ADAPTIVE APPROACH FOR DETECTION IN CHAOS-BASED DIGITAL COMMUNICATION SYSTEMS WITH TRANSMITTED REFERENCE

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Abstract. In this paper, a transmission scheme is proposed for chaos-based communication systems, in which chaotic reference signals and information-bearing signals are transmitted in succession. Chaotic reference signals modulated by a binary training sequence are sent periodically. The same chaotic signals are then modulated by the binary data and transmitted. Two types of receivers are proposed and studied. The two receivers attempt to estimate the chaotic reference signal by using an adaptive filter and a simple inverse-and-average method, respectively. The estimated chaotic signal is then used to correlate with the information-bearing signals for determining the received symbols. The performance bounds of these two schemes are also derived. Finally, the bit error rates of the proposed system are simulated and compared with the widely-studied differential chaos-shift-keying system.

Keywords. Adaptive filter, bit error rates, chaos communications, inverse-and-average method, non-coherent detection.

1 Introduction

Digital communications based on chaotic circuits were first proposed more than a decade ago [1,8]. Since then, various modulation and demodulation schemes have been suggested and studied [2,5,7,10,11,12]. Compared with conventional communication schemes, chaos-based communication systems offer potential advantages such as ease of generation of broadband signal and high security. In coherent chaos-based communication systems, the chaotic carriers need to be reproduced at the receivers in order to perform the demodulation [1,5,8,12]. Unfortunately, at the time of writing, robust chaos synchronization techniques for the practical noise levels concerned are still lacking, the study of coherent detection schemes remains only of academic interest and the performance of coherent systems is used mainly as benchmark indicators. Non-coherent systems, which do not require the reproduction of the chaotic carriers at the receiving end, are more practical and improvements are still being made continually [2,6,10,11]. Among the non-coherent systems, the differential chaos-shift-keying (DCSK) system has been most widely studied [6]. In the DCSK system, each bit duration is divided into
two equal slots. In the first slot, a reference chaotic signal is sent. Dependent upon the symbol being sent, the reference signal is either repeated or multiplied by the factor “−1” and transmitted in the second slot. Note that the DCSK system is employing the transmitted-reference approach because both the reference signal and the information signal are sent [10].

In this paper, we propose another non-coherent transmission scheme for chaos-based digital communication systems utilizing transmitted reference. In the proposed scheme, the user transmits reference chaotic signals, modulated by a training sequence, to the receiver periodically. Afterwards, the same reference signals are modulated by the data sequence to form the information-bearing signals. Two detection methods are studied. In the first method, an adaptive filter is employed. Based on the incoming training signals and the user’s pre-assigned training sequence, the parameters of the adaptive filter are adjusted with an aim to minimizing the error between the estimated symbol and the training symbol. After the training process has been completed, the adaptive filter is used to demodulate the information-bearing signals. For the second detection method, it attempts to recover the reference chaotic signal. The incoming training signals in the time slots are first multiplied by the user’s pre-assigned training sequence. An averaging process is then performed to estimate the reference signal, which is used to correlate with the information-bearing signals for determining the received symbols. The performance of the proposed system, with the use of the two different receivers, is simulated and compared with that of the DCSK system.
2 Description of the Proposed Non-coherent Chaos-Based Communication System

In this section, we make use of the discrete-time baseband equivalent models of the transmitter and receiver to describe the proposed communication system [2,7,9].

2.1 Transmitter Structure

Figure 1 shows the transmitter structure of the proposed non-coherent chaos-based communication system. The transmitter consists of a chaos generator, a number of delay blocks, a switch and a multiplier. The transmitted signal is organized into frames which are further sub-divided into a number of time slots, as shown in Fig. 2. The first \( K_1 \) slots are used to send the reference chaotic samples for training, while the information-bearing chaotic samples are sent in the remaining \( K_2 \) slots. Without loss of generality, we consider the transmitted signal for a single frame period. At the output of the chaos generator, the chaotic signal is first biased before transmission such that its mean value is zero. This avoids transmission of the dc component which carries no useful information but increases transmission power. Suppose \( \beta \) chaotic samples are sent within one slot. For each frame, the same \( \beta \) chaotic samples will be used in each slot. We denote the chaotic samples within this frame by \( \{ x_k : k = 1, 2, \ldots, \beta \} \). For algebraic brevity, we also define a chaotic sample vector \( \mathbf{x} \) as

\[
\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_\beta \end{bmatrix}^T
\]

where \( T \) represents the matrix transposition.

Moreover, the chaotic sample vector will be modulated by the training bits or the data bits before transmission. For the \( m \)th time slot, the sample vector sent by the user, denoted by \( \mathbf{y}_m \), is given by

\[
\mathbf{y}_m = \begin{bmatrix} y_{(m-1)\beta+1} & y_{(m-1)\beta+2} & \cdots & y_{m\beta} \end{bmatrix}^T = d_m \mathbf{x}
\]

where \( d_m \) equals “+1” or “−1”. When \( 1 \leq m \leq K_1 \), \( d_m \) represents the training bit, and when \( K_1 + 1 \leq m \leq K_1 + K_2 \), \( d_m \) denotes the information bit. In other words, if the training/data bit is “+1”, the transmitted samples are the same as the chaotic samples. If the training/data bit equals “−1”, the sign of the chaotic samples will be inverted and then transmitted. A typical transmitted frame structure is shown in Fig. 2. In the following analysis, we further assume that \( K_1 = K_2 = K \) such that on the average, two slots are required to send one data bit. Thus the average number of chaotic samples transmitted per bit (spreading factor) equals \( 2\beta \).
2.2 Receiver Structures

We make the usual assumption that the channel is additive white Gaussian. Thus, during the \( m \)th time slot, the received sample vector, \( \mathbf{r}_m \), is

\[
\mathbf{r}_m = \begin{bmatrix} r_{(m-1)\beta+1} & r_{(m-1)\beta+2} & \cdots & r_{m\beta} \end{bmatrix}^T
\]

\[= \mathbf{y}_m + \Phi_m \tag{3}\]

where

\[
\Phi_m = \begin{bmatrix} \xi_{(m-1)\beta+1} & \xi_{(m-1)\beta+2} & \cdots & \xi_{m\beta} \end{bmatrix}^T \tag{4}\]

and \( \xi_k \) represents the \( k \)th noise sample, the mean and variance (power spectral density) of which are zero and \( N_0/2 \), respectively. The first \( K \) received sample vectors, i.e., \( \{\mathbf{r}_m : m = 1, 2, \ldots, K\} \), are the training signals. Based on these training signals, the receiver needs to update its internal parameters before decoding the information-bearing vectors that follow. Two receiver structures will be investigated. The first one is based on an adaptive transversal filter while the second one estimates the chaotic sample vector from the training signals.

2.2.1 Adaptive Transversal Filter

Figure 3 shows the structure of an adaptive transversal filter, which makes use of the first \( K \) received sample vectors for updating the tap weights. The tap weights are set to zero at the beginning of each training process. During the training period, the estimated training bit corresponding to the \( m \)th slot, denoted by \( \hat{d}_m \), is first computed using

\[
\hat{d}_m = \mathbf{r}_m^T \mathbf{w}_{m-1} \tag{5}\]

where \( \mathbf{w}_m = [w_{m,1}, w_{m,2}, \cdots, w_{m,\beta}]^T \) is a vector containing the tap weights of the adaptive filter after the \( m \)th \((m = 1, 2, \ldots, K)\) iteration (time slot). Then the estimated training bit is compared with the desired symbol.
Based on the error between the desired symbol and the estimated symbol, i.e., $e_m = d_m - \hat{d}_m$, the tap weights in the receiver are updated at the end of each time slot using the normalized least-mean-square (LMS) algorithm. The whole iterative process is summarized in the following steps [3]:

$$w_0 = 0 \quad (6)$$
$$e_m = d_m - \hat{d}_m = d_m - r_m^T w_{m-1} \quad (7)$$
$$w_m = w_{m-1} + \mu \frac{e_m}{\|r_m\|^2} r_m, \quad 0 < \mu < 2, \quad a \geq 0 \quad (8)$$

where $\|r_m\|$ is the Euclidean norm of the input vector $r_m$ defined as $\|r_m\| = \sqrt{\sum_{i=1}^{T} [r_{m-1,\beta+i}]^2}$. At the end of the training period, i.e., after $K$ iterations, we will obtain the tap-weight vector $w_K$ which can then be used to estimate the data symbols embedded in the remaining time slots of the frame. The decoded data symbol, i.e., $\hat{d}_m, m = (K+1), (K+2), \ldots, 2K$, is then determined according to the following rule:

$$\hat{d}_m = \begin{cases} +1 & \text{if } \hat{d}_m = r_m^T w_K > 0, \quad m = (K+1), \ldots, 2K \\ -1 & \text{if } \hat{d}_m = r_m^T w_K \leq 0, \quad m = (K+1), \ldots, 2K. \end{cases} \quad (9)$$

In other words, for the remaining $K$ time slots of each frame, if the estimated data symbol $\hat{d}_m$ is larger than zero, then “+1” is detected. Otherwise, “−1”
During the training period, if the training period is long enough, the error between the desired data and the estimated data will approach zero. To estimate the performance bound, we assume that the error $e_m$ equals zero for all $m > M$, where $M$ is a sufficiently large integer. In other words, $e_m = 0$. Based on (8), it can be further deduced that for all $m > M$, $w_m = w_M$.

When the transversal filter is used to decode the information-bearing chaotic signal, which is now assumed to be corrupted by noise, the system behaves like a coherent antipodal chaos-shift-keying (CSK) system because the reference chaotic signal has been recovered by the adaptive receiver. The bit error rate (BER) of the aforementioned coherent CSK system is bounded by that of the conventional coherent binary phase-shift-keying (BPSK) communication scheme [4] which is given by

$$\text{BER} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right)$$

where $E_b$ denotes the energy of each bit in the demodulation process and $\text{erfc}(.)$ is the complementary error function [9]. In the proposed non-coherent system under study, we assume that the number of training time slots and the number of information-bearing time slots are the same within each frame. As a consequence, if we denote the average bit energy of the user by $E_b$, which equals

$$E_b = 2\beta P_s$$

where $P_s = \mathbb{E}[x_k^2]$ denotes the average transmission power of the user and $\mathbb{E}[.]$ represents the expectation operator, then $E_b$ will be twice as the bit energy spent in the demodulation process, i.e., $E_b = 2\bar{E}_b$. Substituting it into (10), the performance bound of the proposed receiver equals

$$\text{BER} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{2N_0}} \right).$$

In practice, the BER performance is expected to be worse than that given in (12) because the tap-weight vector cannot be estimated perfectly under the influence of noise and with a finite length of the training sequence.

### 2.2.2 Inverse-and-Average Receiver

The structure of the second receiver, the inverse-and-average receiver, is shown in Fig. 4. The roles of the receiver are to estimate the chaotic sample vector during the training period, and to decode the information-bearing chaotic signal by correlating it with the estimated sample vector. To estimate the chaotic sample vector, the modulation due to the training bit at the transmitter is first removed at the receiving end by multiplying the received
signal in each time slot by the corresponding training bit again. Since the training bit equals ±1, multiplying the chaotic sample vector $\mathbf{x}$ twice (once at the transmitter and another time at the receiver) by the same training bit produces no effect on the vector. Afterwards, the resultant vectors in all the training time slots are averaged to produce an estimation of the chaotic sample vector which is represented by $\tilde{\mathbf{x}} = \left[ \tilde{x}_1 \ \tilde{x}_2 \ \cdots \ \tilde{x}_\beta \right]^T$. When the training process is completed, the chaotic signals in the information-bearing time slots will correlate with the estimated chaotic sample vector. When the output of the receiver, denoted by $z_m$, is larger than zero, a “+1” is decoded. Otherwise, a “−1” will be detected.

Using the same notations defined previously, the recovered chaotic sample vector is given by

$$\hat{\mathbf{x}} = \frac{1}{K} \sum_{m=1}^{K} \mathbf{d}_m \mathbf{r}_m = \mathbf{x} + \frac{1}{K} \sum_{m=1}^{K} \mathbf{d}_m \Phi_m.$$  \hspace{1cm} (13)

In (13), the first term is the required chaotic sample vector. The second term, being derived from the noise, cannot be eliminated. The estimated chaotic sample vector $\hat{\mathbf{x}}$ will then correlate with the chaotic signals transmitted in the information-bearing time slots. For the $m$th time slot, denote the input to the detector by $z_m$. Then, $z_m = \hat{\mathbf{x}}^T \mathbf{r}_m$. The received symbol, denoted by
\( \hat{d}_m \), is decoded according to the following rule:

\[
\hat{d}_m = \begin{cases} 
  +1 & \text{if } z_m > 0, \quad m = (K + 1), \ldots, 2K \\
  -1 & \text{if } z_m \leq 0, \quad m = (K + 1), \ldots, 2K 
\end{cases}
\]  

(14)

To estimate the performance bound, we further assume that the number of reference time slots \( K \) is large enough such that the term due to noise can be ignored in (13). Therefore, (13) is simplified to \( \hat{x} \approx x \), implying that a clean chaotic sample vector can be recovered at the receiver. Under such circumstances, during the decoding of the information-bearing time slots, the communication system is equivalent to an antipodal coherent CSK system. Therefore, the overall performance of the inverse-and-average receiver is also bounded by (12).

Comparing the complexity of the two proposed receivers in this section, it can be seen that the receiver based on an adaptive transversal filter has a slightly more complicated structure (mainly because of the LMS algorithm) compared with the inverse-and-average receiver.
3 Results and Discussions

In our simulations, the chaotic samples are generated by the map $x_{k+1} = 4x_k^3 - 3x_k$. With this map, the average transmission power $P_s$ is readily shown equal to 0.5 [7]. It can be substituted into (11) in computing the performance bound equation of the adaptive-transversal-filter (ATF) receiver and the inverse-and-average (IA) receiver, as given in (12). Also, for the ATF receivers, the parameters $\mu$ and $a$ in the LMS algorithm are set to 0.5 and $10^{-20}$, respectively.

Figure 5 plots the simulation results of the proposed system. For the case with $K = 1$, the structure of the transmitted signal is the same as that of a DCSK system [5], the BER performance of which is also shown in the figure for comparison. Also, the structure of the IA receiver is the same as the non-coherent DCSK receiver when $K = 1$. From the results, it can be observed that for $K = 1$, the ATF receiver achieves similar BERs as the DCSK scheme (and consequently the IA receiver). For $K = 4, 8$ and $16$, the ATF and the IA receivers have similar performance. Also, when the length of the training sequence $K$ increases, the BERs of both types of receivers have improved because a larger number of training signals can reduce the effect of noise during the training process.

4 Conclusions

In this paper, we have proposed a non-coherent chaos-based communication system utilizing transmitted reference. The transmission scheme is simple and easy to implement. Essentially, a series of training chaotic signals are sent to train the receiver at the receiving side. Two types of receivers have been proposed and studied. The first one is based on an adaptive transversal filter (ATF). By using the normalized least-mean-square algorithm to update the tap weights of the filter, the mean-square-error between the incoming training symbols and the expected symbols is reduced. The second type of receiver, namely the inverse-and-average (IA) receiver, aims at recovering clean reference chaotic signals by multiplying the signals in the reference time slots by the corresponding training bits, followed by averaging. Simulation results show that the proposed system has the same performance as the DCSK system when there is only one training slot per frame. When the number of training slots increases, the improvement of the proposed system over the DCSK system becomes more prominent.

In our future work, the proposed scheme will be extended to accommodate multiple users. In the multiple access scheme, the users will be assigned with different reference chaotic signals and training sequences. By choosing the training sequences carefully, it is expected that the interference between users can be minimized. Moreover, it is expected that the ATF receiver will outperform the IA receiver and other time-delay-based multiple-access
systems such as multiple access DCSK system because the ATF receiver has the capability to eliminate the inter-user interference.

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6 References


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