin{cases} 
(\hat{u}_{\text{mag}}, \hat{v}_{\text{mag}}), & \text{if } \| \mathbf{a} (s_u^t \hat{u}_{\text{mag}}, s_v^t \hat{v}_{\text{mag}})^H \mathbf{E}_n \|_2 < \| \mathbf{a} (\hat{u}, \hat{v})^H \mathbf{E}_n \|_2, \\
(\hat{u}, \hat{v}), & \text{if } \| \mathbf{a} (s_u^t \hat{u}_{\text{mag}}, s_v^t \hat{v}_{\text{mag}})^H \mathbf{E}_n \|_2 > \| \mathbf{a} (\hat{u}, \hat{v})^H \mathbf{E}_n \|_2.
\end{cases}
\]
then uniaxial velocity sensor is relocated somewhere on the \( z \)-axis. This array unambiguously, despite any sparse noncollocation among azimuth/elevation angle of arrival has been estimated \( \theta \) over a hemisphere defined over \( \{0, \pi/2\} \) and \( \phi \) \( \in (-\pi, \pi) \). The \( \pm \pi \) terms appear in the previously mentioned, because of the following considerations: If \( s^\circ_u = -1 \) and \( s^\circ_v = -1 \), then \( \phi \in (-\pi, -\pi/2) \). If \( s^\circ_u = 1 \), then \( \phi \in (-\pi/2, \pi/2) \). If \( s^\circ_u = -1 \) and \( s^\circ_v = +1 \), then \( \phi \in (\pi/2, \pi) \) If \( s_w = +1 \), then \( \hat{\theta} \in [0, \pi] \). If \( s_w = -1 \), then \( \hat{\theta} \in (-\pi, 0) \).

With prior knowledge of the sign of \( w \), the azimuth/elevation angle of arrival has been estimated unambiguously, despite any sparse noncollocation among the three uniaxial velocity sensors.\(^1\) This array configuration is simpler than the arbitrary configuration to be introduced in Section IV, in that the first two elements here in (2) each depend on only one of the three Cartesian direction cosines (i.e., \( u, v, w \)). Hence, only one sign ambiguity requires disambiguation at each uniaxial velocity sensor, whereas all three sign ambiguities need to be resolved for each uniaxial velocity sensor of the arbitrarily spaced configuration to be introduced in Section IV.

IV. THE NEW SCHEME FOR ANY ARBITRARILY SPREAD TRI-AXIAL VELOCITY SENSOR

This section will show how the algorithmic philosophy in Section III can apply to the arbitrarily general array configuration of Fig. 2, as long as the three uniaxial velocity sensors remain orthogonal among themselves. The algorithm’s basic philosophy here remains as in Section III: To resolve the one-to-many cyclic ambiguity in \( \hat{u}_{\text{phs}} \), by the one-to-two sign ambiguity in \( \hat{u}_{\text{mag}} \).

\(^1\)This prior knowledge of the sign of \( w \) may be replaced by a prior knowledge of the sign of \( u \) (or \( v \)), if the \( x \)-axis- \( (y \)-axis) oriented uniaxial velocity sensor is placed at the Cartesian origin and if the \( z \)-axis-oriented uniaxial velocity sensor is relocated somewhere on the \( z \)-axis.

The symbols \( \alpha_x, \alpha_y, \beta_x, \beta_y \) are graphically defined in Fig. 3. Unlike the simple array manifold (2) in Section III, each complex phase in (14) now depends on all three Cartesian direction cosines \((u,v,w)\). Consequently, the subsequent disambiguation would become more complicated, but the approach remains as before in Section III.

B. The Proposed Algorithm for an Arbitrarily Spread Tri-Axial Velocity Sensor

By eigen-decomposition of the spatial correlation matrix of the collected data, the following steering vector

\[ A_{\text{vec}}(\theta, \phi) \]

\[ = \begin{bmatrix} 1 & e^{j \alpha_x} & e^{j \alpha_y} & e^{j \beta_x} & e^{j \beta_y} \end{bmatrix} \begin{bmatrix} u e^{j (\alpha_x \sin(\beta_x) + \alpha_y \sin(\beta_y))} \cos(\alpha) + v e^{j (\alpha_x \sin(\beta_x) + \alpha_y \sin(\beta_y))} \cos(\alpha) + w e^{j (\alpha_x \sin(\beta_x) + \alpha_y \sin(\beta_y))} \cos(\alpha) \\ v e^{j (\alpha_x \sin(\beta_x) + \alpha_y \sin(\beta_y))} \cos(\alpha) + w e^{j (\alpha_x \sin(\beta_x) + \alpha_y \sin(\beta_y))} \cos(\alpha) + w e^{j (\alpha_x \sin(\beta_x) + \alpha_y \sin(\beta_y))} \cos(\alpha) \\ w \end{bmatrix} \]

The symbols \( \alpha_x, \alpha_y, \beta_x, \beta_y \) are graphically defined in Fig. 3. Unlike the simple array manifold (2) in Section III, each complex phase in (14) now depends on all three Cartesian direction cosines \((u,v,w)\). Consequently, the subsequent disambiguation would become more complicated, but the approach remains as before in Section III.

\[ 2\text{If it is another uniaxial velocity sensor that lies at the origin or if none of the uniaxial velocity sensors lie at the origin, the entire array manifold in (14) would simply be multiplied by a common spatial phase factor that would not materially affect the subsequent development.} \]
estimate will be obtained, instead of the estimate in (15):

\[
\hat{a} \approx \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \overset{\text{def}}{=} \mathbf{a}_{\text{gen}}.
\]  

(15)

With prior knowledge of \( s_u \), defined previously in (5), normalize each of the first three components of (14) by the same \( \hat{c} \) as defined in Section III, thereby yielding

\[
\frac{p_x e^{-j(s_u p_z)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} = u e^{j\frac{\pi}{\Delta_1} h_x}, 
\]

(16)

\[
\frac{p_y e^{-j(s_u p_z)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} = v e^{j\frac{\pi}{\Delta_1} h_y}, 
\]

(17)

where

\[
h_x = u \sin(\alpha_x) \cos(\beta_x) + v \sin(\alpha_x) \sin(\beta_x) + w \cos(\alpha_x),
\]

(18)

\[
h_y = u \sin(\alpha_y) \cos(\beta_y) + v \sin(\alpha_y) \sin(\beta_y) + w \cos(\alpha_y),
\]

(19)

symbolize the non-Cartesian direction cosines obtained by projecting the gradient vector onto the respective axes on

Next, Section IV.B1 will explain how to estimate the non-Cartesian direction cosines \((h_x, h_y)\), from which Section IV.B2 will show how to estimate the Cartesian direction cosines \((u, v, w)\).

1) To Estimate the Non-Cartesian Direction Cosines \((h_x, h_y)\): Consider first the estimation of \( h_x \).

From (16), two complementary estimators of \( h_x \) can be computed simultaneously in parallel (somewhat like the case in Section III):

(i) A many-to-one relationship connects \( h_x \) and

\[
h_x = \frac{1}{2\pi} \frac{\lambda}{\Delta_x} \arg \min \left( \frac{s_u p_x e^{-j(s_u p_z)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right)
\]

\[
= m \frac{\lambda}{\Delta_x} + h_x
\]

(20)

would estimate \( h_x \), ambiguously to within some integer multiple \((m \times)\) of \( \frac{1}{\Delta_x} \), where \( m \) represents a to-be-determined integer.

(ii) The magnitudes

\[
\hat{u}_{\text{mag}} = \frac{p_x}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}},
\]

\[
\hat{v}_{\text{mag}} = \frac{p_y}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}},
\]

\[
\hat{w}_{\text{mag}} = \frac{p_z}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}},
\]

together can estimate \( h_x \) to within a ± sign ambiguity:

\[
\hat{h}_{x,\text{mag}}^{(s_u, s_v)} = s_u \hat{u}_{\text{mag}} \sin(\alpha_x) \cos(\beta_x) + s_v \hat{v}_{\text{mag}} \sin(\alpha_x) \sin(\beta_x) + s_w \hat{w}_{\text{mag}} \cos(\alpha_x),
\]

(21)

with \( s_u, s_v, s_w \in \{1, -1\}, \) \( s_u \) known, but \( s_v \) and \( s_v \) yet to be determined.

Next, estimate \( m \in \left\{ \left\lfloor \frac{1}{\Delta_x} - \frac{\lambda}{\Delta_x} \right\rfloor, \ldots, \left\lfloor \frac{1}{\Delta_x} + \frac{\lambda}{\Delta_x} \right\rfloor \right\} \), in terms of any \( s_u \) and \( s_v \):

\[
\hat{m}^{(s_u, s_v)} = \arg \min_m \left( m \frac{\lambda}{\Delta_x} + \frac{1}{2\pi} \frac{\lambda}{\Delta_x} \arg \min \left( \frac{s_u p_x e^{-j(s_u p_z)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right) \right) - \hat{h}_{x,\text{mag}}^{(s_u, s_v)}
\]

(22)

which \( \Delta_x \) and \( \Delta_y \), respectively, lie. These non-Cartesian direction cosines \((h_x, h_y)\) are counterpart to the Cartesian direction cosines \((u, v)\) in Section III, where all uniaxial velocity sensors lie on some Cartesian axis.

The sign ambiguity in \( s_u \) and the sign ambiguity in \( s_v \) together imply four possible candidates for \( \hat{m}^{(s_u, s_v)} \).

The previously mentioned procedure in (20) to (22) for \( h_x \) can be applied analogously for \( h_y \).

Fig. 3. Geometric illustration of non-Cartesian direction cosines, \( h_x \) and \( h_y \), in three-dimensional space.
To choose among the four candidates of $\hat{m}^{(s_x,s_y)}$, choose

$$(s_x', s_y') \overset{\text{def}}{=} \arg \min_{(s_x, s_y)} \left[ e_x^2 \left( \hat{m}^{(s_x,s_y)} \right) + e_y^2 \left( \hat{m}^{(s_x,s_y)} \right) \right].$$  \hfill (23)

The aforementioned facilitates $\hat{h}_x$ and $\hat{h}_y$ to be unambiguously estimated as

$$\hat{h}_x = \left( \hat{m}^{(s_x',s_y')}_x + \frac{1}{2\pi} \frac{s_x' p_x e^{-j\angle (s_x p_x)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right) \frac{\lambda}{\Delta x},$$  \hfill (24)

$$\hat{h}_y = \left( \hat{m}^{(s_x',s_y')}_y + \frac{1}{2\pi} \frac{s_y' p_y e^{-j\angle (s_y p_y)}}{\sqrt{|p_x|^2 + |p_y|^2 + |p_z|^2}} \right) \frac{\lambda}{\Delta y}. $$  \hfill (25)

The disadvantage of omitting the pressure sensor (which is required in [4]) is that the emitter may now be localized to only within a hemisphere, because only two constraints are available:

1) $m \frac{\lambda}{\Delta x} + \hat{h}_x, \text{phs} = \hat{h}_x, \text{mag}$ from (20), and

$$\begin{pmatrix} \hat{h}_x, \text{mag} \\ \hat{h}_y, \text{mag} \end{pmatrix} = \begin{pmatrix} m \frac{\lambda}{\Delta x} + \hat{h}_x, \text{phs} \\ \hat{h}_y, \text{phs} \end{pmatrix} \frac{\lambda}{\Delta y}.$$  \hfill (26)

The aforementioned defined matrix $M$ would have a full rank of two, if and only if the two non-Cartesian direction cosines $\hat{h}_x$ and $\hat{h}_y$ are nonparallel. Therefore, the Cartesian direction cosines may be estimated from the phase factor as $(\hat{u}, \hat{v})$

$$\hat{u} = \frac{\chi_1 + \left( |b_2| [a_3] - |a_2| [b_3] \right) \hat{h}_x + \left( |a_2| + |a_1| \right) \hat{h}_y}{|x_3|},$$

$$\hat{v} = \frac{\chi_2 + \left( |a_2| + |a_1| \right) \hat{h}_x + \left( |b_2| [a_3] - |a_2| [b_3] \right) \hat{h}_y}{|x_3|},$$

where

$$\chi_1 = [b_2]^2 [a_3] - \left( |a_2| [b_2] + |a_1| [b_3] \right) \hat{h}_x + \left( |a_2| + |a_1| \right) \hat{h}_y,$$

$$+ [a_1]^2 [b_2] \left( |b_2| [a_3] - |a_2| [b_3] \right) \hat{h}_x + \left( |a_2| + |a_1| \right) \hat{h}_y,$$

$$+ [a_1]^2 [b_2] \left( |b_2| [a_3] - |a_2| [b_3] \right) \hat{h}_x + \left( |a_2| + |a_1| \right) \hat{h}_y,$$

$$\chi_2 = [|b_2| [a_3] - |a_1| [b_3]],$$

$$\cdot \left( |a_2| [b_2] [a_3] + |a_2| [a_3] [b_2] + |a_2| [b_2] [a_3] + |b_2| [a_3] [a_2] \right) \hat{h}_x + \left( |a_2| + |a_1| \right) \hat{h}_y,$$

$$+ 2 \left( |a_2| [b_2] + |a_3| [b_3] \right) \hat{h}_x \hat{h}_y - \left( |a_2|^2 + |a_2|^2 \right) [b_2] \hat{h}_x - 2 \left( |a_1| [b_2] + |a_1| [b_3] \right) \hat{h}_x - \hat{h}_y,$$

$$\chi_3 = \left[ |b_2|^3 \left( |a_2|^2 + |a_3|^2 \right) + \left( |b_2|^2 [a_3] - |a_2|^2 [b_3] \right)^2 - 2 |a_1| [b_2] + |a_1| [b_3] \right] \hat{h}_x + \left( |a_2|^2 + |a_3|^2 \right) \hat{h}_y,$$

$$\chi_4 = |b_2|^2 [a_2] \hat{h}_x + |a_2|^2 |b_2|^2 \hat{h}_x + |b_2|^2 \left( |a_2|^2 + |a_3|^2 \right) \hat{h}_x - |a_1| [b_2] \hat{h}_x + |a_2| [b_3] \hat{h}_x - |a_1| [b_2] \hat{h}_x + |a_2| [b_3] \hat{h}_y,$$

$$\chi_5 = |b_2|^2 \left( |a_2|^2 + |a_3|^2 \right) + \left( |b_2|^2 [a_3] - |a_2|^2 [b_3] \right)^2 - 2 |a_1| [b_2] \hat{h}_x + |a_1| [b_2] \hat{h}_y + |a_1| [b_3] \hat{h}_x + \left( |b_2|^2 + |b_3|^2 \right).$$

$a = \left[ \sin(\alpha_x) \cos(\beta_x), \sin(\alpha_y) \sin(\beta_x), \cos(\alpha_x) \right],$

$b = \left[ \sin(\alpha_x) \cos(\beta_y), \sin(\alpha_y) \sin(\beta_y), \cos(\alpha_y) \right].$
Both the magnitude and the phase of the first two entries of $a_{\text{gen}}$ offer information on $u$ and $v$. Analogous to (e) in Section III.B, $(\hat{u}, \hat{v})$ may be estimated as in (28).

\[
(\hat{u}, \hat{v}) = \begin{cases} 
(s_n^H \hat{u}_{\text{mag}}, s_n^H \hat{v}_{\text{mag}}), & \text{if } a(s_n^H \hat{u}_{\text{mag}}, s_n^H \hat{v}_{\text{mag}})^H E_n \|	ext{2} < \| a(\hat{u}, \hat{v})^H E_n \|^2, \\
(\hat{u}, \hat{v}), & \text{if } \| a(s_n^H \hat{u}_{\text{mag}}, s_n^H \hat{v}_{\text{mag}})^H E_n \|^2 > \| a(\hat{u}, \hat{v})^H E_n \|^2.
\end{cases}
\] (28)

The azimuth/elevation arrival angles may then be estimated as in (12) and (13).

Hence, the arbitrarily spread (but still orthogonal) tri-axial velocity sensor may be used with any eigen-based parameter-estimation algorithm for two-dimensional direction finding. This proposed scheme may apply to a single tri-axial velocity sensor or to an array consisting of multiple tri-axial velocity sensors.\(^3\),\(^4\)

V. MONTE CARLO SIMULATIONS TO VERIFY THE PROPOSED SCHEME’S EFFICACY

The proposed scheme may be used with any eigen-based parameter estimation algorithm to estimate the emitters’ azimuth/elevation angles of arrival. To illustrate how, this section will adopt the univector hydrophone ESPRIT algorithm of [8], which was originally developed for a four-component acoustic vector sensor with all its component sensors collocating in a pointlike geometry. To adopt [8] to the spatially spread tri-axial velocity sensor, simply replace (15)–(17) of [8] by the procedure proposed in Section IV of the present work.\(^5\),\(^6\)

Monte Carlo simulations are conducted to verify the proposed scheme’s efficacy and aperture extension, despite its irregular array configuration. The following settings are used: $\alpha_x = 85^\circ$, $\beta_x = 5^\circ$, $\alpha_y = 95^\circ$, $\beta_y = 80^\circ$, and $\Delta = \Delta_x = \frac{1}{2} \Delta_y$. There exist two pure tone signals, at digital frequencies $f_1 = 0.46$ and $f_2 = 0.36$, respectively, with deterministic complex phases $\varphi_1$ and $\varphi_2$ that are randomly generated for each Monte Carlo experiment from a uniform distribution over $[0, 2\pi)$. These two signals impinge, respectively, from $(\theta_1, \phi_1) = (130^\circ, 50^\circ)$ and $(\theta_2, \phi_2) = (45^\circ, -135^\circ)$. All incident signals have unity power. The additive noise is Gaussian, zero mean, white spatiotemporally, with a known power of $\sigma^2 = 25$ dB.

\(^3\)This arbitrarily spread case would degenerate to the simple case in Section III, if $\alpha_x = \pi/2$ and $\beta_x = 0$. Then, (20) would degenerate to (6), and (21) would degenerate to (7).

\(^4\)If $\Delta_x = \Delta_y = 0$ in (14), this arbitrarily spread case will degenerate to the case in which the three univariate velocity sensors are all collocated. Then, (20)–(28) would no longer hold. Instead, $\phi$ and $\theta$ may estimated simply via (12) and (13), with $\hat{u}(\theta)$ replaced by $p_x(\phi)$.\(^7\)

\(^5\)The proposed scheme can handle data models other than that in [8]. This section’s simulations serve only as an illustrative example. For instance, wideband data could first be segmented via a sliding time-domain window, then processed by a short-time discrete Fourier transform (DFT), before each DFT component is individually processed.

\(^6\)This CRMSE is defined as $\sum_{I=1}^{I} \sqrt{\frac{e_{u,k,i}}{n} + \frac{e_{v,k,i}}{m}}$, where $\delta_{u,k,i}$ ($\delta_{v,k,i}$) denotes the error in

\(^7\)The estimation bias is roughly an order of magnitude below the corresponding estimation standard deviation, hence, not shown.

Fig. 4. Monte Carlo simulations verifying aperture extension efficacy of proposed scheme at various apertures.
estimating the $k$th source’s $x$-axis ($y$-axis) direction cosine at the $i$th Monte Carlo experiment. Each figure’s every icon represents $I = 1000$ statistically independent Monte Carlo experiments, each of which involves 80 temporal snapshots. Fig. 4(b) does the same for the second source.

Figs. 4(a), and 4(b) together demonstrate that the proposed scheme successfully resolves the two incident emitters, even if the three uniaxial velocity sensors are not collocated, but spaced apart, sparsely. The collocated case (i.e., $\Delta = 0$) has its estimation error indicated in these figures by a horizontal dotted line to ease comparison with the proposed scheme’s performance. The proposed scheme’s CRMSE drops by about an order of magnitude, as $\Delta$ increases (for the proposed scheme) by an order of magnitude from $\Delta = \lambda_{\text{min}} = 100$ to $\Delta = \lambda_{\text{min}} = 101$. Incidentally, this ESPRIT-based estimation approximates the Cramér-Rao lower bound, to the extent that the parameter estimation algorithm of ESPRIT can do so.

Figs. 5(a), 5(b) are similar to Figs. 4(a), 4(b), but the CRMSE is plotted now versus the signal-to- (Gaussian-) noise ratio (SNR). The simulation scenario is same as in Figs. 4(a), 4(b), except that Figs. 5(a), 5(b) have the sources impinging from $(\theta_1, \phi_1) = (50^\circ, 30^\circ)$ and $(\theta_2, \phi_2) = (40^\circ, 135^\circ)$. Also, in Figs. 5(a), 5(b), the blue dotted curves and the blue “+” are for a triad with collocated uniaxial velocity sensors, whereas the red solid curves and the red asterisks are for a triad with spatially spread uniaxial velocity sensors at $\Delta = \lambda_{\text{min}} = 2.9$. At this modestly extended aperture, the proposed scheme already lowers the CRMSE by almost an order of magnitude, relative to the collocated case.

VI. CONCLUSION

The collocating tri-axial velocity sensor has been used in many eigen-based closed-form algorithms to estimate the azimuth/elevation direction of arrival. This work shows how those earlier eigen-based closed-form algorithms can be modified to apply to any arbitrarily spread tri-axial velocity sensor, even if the three uniaxial velocity sensors are spaced apart anywhere in three-dimensional space, thereby extending their spatial aperture from a point to span a large aperture, refining the tri-axial velocity sensor’s azimuth/elevation resolution.

If this tri-axial velocity sensor is accompanied by a pressure sensor, the scheme in [4] can be used instead. There, the isotropic pressure sensor can serve as a phase reference, facilitating the three uniaxial velocity sensors’ relative complex phases to be estimated. However, the abovementioned approach is not viable in the present work without any pressure sensor. Instead, the presently proposed algorithm exploits the fact that a tri-axial velocity sensor’s data must have a Frobenius norm that is independent of the incident emitter’s azimuth/elevation direction of arrival and that is independent of the incident signal’s frequency.

There exists an electromagnetic counterpart [9], which is predicated on an electric dipole and a magnetic loop’s magnitude responses and phase responses, which differ principally from those in (2) and (14) for the acoustic tri-axial velocity sensor presently under study. On account of the fundamental disparity between acoustics and electromagnetics, the algorithmic steps and the allowable array configurations are basically different between the present work and [9].

REFERENCES


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