A SYSTEMATIC DESIGN PROCEDURE FOR SLIDING MODE CONTROLLED BUCK CONVERTERS

Siew-Chong Tan, Y. M. Lai, Martin K. H. Cheung, and Chi K. Tse

Applied Nonlinear Circuits and Systems Research Group
Department of Electronic and Information Engineering
The Hong Kong Polytechnic University
Hong Kong, China
email: ensctan@eie.polyu.edu.hk

ABSTRACT

This paper presents a systematic design procedure for sliding mode voltage controlled buck converters operating in continuous conduction mode. The discussion covers different practical aspects related to the design. A simple and easy-to-follow design procedure is also described. Experiments were performed to validate the design procedure.

1. INTRODUCTION

Sliding mode (SM) controllers were introduced initially for variable structure systems (VSS) [1], [2]. Although well known for their stability and robustness towards parameter, line, and load variations (ability to handle large transience), SM controllers are seldom used in power converters. This is mainly due to the lack of understanding in its design principle by power supply engineers [3], as well as the lack of a systematic procedure in existing literature, which can be used to develop such controllers. This can be attributed to the fact that much of the work on the subject has been reported from the control's viewpoint, rather than the circuit's viewpoint. Hence, the focus had been on the theoretical aspects of the control, while the practical aspects of the implementation are rarely discussed [3], [4].

Thus, the objective of this paper is to introduce a simple approach that is easily applicable in the development of a sliding mode voltage controlled (SMVC) buck converter, to bridge the gap between the control principle and circuit implementation. To present a complete exposition, mathematical derivations and theoretical analyzes which extend from the work of [4] are firstly performed.

2. THEORETICAL DERIVATION

2.1. Mathematical Model of Ideal Sliding Mode Controlled Buck Converter

Fig. 1 shows the schematic diagram of an SMVC buck converter. Here, the *voltage error* x_1 and the *voltage*

Fig. 1. Basic structure of an SMVC buck converter.

error dynamics (or the rate of change of voltage error) x_2 under CCM, can be expressed as

$$\begin{aligned} x_1 &= V_{\text{ref}} - \beta V_o \\ x_2 &= \dot{x}_1 = \frac{\beta}{C} \left(\frac{V_o}{R_L} - \int \frac{u V_i - V_o}{L} dt \right) \end{aligned} \quad (1)$$

where C, L, R_L are the capacitance, inductance, and load resistance respectively; V_{ref}, V_i , and βV_o are the reference, input, and sensed output voltage respectively; $u = 1$ or 0 is the switching state of power switch S_w . Then, by differentiating (1) with respect to time, the state space model can be obtained as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{R_L C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{V_{\text{ref}}}{LC} - \frac{\beta V_i}{LC} u \end{bmatrix}. \quad (2)$$

For this system's model, it is appropriate to have a control law

$$u = \begin{cases} 1 = \text{'ON'} & \text{when } S > 0 \\ 0 = \text{'OFF'} & \text{when } S < 0 \end{cases} \quad (3)$$

where S is the instantaneous state variable trajectory and is described as

$$S = \alpha x_1 + x_2 \quad (4)$$

and α is the control parameter (termed as sliding coefficient) to be designed. To achieve reachability and asymptotic stability, the system must obey $S \cdot \dot{S} < 0$, which gives $0 < L \left(\frac{1}{R_L C} - \alpha \right) i_C + V_o < V_i$, where i_C represents the capacitor current. Next, it is also known that during SM operation ($S = 0$), α controls the dynamic response of the system with a first order

time constant of $\tau = \frac{1}{\alpha}$. To ensure that α is high enough for fast dynamic response and low enough to maintain a large existence region [4], it is proposed to set $\alpha = \frac{1}{R_L C}$.

2.2. Design of a Practical SM Voltage Controller

2.2.1. Redefinition of Sliding Line

SM controller requires the continuous assessment of the parameters x_1 and x_2 for its control. By substituting (1) into (4) and setting $\alpha = \frac{1}{R_L C}$, we have

$$S = k_1(V_{\text{ref}} - \beta V_o) + k_2 i_C \quad (5)$$

where $k_1 = \frac{1}{R_L C}$ and $k_2 = -\frac{\beta}{C}$. Here, the terms $(V_{\text{ref}} - \beta V_o)$ and i_C are the feedback state variables that should be amplified by gain coefficients k_1 and k_2 respectively, before a summation is performed. However, noting that capacitance C in power converters is usually in the microfarad (μF) range, its inverse term will be significantly higher than β and R_L . When combined, it results in an overall gain coefficients k_1 and k_2 that are too high for practical implementation. If forcibly implemented, the feedback signals may be driven into saturation, making (5) unreliable for the control.

In view of that, it is simpler to reconfigure the switching function to the following description:

$$S = \frac{C}{\beta} \alpha x_1 + \frac{C}{\beta} x_2 = \frac{1}{\beta R_L} (V_{\text{ref}} - \beta V_o) - i_C. \quad (6)$$

Thus, the practical implementation of S becomes independent of C , thereby reducing the amplification of the feedback signals. Although there is modification to the switching function, the maximum existence region and response time is maintained at $\tau = \frac{1}{\alpha}$.

2.2.2. Introduction of Hysteresis Band

Ideally, the converter will switch at infinite frequency with its phase trajectory moving on the sliding line when it enters SM operation. In the presence of switching imperfections (switching time constant and delay), this is not possible. The discontinuity in the feedback control produces a particular dynamic behavior in the vicinity of the surface trajectory known as chattering [1], [2]. If chattering is uncontrolled, the converter system will be self-oscillating at a very high switching frequency corresponding to the chattering dynamics. This is undesirable as high switching frequency results in excessive switching losses, inductor and transformer core losses, and EMI noise issues [5]. Furthermore, with switching frequency being unpredictable, the design of the converter and the selection of the components will be difficult.

To solve these problems, the control law in (3) is redefined as

$$u = \begin{cases} 1 = \text{'ON'} & \text{when } S > \kappa \\ 0 = \text{'OFF'} & \text{when } S < -\kappa \end{cases} \quad (7)$$

where κ is an arbitrarily small value. The reason for introducing a hysteresis band with the boundary conditions κ and $-\kappa$ is to create a dead region $-\kappa \leq S \leq \kappa$ where no changing of switching state can occur. The maximum switching frequency of the SM controller can therefore be controlled by varying κ .

2.2.3. Calculation of Switching Frequency

To control the switching frequency of the converter, the relationship between the hysteresis band κ and switching frequency f_S must be known. From [2], for $\alpha = \frac{1}{R_L C}$, we derive

$$\begin{aligned} \Delta t_{OFF} &= \frac{2\kappa}{-\frac{1}{\beta L} x_1 + \frac{V_{\text{ref}}}{\beta L}} = \frac{2\kappa L}{V_o} \\ \Delta t_{ON} &= \frac{-2\kappa}{-\frac{1}{\beta L} x_1 + \frac{V_{\text{ref}} - \beta V_i}{\beta L}} = \frac{2\kappa L}{V_i - V_o} \end{aligned} \quad (8)$$

where Δt_{OFF} and Δt_{ON} are respectively the turn-off and turn-on time durations of the switch. Therefore, the total period for one switching cycle is

$$T = \Delta t_{OFF} + \Delta t_{ON} = \frac{2\kappa L}{V_o \left(1 - \frac{V_o}{V_i}\right)}. \quad (9)$$

Since the cycle is repeated (cyclic) throughout the SM steady-state operation, the frequency of the converter when it is operating in SM can be expressed as

$$f_S = \frac{1}{T} = \frac{V_o \left(1 - \frac{V_o}{V_i}\right)}{2\kappa L}. \quad (10)$$

Considering that V_i and V_o are non-constant parameter consisting of respectively dc signals of $\bar{V}_i$ and $\bar{V}_o$ and time varying perturbations of $\tilde{V}_i$ and $\tilde{V}_o$, we can resolve (10) into $f_S = \bar{f}_S + \tilde{f}_S$ using small-signal approximation where

$$\bar{f}_S = \frac{\bar{V}_o \left(1 - \frac{\bar{V}_o}{\bar{V}_i}\right)}{2\kappa L} \quad (11)$$

$$\tilde{f}_S = \frac{\tilde{V}_o \left(1 - \frac{2\bar{V}_o}{\bar{V}_i + \bar{V}_i}\right)}{2\kappa L} \quad (12)$$

with $\bar{f}_S$ representing the steady-state (nominal cyclic) switching frequency and $\tilde{f}_S$ representing the ac varying (perturbed) frequency of the converter. For the controller's design, only (11) is required since only nominal steady-state operating conditions will be considered.

3. A STANDARD DESIGN PROCEDURE

3.1. Standard SMVC Converter Model

Fig. 2 shows the proposed SMVC buck converter. Similar to conventional schemes, the feedback sensing network for V_o is provided by the voltage divider circuit, R_1 and R_2 . Additionally, a low resistance current transformer is placed in series with the filter capacitor to obtain the capacitor current, i_C .

Fig. 2. A standard SMVC buck converter.

3.2. Design Steps

The design of the buck converter is well covered in the literature [5]. This discussion starts with the assumption that the parameters are known, and are given in Table 1¹.

Table 1. Specifications of Buck Converter

Description	Parameter	Value
Input voltage	V_i	24 V
Minimum capacitance	C_{min}	4 μ F
Critical inductance	L_{crit}	34.28 μ H
Desired switching frequency	f_{sd}	200 kHz
Load resistance	R_L	6 Ω
Desired output voltage	V_{od}	12 V

Step 1

The current sensing gain, H , is set at a value such that the measured capacitor current, $i_{C(meas)}$ is equal to the actual capacitor current, i_C . Next, setting reference voltage $V_{ref} = 3.3$ V, $\beta = \frac{V_{ref}}{V_{od}} = 0.275$. Since R_1 and R_2 are related by $R_2 = \frac{\beta}{1-\beta} R_1$, choosing $R_1 = 870$ Ω gives $R_2 = 330$ Ω .

Step 2

From (6), the gain required for the amplification of the signal $(V_{ref} - \beta V_o)$ is $\frac{1}{\beta R_L}$. Hence, R_{V1} and R_{V2} are related by $R_{V1} = (\beta R_L) R_{V2}$. Choosing $R_{V2} = 20$ k Ω , we get $R_{V1} = 33$ k Ω . Additionally, the resistors, R_{DIF} , for the difference amplifier circuit: U_D , are chosen as 10 k Ω .

Step 3

The parameter of the hysteresis band, κ , can be obtained from the re-arranged form of (11), i.e.,

$$\kappa = \frac{V_{od} \left(1 - \frac{V_{od}}{V_i}\right)}{2f_{sd}L} \quad (13)$$

¹The parameters are calculated on the basis that the converter is to be operated in CCM for $V_i = 13$ V to 30 V and $i_R = 0.5$ A to 4 A. The maximum peak to peak ripple voltage is 50 mV.

Fig. 3. Calculated κ values for inductances of 50 μ H, 100 μ H, 110.23 μ H (actual inductance), and 150 μ H at switching frequencies of up to 300 kHz.

where V_i , V_{od} , and f_{sd} are the nominal parameters of the converter. A plot giving the calculated κ values for different inductances and switching frequencies is shown in Fig. 3. The actual inductance, L_{act} , used in the design is 110.23 μ H. It should be noted that $L_{act} \geq L_{crit}$ for CCM. Thus, substituting $L = 110.23$ μ H into (13), κ is calculated as 0.136.²

Step 4

The setting of κ for the hysteresis band can be performed by adjusting the ratio of R_{ST1} and R_{ST2} of U_S , using an equation derived from the mathematical description of a non-inverting Schmitt Trigger, i.e.

$$R_{ST2} = \frac{R_{ST1}(V_{CC}^+ - V_{CC}^-)}{2\kappa} \quad (14)$$

where V_{CC}^+ and V_{CC}^- are respectively the positive and negative voltage supplies to Schmitt Trigger U_S . Choosing $R_{ST1} = 110$ Ω gives $R_{ST2} = 12$ k Ω .

4. EXPERIMENTAL RESULTS

4.1. Verification of Design Equation

Fig. 4 shows the graphs of the converter's average switching frequency $\bar{f}_s$ for different κ values at nominal operating condition, that are obtained from calculation, simulation, and experiment. Specifically, the calculation is performed using the proposed design equation (11) and the simulation is carried out in Matlab/Simulink using the circuit expression of the proposed controller. Basically, the simulation and experimental data are in good agreement with the calculated data. The small discrepancy between the experimental data and both the calculated and simulated data is

²The presence of hysteresis band in switching function introduces an error in the output voltage. It is important to limit the hysteresis band κ to a small value to minimize this error. On the other hand, if κ is too small, it may be very sensitive to change of κ (i.e. high $df/d\kappa$). A good value is to set κ in the range $0.1 \leq \kappa \leq 0.2$. Otherwise, a different inductance may be used to keep κ within the range.

Fig. 4. Calculated, simulated, and experimentally measured average switching frequencies $\bar{f}_s$ at nominal operating condition $V_i = 24$ V and $R_L = 6 \Omega$ for different hysteresis band κ settings.

mainly due to component tolerances and finite time delay of practical circuitries. In practical development, fine tuning of κ is still required to achieve the desired $\bar{f}_s$.

Fig. 5. Experimentally measured average output voltage $\bar{V}_o$ at $V_i = 24$ V and $R_L = 3, 6$, and 12Ω for different hysteresis band κ settings.

Fig. 5 shows the graph of measured average output voltage $\bar{V}_o$ against κ for load resistances $R_L = 3, 6$, and 12Ω . From the figure, $\bar{V}_o$ is notably lower with lower R_L . A major point to highlight is that at low κ , i.e., high switching frequency, the voltage regulation is tighter and more accurate. In our design, by setting $0.1 \leq \kappa \leq 0.2$ in the design, we limit the voltage accuracy within ± 0.12 V (i.e. $< 1\%$ of V_{od}) error.

4.2. Steady-State Performance

The controller is fine-tuned to a switching frequency of 200 kHz by replacing R_{ST2} with a 16 k Ω resistor, thereby setting $\kappa = 0.1$. Fig. 6 shows an example of the output waveforms of the SMVC buck converter at steady-state operation. Performing to design expectation, the converter operates at an average switching frequency of $\bar{f}_s = 199$ kHz, with small frequency fluctu-

Fig. 6. Waveforms of $\bar{V}_o$, i_L , and u at steady-state operation under nominal operating conditions $V_i = 24$ V and $R_L = 6 \Omega$.

tations, and the output voltage ripple $\bar{V}_o$ (without considering the ringing oscillation) is around 10 mV (i.e. $< 0.1\%$ of V_{od}), under nominal operating condition.

5. CONCLUSIONS

A detailed analysis of the design principle of a SMVC buck converter is presented. The discussion takes into consideration the practical aspects of the converter. The sliding line for an ideal controller is redefined to meet practical limitations. A hysteresis band is introduced to the sliding line to solve the problem of chattering. The relationship between the hysteresis band and the switching frequency is derived. To facilitate implementation, a standard SMVC converter module is introduced. Its design guidelines are verified through experiment.

6. REFERENCES

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