Tutorial 4  Channel Capacity

1. The binary erasure channel has two inputs and three outputs as described in Fig. 1. The inputs are labeled 0 and 1, and the outputs are labeled 0, 1, and e. A fraction $\alpha$ of the incoming bits are erased by the channel. Find the capacity of the channel.

$$
\begin{align*}
0 & \xrightarrow{1-\alpha} 0 \\
X & \xrightarrow{\alpha} e \\
1 & \xrightarrow{1-\alpha} 1
\end{align*}
$$

Figure 1

2. An analog signal has a 4 kHz bandwidth. The signal is sampled at 2.5 times the Nyquist rate and each sample is quantized into one of 256 equally likely levels. Assume that the successive samples are statistically independent.
   (a) What is the information rate of this source?
   (b) Can the output of this source be transmitted without errors over a Gaussian channel with a bandwidth of 50kHz and S/N ratio of 23 dB?
   (c) What will be the output of the source without errors if the S/N ratio is 10 dB?

3. A black-and-white television picture may be viewed as consisting of approximately $3 \times 10^5$ elements, each of which may occupy one of 10 distinct brightness levels with equal probability. Assume that (1) the rate of transmission is 30 picture frames per second, and (2) the signal-to-noise ratio is 30 dB. Using the information capacity theorem, calculate the minimum bandwidth required to support the transmission of the resulting video signal.
   (Note: As a matter of interest, commercial television transmissions actually employ a bandwidth of 4.2 MHz, which fits into an allocated bandwidth of 6MHz.)