EXIT Chart

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• **EXIT Chart**

  • **EXtrinsic Information Transfer (EXIT) Chart**
  • Use for the analysis of iterative decoding algorithms
  • Predict the behavior of the iterative decoder by looking at the input/output relations of the individual constituent decoders
Basic Assumptions

- The a priori values are independent of the respective channel values
- The probability density function $p_e(\cdot)$ of the extrinsic output values is Gaussian distributed
- The probability density function $p_e(\cdot)$ fulfills the symmetry condition
  \[ p_e(\xi|d) = p_e(-\xi|d) \exp(d\xi) \]
  where the binary random variable $d$ denotes a transmitted symbol (either outer code symbol or inner information symbol) with $d \in \{-1\}$
Interpretation

- Let $L_a$ denote the a priori LLR value corresponding to the transmitted symbol $d$.
- $L_a$ is symmetric Gaussian distributed with mean $\mu_a d$ and variance $\sigma_a^2$.
  - $\mu_a = \sigma_a^2 / 2$.
- Conditional probability density function of $L_a$ is
  $$p_a(\xi | d) = \frac{1}{\sqrt{2\pi \sigma_a^2}} \exp \left[ -\frac{1}{2} \left( \frac{\xi - \mu_a d}{\sigma_a} \right)^2 \right]$$
Interpretation

- The mutual information $I_a = I(d; L_a)$ between the transmitted symbol $d$ and the corresponding LLR value $L_a$ is used as a measure of the information content of the a priori knowledge

$$I_a = \frac{1}{2} \sum_{d \in \{-1, +1\}} \int_{-\infty}^{\infty} p_a(\xi | d) \log_2 \left[ \frac{2p_a(\xi | d)}{p_a(\xi | -1) + p_a(\xi | +1)} \right] \, d\xi$$

- Combining with the conditional PDF equations, we have

$$I_a(\sigma_a) = 1 - \frac{1}{\sqrt{2\pi}\sigma_a^2} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{\xi - \frac{\sigma_a^2}{\sigma_a^2}d}{\sigma_a} \right)^2 \right] \log_2 [1 + \exp(-\xi)] \, d\xi$$

- A function of $\sigma_a$ only

Special Case (usually used in analysis)

- If $d = +1$ are sent all the time,

$$I_a(\sigma_a) = 1 - \frac{1}{\sqrt{2\pi}\sigma_a^2} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{\xi - \frac{\sigma_a^2}{\sigma_a^2}d}{\sigma_a} \right)^2 \right] \log_2 [1 + \exp(-\xi)] \, d\xi$$

- and the inverse function of which is given by [1,2]

$$I_a^{-1}(x) = \begin{cases} \frac{\gamma_1 x^2 + \gamma_2 x + \gamma_3 \sqrt{x}}{\gamma_4 \ln[\gamma_5 (1 - x)] + \gamma_6 x} & \text{if } 0 \leq x \leq 0.3646 \\ \text{otherwise} \end{cases}$$

where $\gamma_1 = 1.09542$, $\gamma_2 = 0.214217$, $\gamma_3 = 2.33737$, $\gamma_4 = -0.706692$, $\gamma_5 = 0.386013$ and $\gamma_6 = 1.75017$

Interpretation

- Similarly, the mutual information $I_e = I(d; L_e)$ between the transmitted symbol $d$ and the corresponding extrinsic LLR value $L_e$ is

$$I_e = I(d; L_e) = \frac{1}{2} \sum_{d \in \{-1, +1\}} \int_{-\infty}^{\infty} p_e(\xi|d) \log_2 \left[ \frac{2p_e(\xi|d)}{p_e(\xi|-1) + p_e(\xi|+1)} \right] d\xi$$

- $p_e(.)$ is estimated by Monte Carlo simulations

- Note that $I_a = I(d; L_a)$ and $I_e = I(d; L_e)$ have no close forms and are evaluated numerically

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Steps to Generate an EXIT Chart

- 1. Generate independent random variables $L_a$ according to

$$p_a(\xi|d) = \frac{1}{\sqrt{2\pi\sigma_a^2}} \exp \left[ -\frac{1}{2} \left( \frac{\xi - \sigma_a^2 d}{\sigma_a} \right)^2 \right]$$

- 2. Apply them as a priori input to the SISO decoder and obtain the distribution of the output extrinsic LLR values
Steps to Generate an EXIT Chart

3. Compute the mutual information $I_a$ at the input of the decoder and the mutual information $I_e$ at the output of the decoder.

4. Apply the same method (Steps 1 to 3) to the two decoders (inner and outer).

5. Plot the EXIT characteristics $I_e=(I_a, E_b/N_0)$ of the two decoders in a single diagram.
   - The axes of the transfer characteristics for the outer decoder are swapped.

EXIT Chart Example

- During the iterative decoding procedure, the extrinsic output values of one decoder become the a-priori values of the other decoder.
  - Indicated by the line between the transfer functions.
  - Each line indicates a single decoding step of the iterative decoding procedure.
  - Successful decoding when $I_a=I_e=1$.

Superscripts $i$ and $o$ denote the inner decoder and outer decoder, respectively.
EXIT Chart

- Provide a visualisation of the exchange of extrinsic information between the two component decoders.
- For very large interleaver sizes (in Turbo code), EXIT charts make it possible to predict the convergence of the iterative decoding procedure.
  - Convergence is only possible if the transfer characteristics do not intersect.
  - In the case of convergence, the average number of required decoding steps can be estimated.

Pinch-off region: the region of low signal-to-noise ratios where the BER hardly improves with iterative decoding.
**Bottleneck region:** the transfer characteristics leave a narrow tunnel. During the iterative decoding the convergence towards low BERs is slow, but possible.

**Wide-open region:** region of fast convergence
Receiver Design for a Partial response (PR) Channel

- Intersymbol interference (ISI) exists
- The decoder structure is formed by one inner SISO detector and one outer SISO decoder
  - the inner SISO detector for the PR channel and the outer SISO decoder for the outer code
- In the concatenated scheme, the detector and the decoder exchange extrinsic LLR messages iteratively so as to increase the accuracy of the decoded messages

Receiver Design for a PR Channel

- E.g., the inner detector and the outer decoder are, respectively, implemented with the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm and the belief propagation (BP) algorithm
Conditional PDFs of the output LLR values $z_i$ of the BCJR detector when the input coded bit $v_i = 0$ and $v_i = 1$.

EXIT bands of two protograph codes and a regular low-density parity-check (LDPC) code in a dicode channel

Expected EXIT curves are shown by solid lines, upper-bound curves and lower-bound curves are represented by dotted lines.
EXIT bands of two protograph codes and a regular LDPC code in an extended class IV PR channel

Expected EXIT curves are shown by solid lines, upper-bound curves and lower-bound curves are represented by dotted lines.

What you have learnt?
- Assumptions used in the EXIT Chart analysis
- Method to construct the EXIT Chart
- Interpretation of the EXIT Chart
Reading