Performance of Frequency-Modulated Differential-Chaos-Shift-Keying Communication System Over Multipath Fading Channels with Delay Spread

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Multipath performance is an important consideration for chaos-based communication systems. In this letter, the performance of the FM-DCSK communication system over multipath fading channels is evaluated by computer simulations. Both Rayleigh fading and Ricean fading are considered, and the low-pass equivalent model of the FM-DCSK system is used in the simulation. Based on this model, we analyze the bit error performance of the system and the effects of system parameters on the bit-error performance.

Keywords: Chaos communication; frequency-modulated differential-chaos-shift-keying system; multipath channel; Rayleigh fading channel; Ricean fading channel.

1. Introduction

The frequency-modulated differential-chaos-shift-keying (FM-DCSK) communication system is a simple and practical noncoherent system utilizing chaotic signals as the information carrier for spread-spectrum (SS) communication applications [Kolumbán et al., 1998]. Such a system under an additive white Gaussian noise (AWGN) environment has been thoroughly studied (see [Kolumbán et al., 2002] for a survey). In wireless communications, however, the transmission environment is much more complex than what is covered by the simple AWGN model [Rappaport, 1996]. The objects and scatterers in a wireless channel reflect a portion of the radio wave, leading to multiple versions of the transmitted signal arriving at the receiver with different amplitudes, phases and time delays. These multipath waves combine at the receiver, causing the received signal to vary greatly in amplitude and phase. Such multipath fading therefore limits the system performance in wireless applications. Due to its wideband property, a spread-spectrum system performs significantly better than a narrowband one in a multipath environment. Since chaos-based systems are spread-spectrum systems, their performance in multipath environments should be taken into important practical consideration.

A multipath channel typically consists of more than one parallel propagation paths, each characterized by its gain and time delay. The shortest time delay is chosen as a reference and the difference in paths is characterized by an excess delay, $\Delta \tau$, which is the difference between the actual delay of the path
in question and the reference delay. In practice, the excess delays are not constants but vary. This phenomenon is known as delay spread.

The multipath performance analysis and data for the chaos-based communication system are generally unavailable. The earliest study of multipath performance was taken by Kennedy et al. [2000] for the FM-DCSK system. Their study was simulation-based and each path in the two-ray channel model was assumed to have a constant gain. In practice, however, each path suffers from random attenuation, which should be duly incorporated in the channel model [Rappaport, 1996]. Recently, Mandal and Banerjee [2003] analyzed the performance of the DCSK system over a channel with Rayleigh fading and Ricean fading. However, the multipath time delay was not considered. In a spread spectrum communication system such as DCSK and FM-DCSK, it is necessary to model the effects of multipath delay spread as well as fading. Furthermore, Xia et al. [2004, 2005] studied the multipath performance of coherent CSK and noncoherent DCSK systems with a two-ray independent Rayleigh fading channel model, including the effects of channel fading and multipath delay spread. For the FM-DCSK system, however, similar data are still unavailable.

In this letter, we study the performance of the FM-DCSK system over multipath fading channels, taking into account the effects of both channel fading and multipath delay spread. In Sec. 2 we describe the basic system operation and the channel models used in the study. In Sec. 3 we report our main results, and in Sec. 4 we give our conclusions.

2. System Model

2.1. FM-DCSK system

To our knowledge, there is no analytical expression available for the performance of the FM-DCSK system over a multipath channel. A computer simulation method has to be used to obtain the bit error rate (BER). The FM-DCSK system is a radio-frequency (RF) band-pass system. Figure 1 shows the RF model of the system. The multipath channel model used in this study is a two-ray fading channel as shown in Fig. 2, where \( \alpha_1 \) and \( \alpha_2 \) denote the gain of the first path and that of the second path, respectively. To simulate such a system directly in the RF domain, we need a high sampling rate and hence a rather long simulation time. For realistic and fast simulations, we consider the low-pass equivalent model of the FM-DCSK system as shown in Fig. 3. In the low-pass equivalent model [Kolumbán, 1998], each RF band-pass signal \( s(t) \) is modeled by its low-pass complex envelope, i.e. \( \tilde{s}(t) = s_I(t) + js_Q(t) \) where \( s_I(t) \) and \( s_Q(t) \) are referred to as the in-phase and quadrature components, respectively.

In the baseband model, the baseband chaotic signal is generated by the chaotic generator with the chip duration \( T_c \), and is fed into the FM modulator. The output of the FM modulator is a band-pass RF signal, consequently, and is characterized by its in-phase and quadrature components:

\[
\begin{align*}
y_I(t) &= \cos \left( 2\pi k_f \int_{0}^{t} x(t) dt \right) \\
y_Q(t) &= \sin \left( 2\pi k_f \int_{0}^{t} x(t) dt \right)
\end{align*}
\]  

where \( k_f \) is the frequency sensitivity of the modulator. These signals are fed into the DCSK receiver.
modulator. The binary information bit, \( b \), modulates the FM chaotic signal in this modulator, as shown in Fig. 3. Thus, every transmitted bit is represented by two FM chaotic signal segments. The first one serves as the reference whereas the second one carries the information. If “+1” is to be transmitted, the information-bearing segment will be identical to the reference segment, while if “−1” is to be transmitted, the information segment will be the inverted version of the reference, i.e.

\[
s(t) = \begin{cases} 
  y(t), & (l - 1)T \leq t < (l - 1)T + \frac{T}{2} \\
  b(t - \frac{T}{2}), & (l - 1)T + \frac{T}{2} \leq t < lT 
\end{cases}
\]

(4)

where \( T \) denotes the bit duration.

The transmitted signal \( s(t) \) passes through the channel and is distorted, as a result of the fading and multipath delay spread. In addition, the received signal is corrupted by AWGN, \( \xi(t) \).

At the receiver, the received signal \( r(t) \) is demodulated by a differentially coherent demodulator, after being filtered by the low-pass filter. The decision variable is obtained by

\[
r_l = \int_{T/2}^{T} r(t)e\left(t - \frac{T}{2}\right)dt.
\]

(5)

Finally the decoded information bit is estimated according to the following decision rule:

\[
\hat{b}_l = \begin{cases} 
  +1, & \text{if } n_l \geq 0 \\
  -1, & \text{if } n_l < 0.
\end{cases}
\]

(6)

### 2.2. Multipath fading channel model

In the analysis of chaos-based communication systems, an AWGN channel model is often assumed. This assumption is valid for some practical communication systems and makes the analysis computationally tractable. However, in mobile or indoor communication systems, the channel is more complex and the transmitted signal will suffer from fading and multipath delay spread in addition to the effect of noise. If delay spread is to be taken into account, the channel model should have more than one path. Moreover, when fading is considered, the attenuation of each path should be a random variable.

The two most commonly used channel models that take into account the effects of delay spread and fading are the multipath Ricean fading channel model and the multipath Rayleigh fading channel model [Rappaport, 1996]. In the Rayleigh fading channel model, the attenuations of all paths are assumed to be Rayleigh distributed. In the Ricean fading channel model, however, there is a dominant stationary signal component, such as a line-of-sight propagation link. Thus, the attenuation of the dominant path is Ricean distributed, and the attenuation distributions of all other paths are Rayleigh distributed [Jeruchim et al., 2000]. In this letter the RF channel model shown in Fig. 2 contains two paths. The output of the multipath channel is given by

\[
s_r(t) = \alpha_1s(t) + \alpha_2s(t - \Delta\tau) + \xi(t)
\]

(7)

where gains \( \alpha_1 \) and \( \alpha_2 \) are independent random variables, and \( \Delta\tau > 0 \) is the excess time delay between the first and the second path.
the two paths. Gain $\alpha_1$ is Rayleigh distributed in Rayleigh fading channel model or Ricean distributed in Ricean fading channel model. Gain $\alpha_2$ is always Rayleigh distributed. Sample function $\xi(t)$ represents the AWGN with mean equal to zero and power spectral density $N_0/2$.

2.3. Multipath nullings

In a narrowband communication system, if the signals from two paths are out of phase, they weaken each other, resulting in a large attenuation. Moreover, they may reinforce each other if the signals are in phase. This effect is called multipath-related nullings and reinforcements [Kolumban et al., 2002]. In the FM-DCSK system, however, one information bit is divided into many chips and the frequency of the radiated signal is constant only for the chip duration. Also, the frequencies of different chips are different as a result of applying FM. Suppose the variance of the carrier frequency is $\sigma_f^2$. It is readily shown that if $\Delta \tau \ll T_c$ and $\sigma_f \Delta \tau \ll 1$, then “approximate” nullings and reinforcements occur when all phase shifts due to the excess delay in the second path are close to $\pi$, $3\pi$, $5\pi$, ..., and $0$, $2\pi$, $4\pi$, ..., respectively. When these conditions are not met (which is the case for most practical situations), no multipath-related nullings or reinforcements would occur. Thus, we can clearly see that the chaotic spread of the carrier frequency among the chips in a bit duration makes the nullings or reinforcements far more unlikely to occur than in the narrowband case. This is one of the reasons why the FM-DCSK system performs better in a multipath environment than narrowband systems do.

3. Simulation Results

3.1. Discrete-time equivalent model

The system described above is a continuous-time system. In order to expedite computer simulations, we employ a simple equivalent model which is a discrete-time version of the original continuous-time system sampled at a rate of 40 MHz. The discrete-time chaotic signal is generated by the logistic map at a clock rate of 20 MHz. Thus, there are two samples per chip.

3.2. BER performance over the multipath Rayleigh fading channel

In a multipath Rayleigh fading channel model, the gain $\alpha$ of each path is Rayleigh distributed, as given by the following probability density function:

$$p_{\text{Rayleigh}}(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right), & 0 \leq \alpha < \infty \\ 0, & \alpha < 0 \end{cases}$$

where $\sigma^2$ is the average power of the signal.

The performance of the FM-DCSK system over a multipath Rayleigh fading channel is shown in Fig. 4, where we set $\Delta \tau = 100$ ns and $T = 2 \mu$s. In the figure two cases with different path gain ratios are considered, and the single-path case (i.e. AWGN channel) is also shown for comparison. To maintain consistency in our comparisons, we assume in the following that the average received energy in the multipath channels is equal to that received in the AWGN channels.

Case I. The two paths have identical average power gain. In this case the average power gain in each path is 0.5, i.e.

$$E[\alpha_1^2] = E[\alpha_2^2] = \frac{1}{2}.$$

Case II. The average power gain of the second path is 10 dB below that of the first path. In
this case, the average power gains of the two paths are
\[ E[\alpha_1^2] = \frac{10}{T} \quad \text{and} \quad E[\alpha_2^2] = \frac{1}{T}. \]  
\[(11)\]

Clearly, Fig. 4 shows that the performance of the FM-DCSK system over a multipath Rayleigh fading channel is much worse than that of the FM-DCSK system over a single-path AWGN channel. When BER = 10⁻³, the performance degradation is about 10 dB in Case I and a bit more in Case II.

3.3. BER performance over the multipath Ricean fading channel

In the multipath Ricean fading channel model, the gain of the first path is Ricean distributed, while that of the second path is Rayleigh distributed. The probability density function of a Ricean channel is given by
\[ P_{\text{Ricean}}(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2} \exp\left( -\frac{\alpha^2 + A^2}{2\sigma^2} \right) I_0\left( \frac{A\alpha}{\sigma^2} \right), & \text{for } A \geq 0, \alpha \geq 0 \\ 0, & \text{otherwise} \end{cases} \]  
\[(12)\]

where \( A \) denotes the amplitude of the dominant stationary signal, \( \sigma^2 \) is the average power of the scattered signals (i.e., signals other than the dominant stationary signal), and \( I_0(\cdot) \) is the modified zero-order Bessel function of the first kind. The Ricean distribution is often described by the Ricean factor \( K \), which is defined as
\[ K = \frac{A^2}{2\sigma^2}. \]  
\[(13)\]

When \( K = 0 \), the dominant path vanishes and the Ricean distribution degenerates to the Rayleigh distribution.

The performance of the FM-DCSK system over a multipath Ricean fading channel is shown in Fig. 5. Here we set \( \Delta \tau = 100 \text{ ns} \) and \( T = 2 \mu s \), and assume that the two paths have an identical average power gain. As expected, the BER curve of \( K = 0 \) is almost identical to that over a two-ray Rayleigh fading channel (see the solid line in Fig. 4). The figure also shows that the bigger the value of \( K \), the better the performance of the system. This is because when the value of \( K \) increases, the stationary signal becomes more dominant, and the overall effect of fading becomes less significant.

3.4. Effect of parameters

In FM-DCSK systems, the selection of system parameters has a strong influence on the system performance. Here we consider two parameters, namely, the bit duration \( T \) and the excess time delay \( \Delta \tau \).

First, we show the effect of bit duration on the system performance in Fig. 6. We consider the multipath Rayleigh fading channel model and assume that the two paths have identical average power gain. The time delay \( \Delta \tau \) is fixed at 100 ns. Figure 6(a) plots the curves of BER versus \( E_b/N_0 \) with \( T = 1, 2, 4 \mu s \). In Fig. 6(b), the effect of \( T \) on the BER performance is shown with \( E_b/N_0 \) fixed to 25 dB. By reducing the bit duration, the BER performance can be improved. Similar results have been obtained for the simple AWGN (single-path) environment [Kolumbán, 2000]. Another benefit of reducing the bit duration is that of the increased data rate. However, when the bit duration decreases, the system will be more sensitive to timing recovery errors. Thus, in practice, a tradeoff of these two factors should be considered.

Another parameter which can affect the system performance is \( \Delta \tau \). It should be noted that in a specific application environment, \( \Delta \tau \) has a typical value. For example, for large warehouses the typical value of \( \Delta \tau \) is 91 ns, and for office buildings the typical value is 75 ns [Kennedy et al., 2000]. Here we do not specify the application. By changing the value of \( \Delta \tau \), we can observe
the general effect of this parameter on the system performance. The results are shown in Fig. 7, where the multipath Rayleigh fading channel model is considered and the two paths have identical average power gain. The bit duration $T$ is fixed to 2 $\mu$s. Figure 7(a) shows the BERs versus $E_b/N_0$, for $\Delta \tau = 50, 100, 200$ ns. In Fig. 7(b) the effect of $\Delta \tau$ on the BER performance is shown explicitly with $E_b/N_0$ fixed to 25 dB. From these two figures we can see that when $\Delta \tau$ increases, the system performance degrades. This result is consistent with the fact that the inter-symbol interference (ISI) increases with $\Delta \tau$. Note that the multipath-related nulls do not occur here, as explained previously in Sec. 2.3.

4. Conclusions

The FM-DCSK system has been considered as a practical candidate for the realization of chaos-based communication systems. In order to be used in a practical wireless communication environment,
the multipath performance of such a system should be thoroughly studied. In this letter, the performance of the FM-DCSK system over a two-ray multipath fading channel is evaluated by computer simulations. Both Rayleigh and Ricean fadings are considered. The degradations of the system performance due to the channel fading and multipath delay spread have been evaluated. This letter also reveals the effects of system parameters on the bit-error performance.

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References


