Circuit Switching: Traffic Engineering

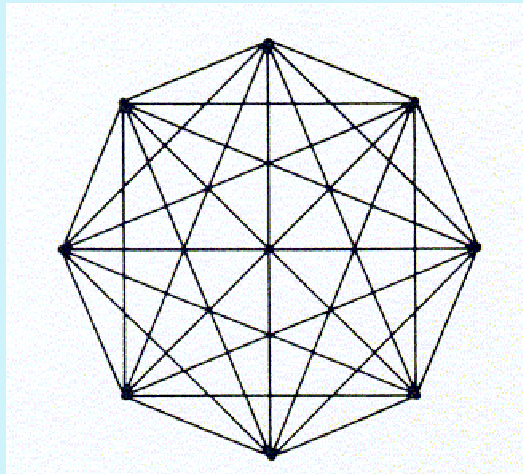
References

- Chapter 1, *Telecommunication System Engineering*, Roger L. Freeman, Wiley.

Introduction: mesh and star

Example:

– mesh connection (full mesh) for an eight-subscriber system



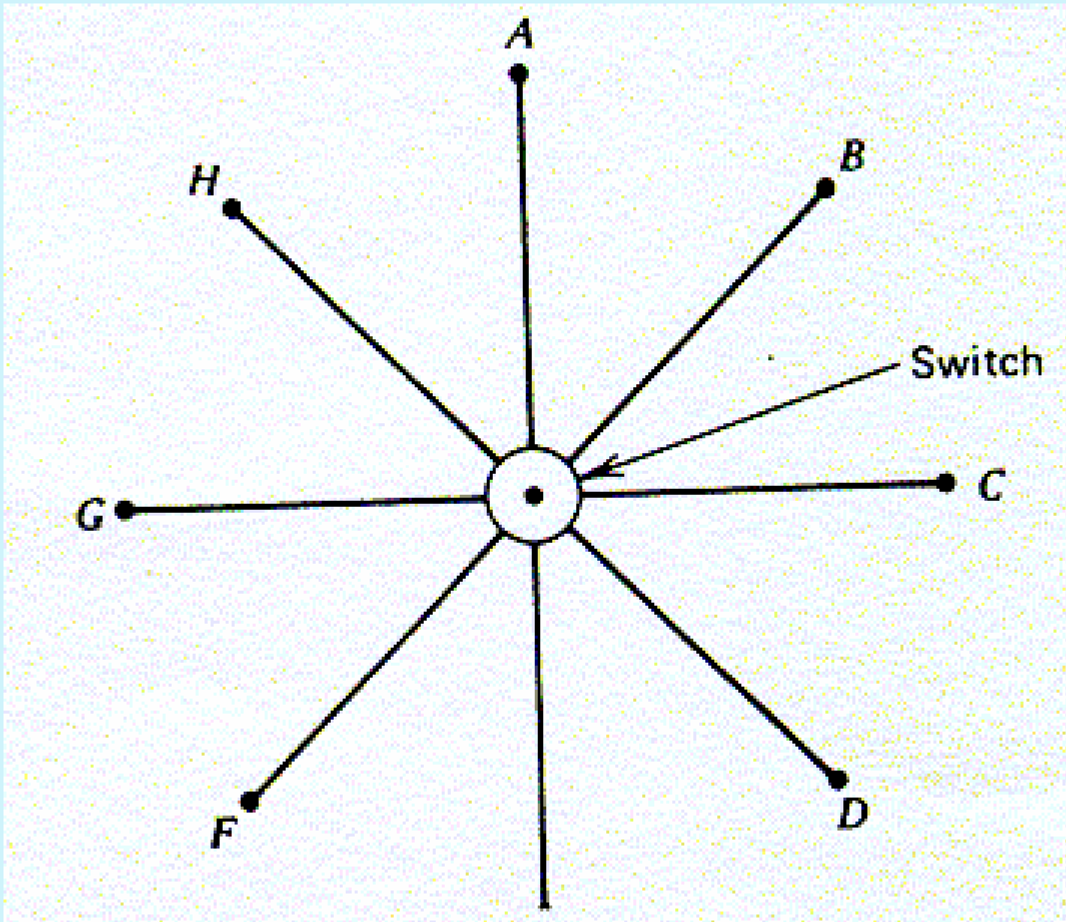
– justify a mesh connection is when each and every subscriber wishes to communicate with <u>every</u> other subscriber in the network for virtually the <u>entire</u> day.

- Most subscriber do not use their telephones on a full-time basis
- the ordinary subscriber will normally talk to only one other subscriber at a time

Introduction: mesh and star

– Star network with a switch at the center
  - switch reduce the number of links between subscribers

4?
3?
2?
1?



Terminology

- Trunk
  - the telephone lines connecting one telephone switch or exchange with another are called trunks.

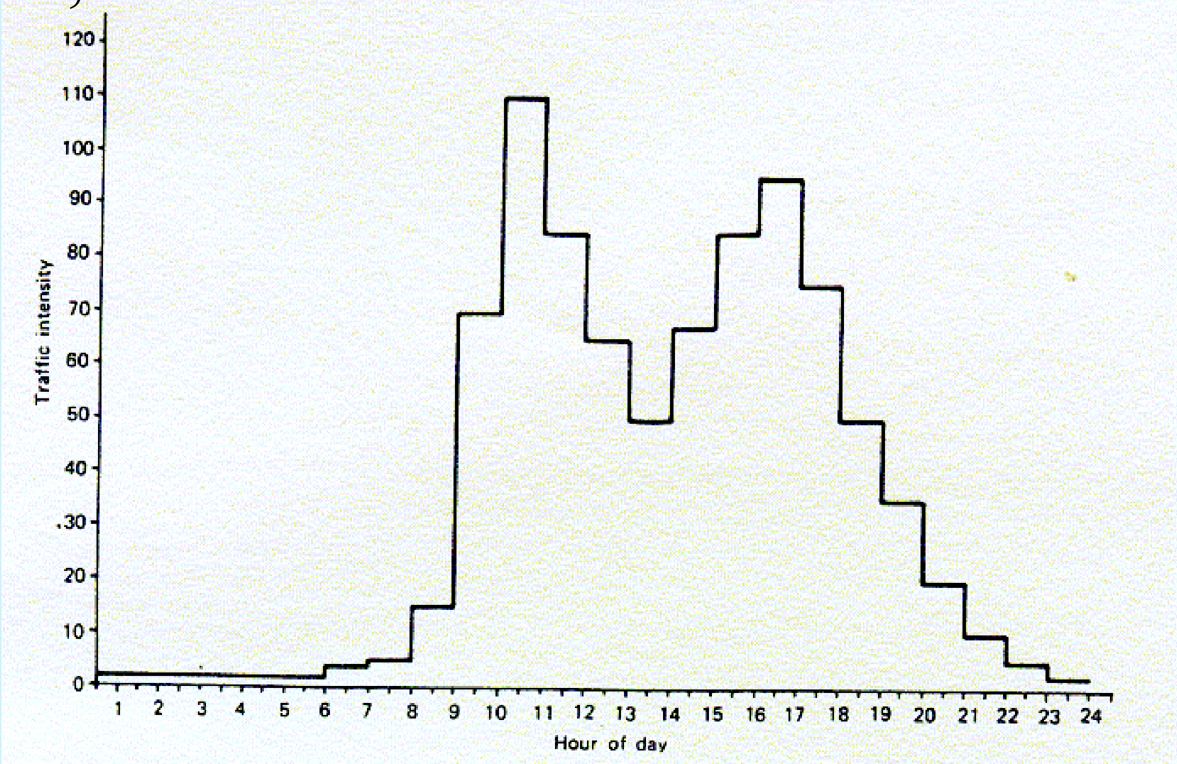
- Calling rate (C)
  - The number of calls which arrive over a time interval
    - Call per unit time

- Holding time (H)
  - The average duration of a call

Busy hour

Telephone traffic may fluctuate throughout the day, and may have a “busy hour” which is the hour that has the most number of calls

- busy hour depends on various factors such as stock market, weather and international events



Measurement of Traffic

– The traffic intensity, more often called the traffic, is defined as the average number of calls in progress.

$$A = Ch / T$$

Unit: Erlang (E)

A: traffic intensity

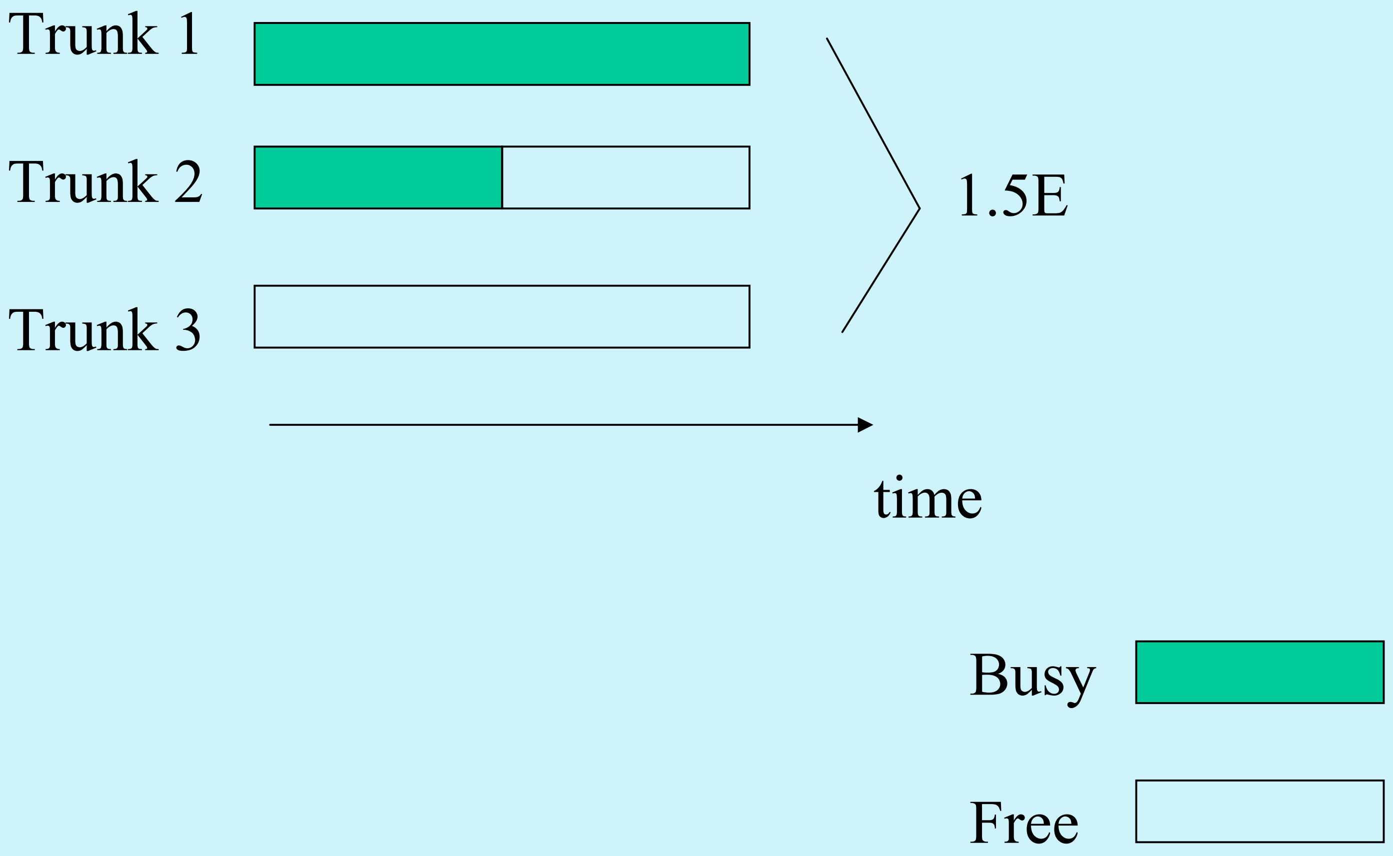
C: number of calls arrivals during time T

h: average holding time

- Example: A <u>single</u> trunk
  - cannot carry more than one call, i.e., $A \leq 1$ for one trunk.
  - The probability of finding the trunk busy is equal to the proportion of time for which the trunk is busy. Thus, this probability equals the occupancy (A) of the trunk.

Measurement of Traffic

Example: 1.5 erlang of traffic carried on three trunks



Measurement of Traffic

Example: On average, during the busy hour, a company makes 120 outgoing calls of average duration 2 minutes. It receives 200 incoming calls of average duration 3 minutes.

Find the outgoing traffic, the incoming traffic and the total traffic.

Solution

$$A = Ch / T$$

where T = 1 hour = 60 minutes

Outgoing traffic = 120 calls x 2 minutes/ 60 minutes = 4 E

Incoming traffic = 200 calls x 3 minutes/ 60 minutes =10 E

Total traffic = 4 E + 10 E = 14 E

Measurement of Traffic

Example:

During the busy hour, on average, a customer with a single telephone line makes three calls and receives three calls. The average call duration is 2 minutes. What is the probability that a caller will find the line engaged?

Total traffic = Occupancy of line = (3+3) x 2 / 60 = 0.1 E

Probability of finding the line engaged = 0.1

Blockage, Lost Calls, and Grade of Service

Lost call or blocked calls

- In a circuit-switched system, all attempts to make calls over a congested group of trunks are unsuccessful. The unsuccessful call is called lost call or blocked call.

Grade of service

- probability of meeting blockage is called the grade of service (*B*)
- Example: On average, one call in 100 will be blocked
  - B=0.01

Blockage, Lost Calls, and Grade of Service

- Grade of service is also the
  - proportion of the time for which congestion exists
  - probability of congestion
  - probability that a call will be lost due to congestion

- If traffic *A* Erlangs is offered by a group of trunks having a grade of service *B*, the traffic lost is *AB* and the traffic carried is *A*(1-*B*) erlangs.

Blockage, Lost Calls, and Grade of Service

Example

During the busy hour, 1200 calls were offered to a group of trunks and six calls were lost. The average call duration was 3 minutes

The traffic offered = $A = C_1 h/T$ = 1200 x 3 /60 = 60 E

The traffic carried = $C_2 h/T$=(1200-6) x 3 / 60 = 59.7 E

The traffic lost = $B = C_3 h/T$ = 6 x 3 / 60 = 0.3 E

Grade of service = B/A = 0.3 / 60 = 0.005

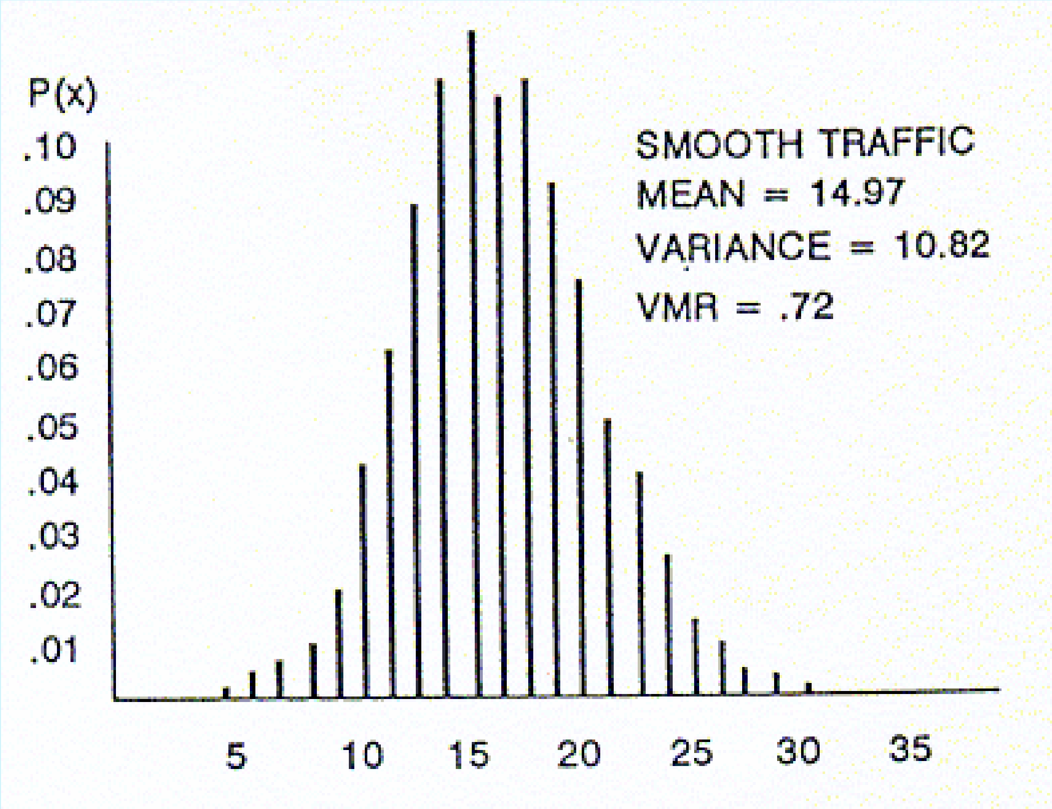
The total duration of the periods of congestion

= B x T = 0.005 x 3600 =18 seconds

Traffic Formulas

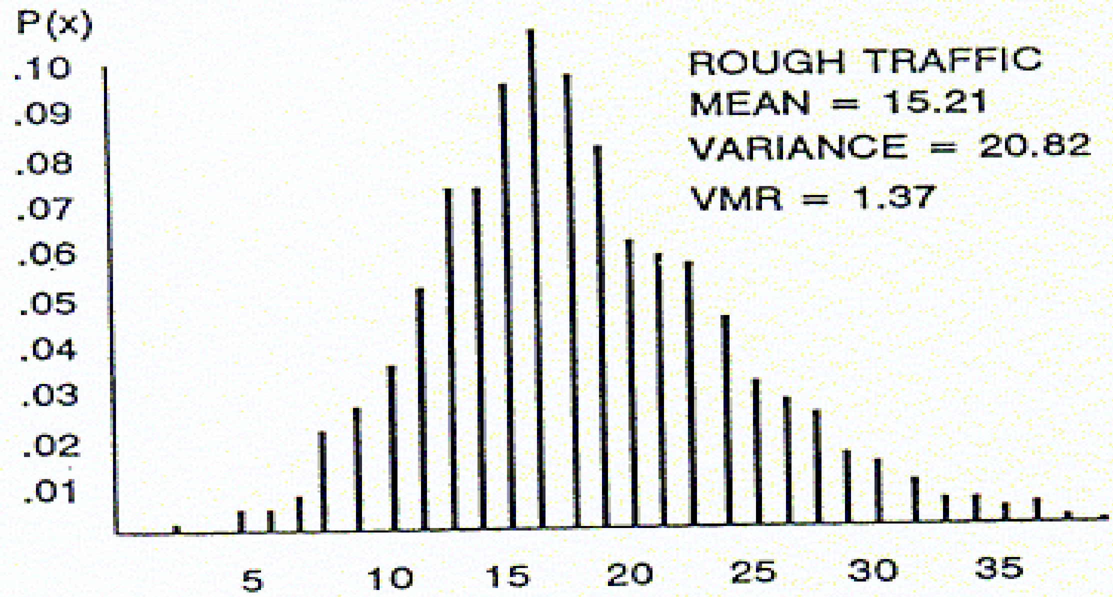
Models of traffic

- Variance-to-mean ratio (VMR) $\alpha = \dfrac{\sigma^2}{\mu}$
  - Smooth: VMR < 1

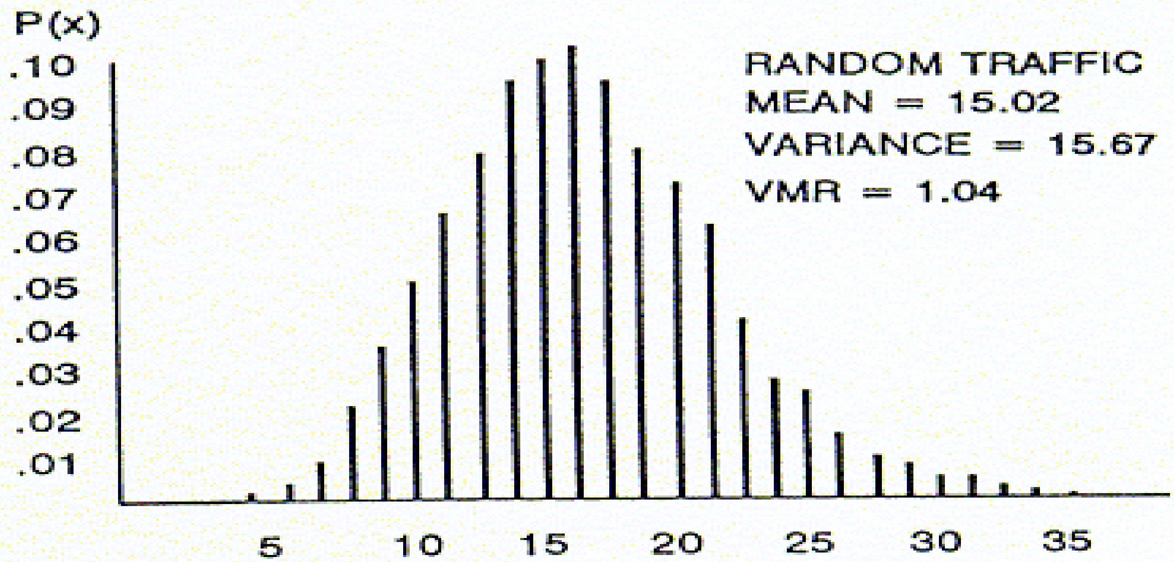


Traffic Formulas

- Rough: VMR > 1



- Random: VMR = 1
- Poisson distribution function is an example of random traffic where VMR = 1



Traffic Formulas

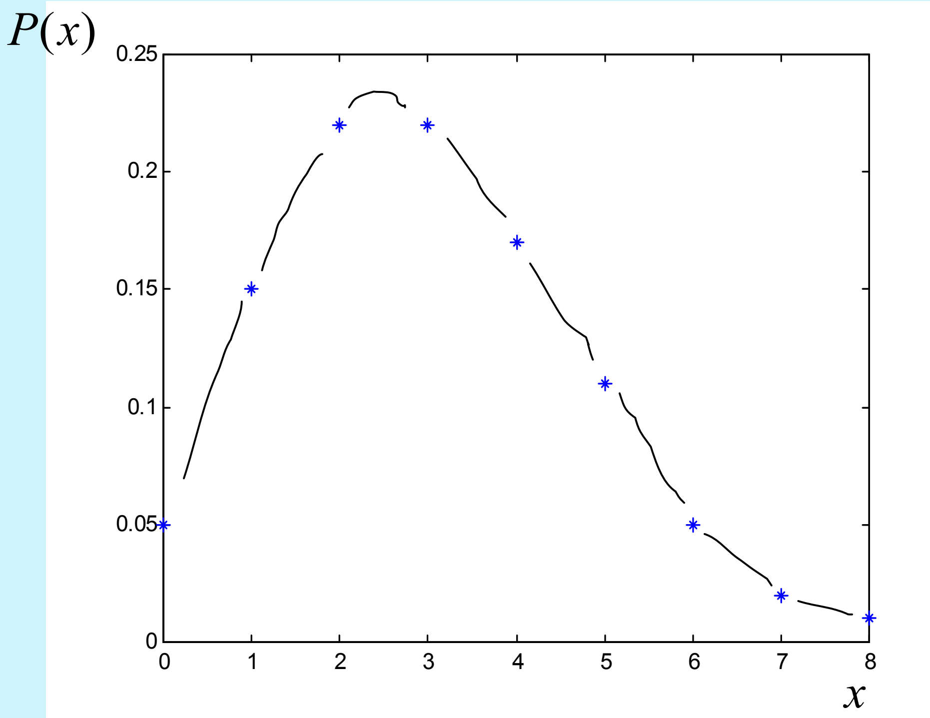
– The number of call arrivals in a given time has a Poisson distribution

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}$$

- x is the number of call arrivals in time T
- μ is the mean number of call arrivals in T

– Example: μ=3

x	$P(x)$
0	0.05
1	0.15
2	0.22
3	0.22
4	0.17
5	0.11
6	0.05
7	0.02
4	0.01



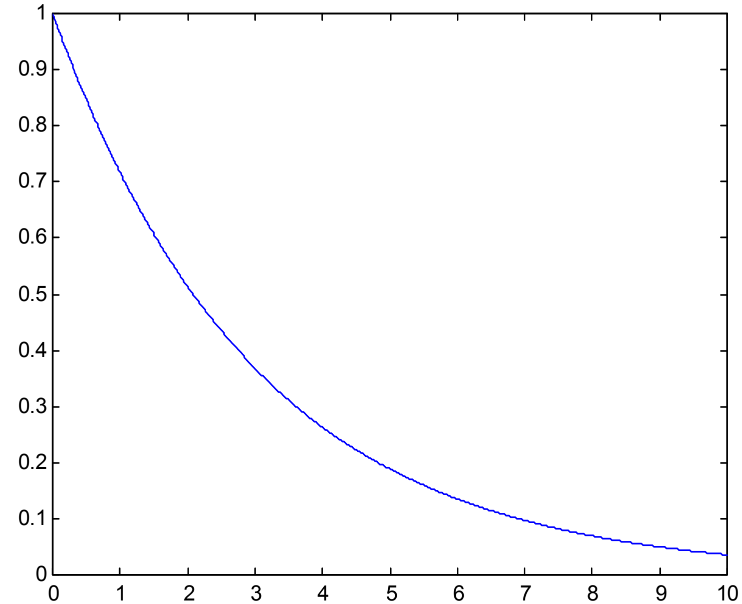
Traffic Formulas

– Call-holding time is modeled by a negative exponential distribution

$$P(T \geq t) = e^{-t/h}$$

- P is the probability of a call lasting longer than t
- h is the mean call duration
- Example: $h = 3$ minutes

$P(T \geq t)$



t

Traffic Formulas

Example: On average one call arrives every 5 seconds. During a period of 10 seconds, what is the probability that

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}$$

a. No call arrives

$$\mu = 2 \text{ calls/10 seconds}$$

$$P(\text{No call arrives}) = P(x = 0) = \frac{2^0}{0!} e^{-2} = 0.135$$

b. One call arrives

$$P(1) = \frac{2^1}{1!} e^{-2} = 0.27$$

c. More than one call arrives

$$P(x > 1) = 1 - P(0) - P(1) = 0.595$$

Traffic Formulas

Example: In a telephone system, the average call duration is 2 minutes. A call has already lasted 4 minutes. What is the probability that

a. the call will last at least another 4 minutes

The probability is independent of the time which as already elapsed.

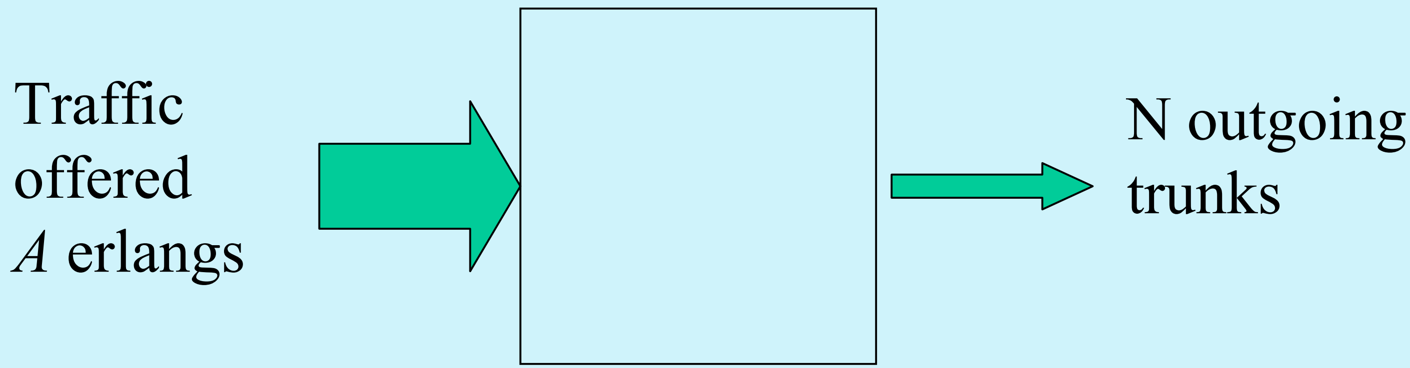
$$P(T \geq 4) = e^{-t/h} = e^{-4/2} = 0.135$$

b. The call will end within the next 4 minutes

$$P(T \leq 4) = 1 - P(T \geq 4) = 1 - 0.135 = 0.865$$

Lost-call systems

Consider that a large number of local loops are served by a small number of trunks in an exchange



when a call demanding a trunk link arrives, it is assigned a free trunk line if one is available, but if all trunks are engaged, that call will be lost since no provision of buffering is made.

Erlang's lost-call formula

– For a lost-call system having N trunks, when offered traffic A, the first Erlang distribution is given by

$$P(x) = \frac{\dfrac{A^x}{x!}}{\displaystyle\sum_{k=0}^{N} \frac{A^k}{k!}}$$

- x is the number of occupied trunks
- $P(x)$ is the probability of x occupied trunks

– The probability of a lost call, which is the grade of service B, is

$$B = P(N)$$

Erlang's lost-call formula

Example

A group of 5 trunks is offered 2 E of traffic. Find

a. The grade of service

$$B = P(x = N) = \frac{\frac{A^N}{N!}}{\sum_{k=0}^{N} \frac{A^k}{k!}} = \frac{\frac{2^5}{5!}}{\sum_{k=0}^{5} \frac{2^k}{k!}} = \frac{0.2667}{7.2667} = 0.037$$

b. The probability that only one trunk is busy

$$P(1) = \frac{\frac{2^1}{1!}}{\sum_{k=0}^{N} \frac{A^k}{k!}} = \frac{2}{7.2667} = 0.275$$

Erlang's lost-call formula

c. The probability that only one trunk is free

$$P(4) = \frac{\frac{2^4}{4!}}{\sum_{k=0}^{N} \frac{A^k}{k!}} = \frac{16/24}{7.2667} = 0.0917$$

d. The probability that at least one trunk is free

$$P(x < 5) = 1 - P(5) = 1 - B = 1 - 0.037 = 0.963$$

Traffic Table

Trunks	Grade of Service 1 in 1000 UC	Grade of Service 1 in 1000 TU	Grade of Service 1 in 500 UC	Grade of Service 1 in 500 TU	Grade of Service 1 in 200 UC	Grade of Service 1 in 200 TU	Grade of Service 1 in 100 UC	Grade of Service 1 in 100 TU
1	0.04	0.001	0.07	0.002	0.2	0.005	0.4	0.01
2	1.8	0.05	2.5	0.07	4	0.11	5.4	0.15
3	6.8	0.19	9	0.25	13	0.35	17	0.46
4	16	0.44	19	0.53	25	0.70	31	0.87
5	27	0.76	32	0.90	41	1.13	49	1.36
6	41	1.15	48	1.33	58	1.62	69	1.91
7	57	1.58	65	1.80	78	2.16	90	2.50
8	74	2.05	83	2.31	98	2.73	113	3.13
9	92	2.56	103	2.85	120	3.33	136	3.78
10	111	3.09	123	3.43	143	3.96	161	4.46
11	131	3.65	145	4.02	166	4.61	186	5.16
12	152	4.23	167	4.64	190	5.28	212	5.88
13	174	4.83	190	5.27	215	5.96	238	6.61
14	196	5.45	213	5.92	240	6.66	265	7.35
15	219	6.08	237	6.58	266	7.38	292	8.11
16	242	6.72	261	7.26	292	8.10	319	8.87
17	266	7.38	286	7.95	318	8.83	347	9.65
18	290	8.05	311	8.64	345	9.58	376	10.44
19	314	8.72	337	9.35	372	10.33	404	11.23
20	339	9.41	363	10.07	399	11.09	433	12.03
21	364	10.11	388	10.79	427	11.86	462	12.84
22	389	10.81	415	11.53	455	12.63	491	13.65
23	415	11.52	442	12.27	483	13.42	521	14.47
24	441	12.24	468	13.01	511	14.20	550	15.29

TU: traffic unit

Traffic Table

Example

On average, during the busy hour, a company makes 120 outgoing calls of average duration 2 minutes. It receives 200 incoming calls of average duration 3 minutes. This company wishes to obtain the grade of service of 0.01 for both incoming and outgoing calls. How many exchanges lines should it rent if

a. Incoming and outgoing calls are handled on separate groups of lines

b. A common group of lines is used for both incoming and outgoing calls.

Traffic Table

a. The outgoing traffic is 120 x 2 / 60 = 4 E

The incoming traffic is 200 x 3 / 60 = 10 E

From the table,

4 E of outing traffic needs 10 lines

10 E of incoming traffic needs 18 lines

b. The total traffic is 14 E

From the table,

14 E of traffic needs 23 lines