Application: Impedance Matching

A transmission line usually connects a generator circuit at one end to a load at the other end. The load may be an antenna or any circuit with an equivalent input impedance $Z_L$.

- The transmission line is said to be matched to the load when its characteristic impedance $Z_o = Z_L$.
  - **No** reflection at the load.

The primary uses of transmission lines are to transfer power and to transmit coded signal (such as digital data), a matched load ensures that the power delivered to the load is a **maximum**.

![Diagram of a transmission line with matched load](attachment:diagram.png)
The simplest solution to matching a load to a transmission line is to design the load circuit such that its impedance $Z_L = Z_o$.

- Unfortunately, this may not be possible in practice because the load circuit may have to satisfy other requirements.
- An alternative solution is to place an **impedance-matching network** between the load and the transmission line.

$$\frac{V_{A-A'}}{I_{A-A'}} = Z_L \Rightarrow Z_{in} = \frac{V_{A-A'}}{I_{A-A'}} = Z_o$$
Quarter-wave Transformer

For a **resistive** load, a quarter-wave transformer can achieve the matching.

- To achieve impedance matching, we want \( Z_{in} = Z_{o1} \)

- In general, \( Z_{in} = Z_{o2} \frac{Z_L + jZ_{o2} \tan \beta l}{Z_{o2} + jZ_L \tan \beta l} \)

- If \( l = \frac{\lambda}{4} \), \( \tan \beta l = \tan(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}) \to \infty \)

\[
Z_{in} = \frac{Z_{o2}^2}{Z_L} \\
\therefore \ Z_{o2} = \sqrt{Z_{o1}Z_L}
\]
Example 2-9

A 50Ω lossless transmission line is to be matched to a resistive load impedance with $Z_L=100 \, \Omega$ via a quarter-wave transformer. Find the characteristic impedance of the quarter-wave transformer.

$$Z_{o2} = \sqrt{50 \times 100} = 70.7 \, \Omega$$

Suppose the signal frequency is 3GHz and the phase velocity of the signal inside the transmission line is $3 \times 10^8 \, m/s$. Find the length of the quarter-wave transformer.

$$\lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.1$$

$\therefore \lambda / 4 = 0.25 \, m$
Single-stub matching network

To match a load impedance $Z_L = R_L + jX_L$ to a lossless transmission line with characteristic impedance $Z_o$.

- The network has to transform the real part of the load impedance from $R_L$ at the load to $Z_o$ and
- To transform the reactive part from $X_L$ at the load to zero.

- To achieve these two transformations, the matching network has to have at least two degree of freedom.
Single-stub matching network

Aim: \( Y_{in} = Y_d + Y_s = Y_o = 1 / Z_o \)

\[
Y_d = Y_o \frac{Y_L + jY_o \tan \beta d}{Y_o + jY_L \tan \beta d} = Z_o + jB
\]

\[
Y_s = \lim_{Y_L \to \infty} Y_o \frac{Y_L + jY_o \tan \beta l}{Y_o + jY_L \tan \beta l} = -jY_o \cot \beta l
\]

Therefore, we have

\[
Y_o \frac{Y_L + jY_o \tan \beta d}{Y_o + jY_L \tan \beta d} - jY_o \cot \beta l = Y_o
\]

(we have two equations and two unknowns)