Circuit Switching: Traffic Engineering

References

• Chapter 1, *Telecommunication System Engineering*, Roger L. Freeman, Wiley.
Introduction

Example:
- mesh connection (full mesh) for an eight-subscriber system
Introduction

– justify a mesh connection is when each and every subscriber wishes to communicate with every other subscriber in the network for virtually the entire day.

– Most subscriber do not use their telephones on a full-time basis
– the ordinary subscriber will normally talk to only one other subscriber at a time
  • will not need to talk to all other subscribers simultaneously
Introduction

- Star network with a switch at the center
  - switch reduce the number of links between subscribers
Introduction

Terminology

– Trunk

• the telephone lines connecting one telephone switch or exchange with another are called trunks.

• one of the most important steps in telecommunication engineering practice is to determine the number of trunks required between exchanges.

↑ Dimensioning
Terminology

– Calling rate \((C)\)
  • The number of calls which arrive over a time interval

– Holding time \((H)\)
  • The average duration of a call

– Telephone traffic may fluctuate throughout the day, and may have a “busy hour” which is the hour that has the most number of calls
  • busy hour depends on various factors such as stock market, weather and international events
Introduction

Example
Measurement of Traffic

- The traffic intensity, more often called the traffic, is defined as the average number of calls in progress. The unit of traffic is Erlang (E).

\[ A = \frac{Ch}{T} \]

- \( A \): traffic intensity
- \( C \): number of calls arrivals during time \( T \)
- \( h \): average holding time
Measurement of Traffic

– Since a single trunk cannot carry more than one call, we can write

\[ A \leq 1 \]

– The probability of finding the trunk busy is equal to the proportion of time for which the trunk is busy. Thus, this probability equals the occupancy \( A \) of the trunk.
Measurement of Traffic

Example: 1.5 erlang of traffic carried on three trunks

Trunk 1

Trunk 2

Trunk 3

1.5E

time

Busy

Free
Measurement of Traffic

Example:

- On average, during the busy hour, a company makes 120 outgoing calls of average duration 2 minutes. It receives 200 incoming calls of average duration 3 minutes.
  - Find the outgoing traffic, the incoming traffic and the total traffic

  Outgoing traffic = 120 x 2 / 60 = 4 E
  Incoming traffic = 200 x 3 / 60 = 10 E
  Total traffic = 4 + 10 = 14 E
Measurement of Traffic

Example:
– During the busy hour, on average, a customer with a single telephone line makes three calls and receives three calls. The average call duration is 2 minutes. What is the probability that a caller will find the line engaged?

Total traffic = Occupancy of line = \((3+3) \times \frac{2}{60} = 0.1\) E

Probability of finding the line engaged = 0.1


**Blockage, Lost Calls, and Grade of Service**

**Lost call or blocked calls**
- In a circuit-switched system, all attempts to make calls over a congested group of trunks are unsuccessful. The unsuccessful call is called lost call or blocked call.

**Grade of service**
- Probability of meeting blockage is called the grade of service \( (B) \)
- Example: On average, one call in 100 will be blocked
  - \( B=0.1 \)
Lost call or blocked calls
– In a circuit-switched system, all attempts to make calls over a congested group of trunks are unsuccessful. The unsuccessful call is called lost call or blocked call.

Grade of service
– probability of meeting blockage is called the grade of service ($B$)

$$B = \frac{\text{Number of lost calls}}{\text{Total number of offered calls}}$$
Grade of service is also the
• proportion of the time for which congestion exists
• probability of congestion
• probability that a call will be lost due to congestion

If traffic $A$ erlangs is offered by a group of trunks having a grad of service $B$, the traffic lost is $AB$ and the traffic carried is $A(1-B)$ erlangs.
Example

– During the busy hour, 1200 calls were offered to a group of trunks and six calls were lost. The average call duration was 3 minutes

– The traffic offered = \( A = \frac{1200 \times 3}{60} = 60 \text{ E} \)
– The traffic carried = \( A-B = (1200-6) \times 3 / 60 = 59.7 \text{ E} \)
– The traffic lost = \( B = \frac{6 \times 3}{60} = 0.3 \text{ E} \)
– Grade of service = \( B/A = \frac{0.3}{60} = 0.005 \)
– The total duration of the periods of congestion
  \[ = B \times T = 0.005 \times 3600 = 18 \text{ seconds} \]
Traffic Formulas

Models of traffic

– call arrivals at an exchange are random in nature.
– It fits a family of probability-distribution curves following a **Poisson distribution**

– Variance-to-mean ratio (VMR)

\[ \alpha = \frac{\sigma^2}{\mu} \]
Traffic Formulas

Traffic probability distribution (Smooth, Rough and Random)

- Smooth: VMR < 1

![Histogram showing smooth traffic distribution with mean and variance values.]
Traffic Formulas

Traffic probability distribution
- Rough: VMR > 1
Rough traffic

- tends to be “peakier” than random and smooth traffic.

- For a given grade of service, more circuits are required for rough traffic because of the greater spread of distribution curve.
Traffic probability distribution

- Random: VMR = 1
- Poisson distribution function is an example of random traffic where VMR = 1
Traffic Formulas

- The number of call arrivals in a given time has a Poisson distribution

\[ P(x) = \frac{\mu^x}{x!} e^{-\mu} \]

- \( x \) is the number of call arrivals in time \( T \)
- \( \mu \) is the mean number of call arrivals in \( T \)
– Consider call-holding times to have a negative exponential distribution

\[ P(T \geq t) = e^{-t/h} \]

- \( P \) is the probability of a call lasting longer than \( t \)
- \( h \) is the mean call duration
Example

– On average one call arrives every 5 seconds. During a period of 10 seconds, what is the probability that
– a. No call arrives

\[ P(0) = \frac{2^0}{0!} e^{-2} = 0.135 \]

– b. One call arrives

\[ P(1) = \frac{2^1}{1!} e^{-2} = 0.27 \]

– c. More than one call arrives

\[ P(x > 1) = 1 - P(0) - P(1) = 0.595 \]
Example

– In a telephone system, the average call duration is 2 minutes. A call has already lasted 4 minutes. What is the probability that
– a. the call will last at least another 4 minutes
  • the probability is independent of the time which has already elapsed.
    \[ P = e^{-t/h} = e^{-4/2} = 0.135 \]
– b. The call will end within the next 4 minutes
  \[ P(T \leq t) = 1 - P(T \geq t) = 1 - 0.135 = 0.865 \]
Lost-call systems

– Consider that a large number of local loops are served by a small number of trunks in an exchange

Traffic offered $A$ erlangs

$N$ outgoing trunks

– when a call demanding a trunk link arrives, it is assigned a free trunk line if one is available, but if all trunks are engaged, that call will be lost since no provision of buffering is made.
For a lost-call system having $N$ trunks, when offered traffic $A$, the first Erlang distribution is given by

$$P(x) = \frac{A^x}{\sum_{k=0}^{N} \frac{A^k}{k!}}$$
The probability of a lost call, which is the grade of service $B$, is

$$B = \frac{A^N}{\sum_{k=0}^{N} \frac{A^k}{k!}}$$
Erlang’s lost-call formula

Example

– A group of 5 trunks is offered 2 E of traffic. Find
– a. The grade of service

\[ B = \frac{A^N}{N!} = \frac{2^5}{5!} = \frac{0.2667}{7.2667} = 0.037 \]

– b. The probability that only one trunk is busy

\[ P(1) = \frac{2^1}{1!} = \frac{2}{7.2667} = 0.275 \]
Erlang’s lost-call formula

– c. The probability that only one trunk is free

\[ P(4) = \frac{2^4}{4!} = \frac{16}{24} \cdot \frac{7.2667}{7.2667} = 0.0917 \]

– d. The probability that at least one trunk is free

\[ P(x < 5) = 1 - P(5) = 1 - B = 1 - 0.037 = 0.963 \]
# Traffic Table

<table>
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TU: traffic unit
Example

– ON average, during the busy hour, a company makes 120 outgoing calls of average duration 2 minutes. It receives 200 incoming calls of average duration 3 minutes. This company wishes to obtain the grade of service of 0.01 for both incoming and outgoing calls. How many exchanges lines should it rent if

– a. Incoming and outgoing calls are handled on separate groups of lines
– b. A common group of lines is used for both incoming and outgoing calls.
Traffic Table

– a. The outgoing traffic is $120 \times 2 / 60 = 4 \text{ E}$
The incoming traffic is $200 \times 3 / 60 = 10 \text{ E}$

From the table,
4 E of outgoing traffic needs 10 lines
10 E of incoming traffic needs 18 lines

– b. The total traffic is 14 E

From the table,
14 E of traffic needs 23 lines