Quasi-Linear Modeling and Control of DC–DC Converters

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Abstract—A quasi-linear approach is proposed for modeling and control of dc–dc converters. The method presented in this paper is derived by perturbing an approximate large-signal equation around a “varying” operating point in a reduced variable space. This differs from the usual practice of applying the perturbation technique around a fixed operating condition. In the proposed algorithm, the control equation as well as the control parameter are constantly adjusted according to the environmental changes to meet the specified dynamical requirement.

I. INTRODUCTION

In its simplest terms, the operation of a dc–dc converter can be described as an orderly repetition of a fixed sequence of circuit topologies. The conversion function of the converter is determined by the constituent topologies and the order in which they are repeated. Such toggling between circuit topologies is achieved by placing switches at suitable positions and turning them on and off in such a way that the required topological sequence is produced. Fig. 1 shows the circuit diagram of three basic switching cells.

Clearly, the absence of a fixed circuit configuration poses a serious problem to the analysis and modeling of dc–dc converters. The major difficulty lies in the fact that the manner in which the system operates is highly nonlinear. In the last decade, various techniques of linearizing the dc–dc converters have been proposed [1]–[4]. Their common goal is to provide a small-signal linear model that is capable of describing accurately the behavior of the system in the neighborhood of a fixed operating point, thereby offering some simple solutions to the control problem. Of course, a major drawback would be the very limited range of fluctuation of variables for which the linearized model is valid.

In this paper, we propose a simple quasi-linear approach for modeling the PWM dc–dc converters. The essence of this approach, as opposed to conventional linear approaches, lies in the application of the technique of perturbation to the system around a “varying” operating point in a reduced variable space. This method also leads to a control scheme whose validity extends over a wider range of input fluctuation. The essential feature of this control scheme consists in modifying the feedback equation continuously in time, according to the variation of the input voltage. In this sense, it is a kind of adaptive control. The price to pay is a slightly more complex implementation of the controller.

In terms of its structural organization, the proposed quasi-linear controller represents a combined feedforward-feedback configuration. However, it should not be confused with the small-signal feedforward-feedback control proposed in reference [5] or [6], which is derived using a linearized injected-absorbed-current model with the inductance current serving as the control quantity. In order to show the relationship between the quasi-linear scheme described in this paper and the feedforward-feedback schemes, we present a state-space version of small-signal feedforward-feedback, prior to our main discussion of a quasi-linearization, and show how this feedforward-feedback concept leads to an adaptive control scheme.

II. REVIEW OF THE DISCRETE-TIME LARGE-SIGNAL MODEL

We start our discussion by obtaining a discrete-time large-signal state equation for the class of dc–dc converters under investigation. The general procedure for deriving this equation essentially follows that of reference [7] and is summarized as follows. During a switching period, several circuit configurations can be identified. In the case of a switching cell operating in continuous mode, there exist two circuit configurations corresponding to two sub-intervals of a switching cycle. In addition, a third circuit configuration exists if the system operates in discontinuous mode. For each of these circuits, a state equation can be written. The solution to each of these equations can be expressed in terms of their respective transition matrices, and the consecutive solutions are then “stacked” over a switching period, resulting in a dis-
crete-time large-signal equation of the form
\[ x_{n+1} = \Phi(d)x_n + \Psi(d)e_n \]  
(1)
for continuous mode and
\[ x_{n+1} = \Phi(d, h)x_n + \Psi(d, h)e_n \]  
(2)
for discontinuous mode, where \( x \) is the state vector comprising the capacitance voltage \( v \) and the inductance current \( i \), i.e., \( x = [v_i] \). \( \Phi(\cdot) \) is a square matrix of order 2, \( \Psi(\cdot) \) is a 2-dim vector, \( e_n \) is the input voltage, \( d \) is the duty ratio, \( h \) (only applicable for discontinuous mode) is the fraction of the period during which the inductance current is identically zero, and subscript \( n \) denotes the value at the \( n \)th switching instant, i.e. \( x_n = x(nT) \).

When the converter is operating in discontinuous mode, two additional conditions apply. First, the inductance current is zero for all \( t = nT \). This condition prevents the inductance current from behaving as a state variable, thus reducing the order of the system by one. Second, continuity of the inductance current implies that
\[ \lim_{t \to nT^+} i(t) = \lim_{t \to nT^-} i(t) \]  
(3)
where \( t_n = t_n + dT \). This leads effectively to an expression relating \( d \) and \( h \), thus eliminating the dependence of \( \Phi(\cdot) \) and \( \Psi(\cdot) \) upon \( h \). Taking into account these two conditions, the discrete-time state equation for the discontinuous-mode case is of the form
\[ v(x, n) + f(x, n, d, e) \]  
(4)
As a consequence of the strict passivity of the resistive elements and the absence of capacitor-inductor-current-source cutsets and capacitor-inductor-voltage-source loops, the converter circuit is guaranteed to have a unique steady-state solution [10]. Hence, the steady-state solution can be obtained by enforcing periodicity. For the system operating in continuous mode, putting \( x_{n+1} = x_n \) into (1) gives
\[ X = (1 - \Phi(D))^{-1}\Psi(D)E \]  
(5)
where \( X \), \( D \), and \( E \) denote the steady-state values of \( x \), \( d \), and \( e \), respectively. For the system operating in discontinuous mode [9], however, the steady-state solution is implicit in
\[ X = f(X, D, E) \]  
(6)
For the purpose of illustration, we shall concentrate on the system operating in continuous mode, although the technique is easily extended to systems operating in discontinuous mode, and even further to a general class of nonlinear systems.

III. APPLICATION OF LINEAR METHODS TO THE CONTROL OF DC-DC CONVERTERS

A. Linearization via Perturbation Around a Fixed Point

Clearly, the absence of a linear term in \( d \) in the state equation represents some difficulties in dealing with this system. A conventional approach to overcome this problem is to linearize (1) around a fixed operating point, leading to a small-signal difference equation:
\[ \dot{x}_{n+1} = \Phi(D)\dot{x}_n + [\Phi'(D)X + \Psi'(D)E]d_n + \Psi(D)\dot{e}_n \]  
(7)
where
\[ \dot{x} = x - X \]  
(8)
\[ \dot{d} = d - D \]  
(9)
\[ \dot{e} = e - E \]  
(10)
\[ \Phi'(D) = \frac{\partial \Phi(d)}{\partial d} \]  
(11)
\[ \Psi'(D) = \frac{\partial \Psi(d)}{\partial d} \]  
(12)
Equation (7) represents a discrete-time small-signal linear model for the system under investigation. This model differs from others commonly used time-averaged models, such as the state-space-averaged model [1], [2] and the absorbed-injected-current model [3], in two respects. First, it is a discrete-time model whereas the time-averaged models are continuous-time models. Second, it is derived from linearizing an exact nonlinear equation and any approximation introduced thereafter can be made to any desired degree of accuracy. The averaged models, on the other hand, make some simplifying assumptions at the very beginning of their derivations, and hence impose some inherent restrictions to all subsequent analyses.

The dynamics of the open-loop system in the neighborhood of \( (X, D, E) \) is characterized by the eigenvalues of the matrix \( \Phi(D) \), i.e., the roots of the equation \( \det [zI - \Phi(D)] = 0 \). It is worth noting that since the open-loop system is globally asymptotically stable, the eigenvalues of \( \Phi(D) \) must lie within the unit circle in the complex \( z \) plane.

B. Linear Pole-Placement Control via State Feedback

Arbitrary dynamics can be assigned to the linearized system by a linear feedforward-feedback control law of the following form:
\[ \dot{d} = K\dot{e} + \mu\dot{e} \]  
(13)
where
\[ K = [k_1 \ k_2] \]  
(14)
and \( k_1, k_2 \), and \( \mu \) are constants for a given steady-state operating point.

Putting (13) into (7) gives
\[ \dot{x}_{n+1} = [\Phi(D) + \Gamma(D, X, E)K]\dot{x}_n + [\Psi(D) + \mu\Gamma(D, X, E)]\dot{e}_n \]  
(15)
where
\[ \Gamma(D, X, E) = \Phi'(D)X + \Psi'(D)E \]  
(16)
Here we want to find the values of $\mu$ and $K$ such that the effect of the disturbance is minimized, and the resulting eigenvalues are placed at some desired locations. Suppose a value of $\mu$ is chosen which minimizes the effect of the second term of the right-hand side of (15). Then (15) becomes

$$\dot{x}_{n+1} = [\Phi(D) + \Gamma(D, X, E)K]x_n$$  \hspace{1cm} (17)

and the characteristic equation of the system represented by (17) becomes

$$\text{det}[zI - \Phi(D) - \Gamma(D, X, E)K] = 0.$$  \hspace{1cm} (18)

Further suppose that a pair of desired eigenvalues ($z$-plane poles) have been specified, say $\lambda_1$ and $\lambda_2$. Then (18) is equivalent to

$$z^2 - (\lambda_1 + \lambda_2)z + \lambda_1\lambda_2 = 0$$  \hspace{1cm} (19)

and hence $K$ may be obtained by comparing coefficients of (18) and (19). By writing $\Phi$ and $\Gamma$ as

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \quad \text{and} \quad \Gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$$  \hspace{1cm} (20)

the feedback factors are found to be

$$k_1 = \frac{(\lambda_1 + \lambda_2 - \phi_{11} - \phi_{22})(\gamma_2\phi_{11} - \gamma_1\phi_{12}) - \gamma_2(\lambda_1\lambda_2 + \phi_{12}\phi_{21} - \phi_{11}\phi_{22})}{\gamma_1(\gamma_1\phi_{11} - \gamma_2\phi_{22}) - \gamma_2(\gamma_1\phi_{12} + \phi_{11}\phi_{21} - \phi_{12}\phi_{22})}$$  \hspace{1cm} (21)

$$k_2 = \frac{-(\lambda_1 + \lambda_2 - \phi_{11} - \phi_{22})(\gamma_2\phi_{12} - \gamma_1\phi_{21}) + \gamma_1(\lambda_1\lambda_2 + \phi_{12}\phi_{21} - \phi_{11}\phi_{22})}{\gamma_2(\gamma_1\phi_{11} - \gamma_2\phi_{22}) - \gamma_1(\gamma_1\phi_{12} + \phi_{11}\phi_{21} - \phi_{12}\phi_{22})}$$  \hspace{1cm} (22)

provided that $\gamma_1\gamma_2(\phi_{11} - \phi_{22}) - \gamma_2^2\phi_{21} + \phi_{11}\phi_{12} \neq 0$, i.e., the controllability matrix $[\Gamma\Phi\Gamma]$ is of full rank [8]. Notice that $k_1$, $k_2$, and $\mu$ are constants for any given steady-state condition.

Since all state variables are accessible, the implementation of the controller is straightforward: the values of $\dot{x}$ and $x$ are sampled every period and the duty ratio is obtained using the linear law given by (13). Fig. 2 shows the block diagram of this controller.

IV. QUASI-LINEAR MODELING AND CONTROL

For the system under investigation, the variables of interest are $x$, $d$, and $e$, implying a four-dimensional space in which the perturbation procedure is carried out. (Note that $x$ is a two-dimensional vector.) The result of perturbing the system in this four-dimensional space is exactly the linear small-signal model represented by (7). This model describes accurately the behavior of the system only if the system is operating in a small neighborhood of the fixed point $(X, D, E)$ around which the system has been linearized.

The preceding section also illustrates a kind of pole-placement technique that is common in linear control systems. It provides satisfactory regulation only if the fluctuations of the variables about the operating point are not too large. In the following subsection, a similar procedure is followed, but (1) will not be perturbed around a fixed point. Instead, the operating point is varying as a function of the input voltage $e$, and the perturbation technique is applied around this changing operating point.

A. Linearization Except One Variable

The key concept in quasi-linear modeling is the reduction of the number of variables that are to be linearized. The dimension of the resulting variable space is thus less than that when all variables are linearized. In this reduced space, the steady-state operating point is no longer fixed but is varying with the variables not being included in the reduced space.

For the system under investigation, it is possible to choose a reduced space that excludes the variable $e$. Thus, the variables involved are $x$ and $d$ only, and the steady-state operating point is $(x_{op}(e), d_{op}(e))$, which varies with $e$.

Because the operating point is no longer fixed, the definition of a small-signal quantity should be rephrased as the difference between the instantaneous value and the "varying" operating value, i.e.,

$$\dot{x} = x - x_{op}(e)$$  \hspace{1cm} (23)

$$d = d - d_{op}(e).$$  \hspace{1cm} (24)

The technique of perturbation may now be applied to (1) in the reduced variable space, resulting in the following quasi-linear "small-signal" equation:

$$\dot{x}_{n+1} = \Phi(d_{op})x_n + \Gamma(d_{op})d_n$$  \hspace{1cm} (25)

where

$$\Gamma(d_{op}) = \Phi'(d_{op})x_{op}(e) + \Psi'(d_{op})e.$$  \hspace{1cm} (26)

Notice that there is no $\dot{e}$ term in (25). In fact, the input disturbance has been absorbed into $\Phi$ and $\Gamma$, and the system is essentially linear with varying coefficients.
B. The Varying Operating Point

Suppose that for all basic switching cells, \( d_{op} \) can be expressed explicitly as a function of \( e \). Then, the expression for \( x_{op} \) can be derived from (5), namely,

\[
x_{op} = \left[ \frac{v_e}{i_{loop}} \right] = [1 - \Phi(d_{op})]^{-1} \Psi(d_{op}) e.
\] (27)

For all practical purposes, the output voltage always needs to be maintained at a fixed level, say \( U_{ref} \). Assuming that the capacitor-series-resistance is small, the capacitance voltage is in fact the output voltage, i.e.,

\[
x_{1,op} = U_{ref}.
\] (28)

In general, \( x_{op} \) will vary with \( e \), but in this particular case, \( x_{1,op} \) is also the steady-state output voltage, which is required to be constant.

It should be noted that, in deriving the operating point, the existence of an explicit expression for \( d_{op} \) has been assumed. This assumption is justified by inspection of the basic dc voltage transfer function. For example,

\[
d_{op}(e) = 1 - \frac{e}{U_{ref}}
\] (29)

For the boost cell, and

\[
d_{op}(e) = \frac{U_{ref}}{e}
\] (30)

for the buck cell. Since the dc voltage transfer function is indeed implicit in (27), \( x_{1,op} = U_{ref} \) should follow consistently from the first row of (27). Finally, \( x_{2,op} \) is given by the second row of (27). An illustrative example will be given in Section VI.

C. An Adaptive Control from the Quasi-Linear Model

The same technique as illustrated in Section III-B can be applied to (25) to yield a control law analogous to (13). However, since the quasi-linear equation does not contain any \( \dot{e} \) term, the corresponding term in the control equation can be omitted. Thus, the resulting control law takes the form

\[
d = d_{op}(e) + K(e)(x - x_{op})
\] (31)

where \( K(e) = [k_1(e) \ k_2(e)] \). The values of \( k_1 \) and \( k_2 \) are still given by (21) and (22), but now \( \Phi \) and \( \Psi \) are functions of \( e \), and so are \( k_1 \) and \( k_2 \). This requires a slightly more complex implementation of the controller due to the need to modify the feedback equation continuously in time. Reference to Fig. 3 shows that an additional processor and a feedback loop are needed to locate the operating point and update the feedback factors accordingly.

To further substantiate the concept of feedback as applied to this particular situation, we consider a simple small-signal state-feedback system in which the steady-state operating point is assumed to be fixed. Clearly, since the values of the feedback factors relate closely to the steady-state operating point, this system is satisfactorily regulated only if the assumption of a fixed operating point remains valid. Now the idea of quasi-linear control essentially means that the system utilizes an additional feedforward path to detect the disturbance, uses the disturbance to update our knowledge of the location of the operating point, and continuously recomputes the values of the feedback factors.

An obvious advantage of the quasi-linear approach is that the resulting model is accurate for a much wider fluctuation of the input voltage. This is because the input voltage has not been treated as a small-signal quantity. The nonlinear dependence of the system on the input voltage is thus well preserved in the resulting quasi-linear model.

D. Treatment for Discontinuous-Mode Operation

The preceding developments deal only with the system operating in continuous mode. All results obtained so far apply only to this particular operating mode and appear totally irrelevant to the discontinuous-mode case. Nevertheless, the basic principle underlying the kind of technique presented in the foregoing subsections is valid for all nonlinear systems.

It has been shown that the system operating in discontinuous mode is described by the following state equation:

\[
u_e(t_{n+1}) = f(u_e(t_n), d, e).
\] (32)

To apply the quasi-linear approach, the variable \( e \) is again excluded. The operating point now varies with \( e \) and can be derived by putting \( u_e(t_{n+1}) = u_e(t_n) \) into the above state equation. Perturbing the system around the operating point \( (u_{e,op}, d_{op}) \) yields

\[
\dot{e}_e(t_{n+1}) = f_e(u_e, d_{op}) \dot{e}_e(t_n) + f_d(u_{e,op}, d_{op}) \dot{d}
\] (33)

where

\[
\dot{e}_e = e_e - u_e, \quad \dot{d} = d - d_{op},
\]

\[
f_e(u_{e,op}, d_{op}) = \frac{\partial f_e}{\partial u_e}
\] (34)
and

$$f_d(v_{e, \text{op}}, d_{\text{op}}) = \left. \frac{\partial f(\cdot)}{\partial d} \right|_{(v_{e, \text{op}}, d_{\text{op}})},$$  \hspace{1cm} (35)$$

As explained before, the condition $v_{e, \text{op}} = U_{e, \text{op}}$ holds if the capacitor-series-resistance is neglected.

Now suppose that the desired eigenvalue is $\lambda$. An equivalent representation of this requirement is

$$v_e(t_n + 1) = \lambda v_e(t_n).$$  \hspace{1cm} (36)$$

In order to meet this requirement, $\tilde{d}$ must be assigned according to

$$\tilde{d} = \frac{\lambda - f_e(\cdot)}{f_e(\cdot) - v_e}.$$  \hspace{1cm} (37)$$

Notice that $d_{\text{op}}$ varies with $e$, and so do $f_e$ and $f_e$. Thus the feedback factor also varies with $e$ and is identified as

$$k(e) = \frac{\lambda - f_e(\cdot)}{f_e(\cdot)}.$$  \hspace{1cm} (38)$$

Needless to say, the algorithm for regulating the switching cell when operated in discontinuous mode is much simpler than in the continuous-mode case. However, the choice of operating mode is dictated by the power level and device capability [11]. In most practical situations, continuous mode is the preferred mode of operation, especially for high-power applications.

V. TOWARD NONLINEAR CONTROL

The method of control described in Section IV-C differs from the one in Section III-B in that the variable $e$ remains unperturbed. The result is a better description and an improved control of the system, as will be seen from the simulation results presented in Section VI. One may now be inspired to keep one more variable unperturbed and attempt to control the system based on the following equation:

$$\dot{x}_{n+1} = \left. \frac{\partial \Phi(\cdot)}{\partial x} \right|_{(x_{n}, e, d)} \dot{x}_{n},$$  \hspace{1cm} (39)$$

where $x_{\text{op}}$ is a function of $d$ and $e$. The gradual reduction in the number of small-signal quantities from (7) and (25) to (39) is accompanied by an increase in the dependence of the coefficients of the small-signal equations on other nonlinearized quantities. For example, in (7) all coefficients are independent of any varying quantities, whereas in (25) the coefficients are functions of $e$.

Control based on models of (25) and (39) can be regarded as an intermediate step toward a nonlinear control. In fact, we are moving back to the original large-signal equation (1). It can be shown that the kind of control derived from (1) is superior to those based on (7), (25), or (39). Results for the discontinuous-mode case can be found in [9], and the continuous-mode case in [12]. Our purpose here, however, is to show how the technique of perturbation can be applied to a nonlinear system with a reduced number of small-signal quantities, leading to a more accurate model at the expense of varying coefficients.

VI. NUMERICAL EXAMPLE AND SIMULATION RESULTS

A. System Specifications

In this section, we present an example to demonstrate the control scheme discussed in the preceding section. The converter to be designed consists of a basic boost converter regulated by the quasi-linear feedback controller. The converter is to operate in continuous mode, supplying a constant output voltage of 25 V. The switching frequency is 30 kHz, and the power delivered is 50 W. The input voltage has a nominal value of 18 V. For convenience, we choose some simple figures for the normalized natural frequencies, say $RT/L = 10$ and $T/(R + r_c)C = 1/10$. The values of the various circuit components are found in Table I.

B. Evaluating the Matrix Coefficients

In general, matrices $\Phi(\cdot)$ and $\Psi(\cdot)$ can be expressed in terms of the duty ratio and the transition matrices corresponding to the two (three for light-mode) switch states. The transition matrices themselves can be written in the form of an exponential matrix. Thus $\Phi(\cdot)$ and $\Psi(\cdot)$ are given as

$$\Phi(d) = e^{\Phi(1-d)T}e^{AdT}$$  \hspace{1cm} (40)$$

$$\Psi(d) = e^{\Psi(1-d)T}e^{AdT} + e^{AdT}B_0$$  \hspace{1cm} (41)$$

where $A_1$, $B_1$, and $A_2$, $B_2$, are the standard system matrices corresponding to the two switch states respectively. (A linear system is characterized by the equation $\dot{x} = Ax + B_0$.) In evaluating $\Phi(\cdot)$ and $\Psi(\cdot)$, all exponential matrices are approximated as the sum of the first $(N + 1)$ terms of the power series:

$$e^{A\xi} = 1 + \sum_{i=1}^{N} \frac{A^i\xi^N}{N!}$$  \hspace{1cm} (42)$$

where $k = 1, 2$. If the capacitor-load resistance time constant is much longer than the switching period, it can be shown [13] that taking $N = 2$ gives a reasonably good approximation to the involving transition matrices. Results are given in the Appendix.

C. Open-Loop System Dynamics

Dynamical performance of a nonlinear system is often discussed in terms of the eigenvalues of the linearized model, i.e., the roots of the equation det $(\xi - \Phi(D)) = 0$. The eigenvalues essentially serves as an indication of how rapidly the system converges to the steady-state operating point after a small disturbance is introduced. For discrete-time systems in general, the smaller the magni-
TABLE I
VALUES OF CIRCUIT COMPONENTS

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance C</td>
<td>26.67 μF</td>
</tr>
<tr>
<td>Inductance L</td>
<td>41.67 μH</td>
</tr>
<tr>
<td>Capacitor-series-resistance r_c</td>
<td>0.05 Ω</td>
</tr>
<tr>
<td>Load resistance R</td>
<td>12.5 Ω</td>
</tr>
</tbody>
</table>

...values of the eigenvalues are, the faster the system converges to its steady state.

The steady-state operating values for the boost converter is given in the Appendix and is repeated here for convenience.

\[ D = 1 - \frac{E}{U_{\text{ref}}} \]  

\[ X = \frac{U_{\text{ref}}}{RE} \left[ 1 - \frac{E^2RT}{2LU_{\text{ref}}^2} \left( 1 - \frac{E}{U_{\text{ref}}} \right) \right] \]  

The steady-state requirements are \( U_{\text{ref}} = 25 \, \text{V} \), \( E = 18 \, \text{V} \), and \( T = 1/30,000 \, \text{s} \), giving \( D = 0.28 \), \( X_1 = 25 \, \text{V} \) and \( X_2 = 0.76 \, \text{A} \). The values of the elements of \( \Phi \) as given in the Appendix are computed as

\[ \Phi(0.28) = \begin{bmatrix} 0.6458 & 0.8546 \\ -0.5308 & 0.7124 \end{bmatrix} \]  

Thus the characteristic equation is

\[ z^2 = 1.3582z + 0.9137 = 0 \]  

giving a pair of roots at \( 0.6791 \pm 0.6727j \). This represents a rather poor dynamical response. Fig. 4 shows the input and output voltage waveforms of the uncontrolled system.

D. Quasi-Linear Model

Following the procedure described in Section IV-B, the operating point in the reduced variable space is expressed as

\[ d_{qp} = 1 - \frac{e}{U_{\text{ref}}} \]  

\[ x_{qp} = \begin{bmatrix} U_{\text{ref}} \\ U_{\text{ref}}^2 \left( 1 - \frac{e^2RT}{2LU_{\text{ref}}^2} \left( 1 - \frac{e}{U_{\text{ref}}} \right) \right) \end{bmatrix} \]  

The quasi-linear state equation is evaluated as

\[ \dot{x}_{n+1} = \Phi(\cdot)\dot{x}_n + \Gamma(\cdot)\dot{d} \]  

where

\[ \Phi(\cdot) = \begin{bmatrix} 0.9050 & 0.0000 \\ 0.0000 & 1.0000 \end{bmatrix} + \begin{bmatrix} 0.0000 & 1.2500 \\ -0.7200 & -0.4000 \end{bmatrix} \left( \frac{e}{U_{\text{ref}}} \right) + \begin{bmatrix} -0.5000 & -0.0875 \\ -0.0240 & -0.4992 \end{bmatrix} \left( \frac{e}{U_{\text{ref}}} \right)^2 \]  

\[ \Psi(\cdot) = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix} + \begin{bmatrix} 1 \\ -0.032 \end{bmatrix} \left( \frac{e}{U_{\text{ref}}} \right) + \begin{bmatrix} -0.5 \\ 0.016 \end{bmatrix} \left( \frac{e}{U_{\text{ref}}} \right)^2 \]  

and

\[ \Gamma(\cdot) = \Phi'(d_{qp})x_{qp}(e) + \Psi'(d_{qp})e \]  

\[ = -U_{\text{ref}} \left[ \frac{d\Phi(\cdot)}{de} \left( e \right) + \frac{d\Psi(\cdot)}{de} \left( e \right) \right] \]  

It is worth emphasizing that the quantities \( \dot{x} \) and \( \dot{d} \) are "moving" small-signal quantities, representing the differences between the instantaneous values and the expected operating values.

Note that we may also generate the fully linearized model by putting \( e = E \) in the above quasi-linear equation and introducing the \( \dot{e} \) term, i.e.,

\[ \dot{x}_{n+1} = \Phi(E)\dot{x}_n + \Gamma(X, D, E)\dot{d} + \Psi(E)\dot{e} \]  

As a minor observation, the matrices \( \Phi(\cdot) \) and \( \Psi(\cdot) \), which were originally given in Section III-A as functions of \( D \), are now functions of \( E \). This is simply a conse-
E. Computer Simulation Results

A computer program, written in the C language and implemented on a SUN4 machine, is specially developed for simulating switched-mode converter circuits. This program utilizes an exact piecewise switched model to simulate the process. Essentially it contains a collection of linear system descriptions corresponding to all possible circuit configurations and selects the appropriate configuration according to the state of the switch and the value of the inductance current. Waveforms of various quantities are computed continuously for the particular time interval in which the selected configuration is valid. In short, the algorithm involves toggling between configurations as well as numerically solving the linear network equations. The numerical method employed in solving state equations is a variation of the second-order Runge-Kutta algorithm.

Two different controllers can now be designed, one from the linear model (Fig. 2), and the other from the quasi-linear model (Fig. 3). Their algorithms are assimilated into two separate computer subroutines that are then incorporated with the aforementioned computer program to simulate the controlled processes. As mentioned previously, the closed-loop dynamics of the system can be arbitrarily assigned by choosing suitable eigenvalues. In this example, we simply choose \( \lambda_1 = 0.2 \) and \( \lambda_2 = 0.25 \), which guarantee a fairly rapid convergence of the variables to their steady-state values.

To compare the two controllers, a voltage of 18 V superposed by a 1-kHz ac small signal is applied to the input, i.e.,

\[
e(t) = 18 + \frac{1}{2} e_{pp} \sin 2000\pi t
\]

where \( e_{pp} \) is the peak-to-peak voltage fluctuation. For the purpose of comparison, two different degrees of fluctuations are applied to the input: (1) 6 V peak-to-peak; (2) 10 V peak-to-peak. In all cases, no special arrangement is made to deal with the initial startup phase, and the capacitance voltage and the inductance current are initially set at 12 V and 0 A, respectively, to avoid excessive over-shootings to be shown in the simulated waveforms. Thus the comparisons purely reflect the performance of the controllers in respect to their ability to regulate the system subject to a slowly varying disturbance. Figs. 5 and 6 show the input and output voltage waveforms when the system is controlled by the linear and quasi-linear schemes, depicted by Figs. 2 and 3, respectively. Comparing Figs. 5(b) and 6(b) confirms that, as far as output regulation is concerned, the quasi-linear scheme is better than the linear one. However, the difference is only marginal when the input fluctuation is reduced to 6 V, as can be seen by comparing Figs. 5(a) and 6(a). These results are expected since the linear control assumes small deviation of the input voltage about its nominal value whereas the quasi-linear control accommodates to a greater extent the variation of the input voltage.

VII. Conclusion

A quasi-linear approach, which is derived from linearising a large-signal model around a "changing" operating
point, has been proposed for modelling and controlling dc–dc converters. The technique involves taking small variations about an operating point, which is itself a function of a varying input voltage. The control strategy is developed from a linear feedback scheme in which the coefficients are also functions of the varying input voltage. This scheme illustrates the idea of constantly modifying the control equation in accordance with the environmental changes. The results show that a significant improvement in performance over a linear approach can be gained.

**APPENDIX: VALUES OF $\Phi$ AND $\Psi$ FOR THE BOOST CELL**

Matrices $\Phi(d)$ and $\Psi(d)$ can be evaluated by applying finite-series approximation to the transition matrices $e^{aT_{ef}}$ and $e^{aT_{ef}}$ as described in Section VI-B. We shall use $\omega_p$ and $\omega_n$ to denote $1/C(r_e + R)$ and $R/L$, respectively, where $r_e$ is the capacitor-series-resistance, and all other symbols are ad defined in the paper.

\[
d_{op} = 1 - \frac{e}{U_{ref}}
\]

\[
x_{op} = \left[ \frac{U_{ref}}{\frac{U_{ref}^2}{Re} \left( 1 - \frac{e^2}{2U_{ref}^2} \right) \left( 1 - \frac{e}{U_{ref}} \right) } \right]
\]

\[
\phi_1(d_{op}) = 1 - \frac{\omega_p T}{2} + \frac{1}{2} \omega_p^2 T^2 - \frac{1}{2} \left( 1 - d_{op} \right)^2 \omega_p \omega_n T^2
\]

\[
\phi_2(d_{op}) = R \left( 1 - d_{op} \right) \omega_p T - \frac{1}{2} R \left( 1 - d_{op} \right)^2 \omega_p^2 T^2
\]

- \frac{1}{2} r_e \left( 1 - d_{op} \right)^2 \omega_p \omega_n T^2
\]

\[
\phi_3(d_{op}) = -\frac{1}{R} \left( 1 - d_{op} \right) \omega_n T + \frac{1}{2R} \left( 1 - d_{op} \right)^2 \omega_n \omega_p T^2
\]

+ \frac{r_e}{2R^2} \left( 1 - d_{op} \right)^2 \omega_n^2 T^2
\]

\[
\phi_4(d_{op}) = 1 - \frac{r_e}{R} \left( 1 - d_{op} \right) \omega_n T - \frac{1}{2} \left( 1 - d_{op} \right)^2 \omega_p \omega_n T^2
\]

+ \frac{r_e}{2R^2} \left( 1 - d_{op} \right)^2 \omega_n^2 T^2
\]

\[
\Psi(d_{op}) = \left[ \frac{1}{2} \left( 1 - d_{op} \right) \omega_p \omega_n T^2 \right]
\]

\[
\left[ \frac{1}{2} \left( 1 - d_{op} \right) \omega_p \omega_n T^2 \right]
\]

\[
\left[ \frac{1}{2} \left( 1 - d_{op} \right) \omega_p \omega_n T^2 \right]
\]

\[


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