Passband Data Transmission

FSK
1. A coherent binary FSK system transmits binary data at the rate of 2.5x10^6 bits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} W/Hz is added to the signal. In the absence of noise, the amplitude of the received sinusoidal wave for digit 1 or 0 is 1mV. Determine the average probability of symbol error.

   Given: \( \text{erfc}(x) \approx \frac{e^{-x^2}}{\sqrt{\pi x}} \) for large \( x \)

QAM
2. Determine the transmission bandwidth reduction of 256-QAM, compared to 64-QAM. (The transmission bandwidth is defined as the null-to-null bandwidth)
3. Two passband data transmission systems are to be compared. One system uses 16-PSK, and the other uses 16-QAM. Both systems are required to produce an average probability of symbol error equal to 10^{-3}. Compare the signal-to-noise ratio requirement of these two systems.

Comparison of Digital Modulation Schemes
4. Binary data are transmitted over a microwave link at the rate of 10^6 b/s, and the power spectral density of the noise at the receiver input is 10^{-10} W/Hz. Find the average carrier power required to maintain an average probability of error \( P_e \leq 10^{-4} \) for (a) coherent binary PSK and (b) DPSK

   Given: The average BER for DPSK is \( \frac{1}{2} e^{-E_b/N_0} \)

5. The values of \( E_b / N_0 \) required to realize an average probability of symbol error \( P_e = 10^{-4} \) using coherent binary PSK is 7.2. Using the approximation

   \( \text{erfc}(u) = \frac{1}{\sqrt{\pi u}} \exp(-u^2) \)

   determine the separation in the values of \( E_b / N_0 \) for \( P_e = 10^{-4} \), using coherent binary PSK and DPSK
Solution:

1. \( T_b = 1/2.5 \times 10^6 = 0.4 \mu s \)

\[ E_b = \frac{1}{2} A^2 T_b = \frac{1}{2} \left(10^{-1}\right)^2 0.4 \times 10^{-6} = 2 \times 10^{-13} J \]

\[ P_e = \frac{1}{2} \text{erfc}\left(\sqrt{E_b / 2 N_o}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{2 \times 10^{-13} / 2 \times 10^{-20}}\right) = ... = 0.85 \times 10^{-3} \]

2. The transmission bandwidth of M-ary QAM signal is

\[ \frac{2R_b}{\log_2 M} \]

Thus the bandwidth reduction is

\[ B_{64} - B_{256} = \frac{2R_b}{\log_2 64} - \frac{2R_b}{\log_2 256} = \frac{R_b}{4} - \frac{R_b}{8} = \frac{R_b}{8} \]

3. The probability of symbol error of M-ary QAM is given by

\[ P_e = 2 \left(1 - \frac{1}{\sqrt{M}}\right) \text{erfc}\left(\sqrt{\frac{3E_{av}}{2(M-1)N_o}}\right) \]

Setting \( P_e = 10^{-3} \) and \( M=16 \), we get \( \frac{E_{av}}{N_o} = 58 = 17.6 \text{dB} \).

The probability of symbol error of M-ary PSK is given by

\[ P_e = \text{erfc}\left(\sqrt{\frac{E}{N_o}} \sin(\pi / M)\right) \]

Setting \( P_e = 10^{-3} \) and \( M=16 \), we get \( \frac{E}{N_o} = 142 = 21.5 \text{dB} \).

Hence, on the average, the 16-PSK demands 21.5 - 17.6 = 3.9 dB more symbol energy than the 16-QAM for \( P_e = 10^{-3} \).
4. For $P_e = 10^{-4}$, the $E_b / N_o$ of the binary PSK system is 7.0. Hence $E_b = 3.5 \times 10^{-10}$. The required average carrier power is 0.35 mW.

For $P_e = 10^{-4}$, the $E_b / N_o$ of the DPSK system is 8.5. Hence $E_b = 4.3 \times 10^{-10}$. The required average carrier power is 0.43 mW.

5. For a coherent PSK system, the average probability of error is

$$P_e = \frac{1}{2} \text{erfc} \left[ \sqrt{(E_b / N_o)_1} \right]$$

For a DPSK system, we have

$$P_e = \frac{1}{2} \exp[-(E_b / N_o)_2]$$

For $P_e = 10^{-4}$, $(E_b / N_o)_1 = 7.2 = 8.6$ dB and $(E_b / N_o)_2 = 8.5 = 9.3$ dB. The separation is 0.7 dB.